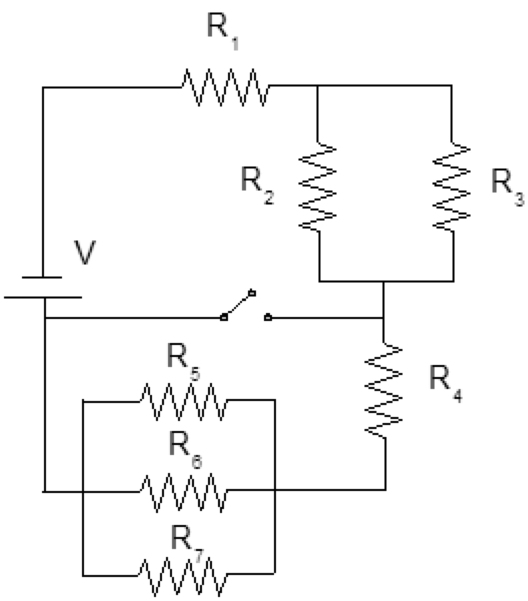
BASIC DC CIRCUIT ANALYSIS

STEP 1 Find the equivalent resistance of the circuit.

To do this we combine the parallel and series resistors as we go around the circuit, adding up all the resistances to get an overall, "effective" resistance for the whole circuit. Since the parallel resistors are more complicated to combine, it is a good idea to do those sections first. Sometimes, the way the circuit is diagrammed can make it difficult to see where the parallel sections actually are.

The process is best illustrated with an example. Consider the circuit below, with the switch open, as shown. R1 is in series with the parallel group R2 and R3, and that in turn is in series with R4, and that in turn is in series with the parallel group {R5 R6 R7}. (Many variations on this are of course possible.)

When resistors are in series, we just add the resistances. When they are in parallel, we use the reciprocal rule. (We assume a single voltage source.)



$$R_{series} = \sum_{i=1}^{n} R_i$$

$$R_{parallel} = \frac{1}{\sum_{j=1}^{m} \frac{1}{R_j}}$$ always works

$$R_{parallel} = \frac{R_1 R_2}{R_1 + R_2}$$ m = 2 only

$$R_{parallel} = \frac{R_{equal}}{m}$$ only if all m are equal

So, for this circuit, we can immediately write, for the total resistance up to the group {R5 R6 R7}:

$$R_{eff} = R_1 + \frac{R_2 R_3}{R_2 + R_3} + R_4$$

To get the total resistance of the last group, we have

$$R_{5,6,7} = \frac{1}{\frac{1}{R_5} + \frac{1}{R_6} + \frac{1}{R_7}}$$

This can of course be expanded out algebraically, but that accomplishes little, and could easily lead to a mistake; we're just as well off to use a calculator to crunch this directly. This can be done either by typing in the expression just as it is written above, using parentheses carefully, or we can be a little more clever and use the x^{-1} key, like this (NOTE THE PARENTHESES!):

$$\left(R_5^{-1} + R_6^{-1} + R_7^{-1}\right)^{-1}$$

where you would use the numerical values for the resistances, of course. Let's put in some values (in ohms) for the resistors and do the calculations.

$R_1 := 30 \qquad R_2 := 100 \qquad R_3 := 75 \qquad R_4 := 200 \qquad R_5 := 300 \qquad R_6 := 50 \qquad R_7 := 150$

$$R_{eff} := 30 + \frac{100\,(75)}{100 + 75} + 200 + \frac{1}{\frac{1}{300} + \frac{1}{50} + \frac{1}{150}} \qquad \boxed{R_{eff} = 306.19} \quad \Omega$$

Check your calculator skills; try this:

$$\left(300^{-1} + 50^{-1} + 150^{-1}\right)^{-1} = 33.333$$

Suppose the switch is closed. Then the effective resistance will be, since resistors R4-7 are no longer part of the circuit (no current will flow through those resistors, since closing the switch provides a zero-resistance path back to the voltage source; also, there will be no potential difference to drive any current through that path):

$$R_{eff} := 30 + \frac{100\,(75)}{100 + 75} \qquad R_{eff} = 72.857 \quad \Omega$$

STEP 2 Find the total current.

Now that we have the effective resistance, if we know the voltage, we can find the current, using Ohm's Law. Thus,

$$I_{total} = \frac{V}{R_{eff}}$$

In the example, suppose the voltage source is a 9V battery. Then the current with the switch open is

$$I_{total} := \frac{9\ \mathrm{V}}{306.2\ \mathrm{ohm}} \qquad \boxed{I_{total} = 0.029\,\mathrm{A}}$$

With the switch closed the current is

$$I_{total} := \frac{9\ \mathrm{V}}{72.9\ \mathrm{ohm}} \qquad I_{total} = 0.123\,\mathrm{A}$$

STEP 3 Find the total power.

We know that the power in the circuit is just the product of the voltage and current, so that

$$P := I_{total}\, V \qquad P := 0.029\ \text{A}\ (9\ \text{V}) \qquad P = 0.261\,\text{W} \qquad \text{open}$$

$$P := 0.123\ \text{A}\ (9\ \text{V}) \qquad P = 1.107\,\text{W} \qquad \text{closed}$$

Of course we could find the power without finding the current, if need be:

$$P := \frac{V^2}{R_{eff}} \qquad P := \frac{(9\ \text{V})^2}{306.2\ \text{ohm}} \qquad P = 0.265\,\text{W} \qquad \text{open}$$

$$P := \frac{(9\ \text{V})^2}{72.9\ \text{ohm}} \qquad P = 1.111\,\text{W} \qquad \text{closed}$$

The results differ slightly, due to roundoff.

STEP 4 Find the voltage drops (potential differences) around the circuit.

We also know that the sum of the voltage drops around the circuit must equal the source voltage. This can provide a check of the calculations we've already done.

The voltage drop across R1 will be (all this is with the switch open):

$$\Delta V_1 := I_{total}\, R_1 \qquad \Delta V_1 := 0.029\ \text{A}\ (30\ \text{ohm}) \qquad \Delta V_1 = 0.87\,\text{V}$$

An easy way to do the rest of the circuit is to treat each parallel group as a single resistor, at the combined resistance of the group. This makes the entire circuit just a set of resistors in series, the current through which is the same for each. So, for R2 and R3 we have

$$R_{2,3} = \frac{100\ (75)}{100 + 75} \qquad R_{2,3} = 42.857 \qquad \Omega$$

Then the voltage drop across here will be (note that it's the same drop for both R2 and R3)

$$\Delta V_{2,3} = I_{total}\, R_{2,3} \qquad \Delta V_{2,3} = 0.029\ \text{A}\ (42.857\ \text{ohm}) \qquad \Delta V_{2,3} = 1.243\ \text{V}$$

For R4 we have

$$\Delta V_4 = I_{total}\, R_4 \qquad \Delta V_4 = 0.029\ \text{A}\ (200\ \text{ohm}) \qquad \Delta V_4 = 5.8\ \text{V}$$

For {R5 R6 R7} we already found the combined resistance, so we can write

$$\Delta V_{5,6,7} = I_{total}\, R_{5,6,7} \qquad \Delta V_{5,6,7} = 0.029\ \text{A}\ (33.33\ \text{ohm}) \qquad \Delta V_{5,6,7} = 0.967\ \text{V}$$

The sum of these voltage drops is 8.88 volts, which is essentially the supply voltage of 9 V, considering the various rounding done in the calculations (carrying more decimals does yield 9 V).

STEP 5 Find the current flows around the circuit.

The current flow through series resistors is just the total current in the circuit. So for resistors R1 and R4, we have 0.029 A through them. For parallel resistors we can find the current in each resistor according to the voltage drop ΔV across the parallel group, using

$$I_j = \frac{\Delta V_{parallel}}{R_j} = I_{total} \frac{R_{parallel}}{R_j}$$

In other words, the current in the j-th parallel resistor will be the total current in the circuit times the ratio of the combined-group resistance to that j-th resistance. Then for R2 and R3 we will have

$$I_2 = 0.029 \frac{\left[\frac{100\ (75)}{100 + 75}\right]}{100} = 0.012 \text{ A} \qquad I_3 = 0.029 \frac{\left[\frac{100\ (75)}{100 + 75}\right]}{75} = 0.017 \text{ A}$$

and the sum of these is of course the total current 0.029 A. For the {R5 R6 R7} group we find that

$$I_5 = 0.029 \frac{33.33}{300} = 0.0032 \text{ A} \qquad I_6 = 0.029 \frac{33.33}{50} = 0.019 \text{ A} \qquad I_7 = 0.029 \frac{33.33}{150} = 0.0064 \text{ A}$$

STEP 6 Find the power dissipation in the resistors around the circuit.

Power dissipation is an important consideration in circuit design. Resistors are designed to be able to withstand a certain amount of heat; too much heat and the resistor will fail, usually catastrophically. (This happened to Mr. Evans once, when playing guitar long ago in a club on M Street in Georgetown, when a resistor in the amp he was using literally turned to ash during his solo in *Day Tripper*.) Resistors are rated by wattage; for most electronic circuits, 0.25 watt is a typical rating.

The power in the k-th resistor is just the product of the current and voltage across that resistor:

$$P_k = I_k \Delta V_k$$

so for the example circuit we will find

P_1	$0.029 \text{ A } (0.87 \text{ V}) = 0.025 \text{W}$	P_5	$0.0032 \text{ A } (0.967 \text{ V}) = 3.094 \times 10^{-3} \text{W}$
P_2	$0.012 \text{ A } (1.243 \text{ V}) = 0.015 \text{W}$	P_6	$0.019 \text{ A } (0.967 \text{ V}) = 0.018 \text{W}$
P_3	$0.017 \text{ A } (1.243 \text{ V}) = 0.021 \text{W}$	P_7	$0.0064 \text{ A } (0.967 \text{ V}) = 6.189 \times 10^{-3} \text{W}$
P_4	$0.029 \text{ A } (5.8 \text{ V}) = 0.168 \text{W}$		

The sum of these is, within roundoff, the same as the total power we found above. Since no resistor dissipates more than 0.25 watt, this circuit could be safely built with 0.25 watt resistors.

$$R_{series} = \sum_{i=1}^{n} R_i$$

RESISTANCE OF N RESISTORS IN SERIES

RESISTANCE OF M RESISTORS IN A PARALLEL GROUP

$$R_{parallel} = \frac{1}{\sum_{j=1}^{m} \frac{1}{R_j}}$$

always works; use x^{-1} key on calculator to do this directly

$$R_{parallel} = \frac{R_1 R_2}{R_1 + R_2}$$

m = 2 only

$$R_{parallel} = \frac{R_{equal}}{m}$$

only if all m are equal

EFFECTIVE RESISTANCE OF CIRCUIT

$$R_{eff} = R_{series} + \sum_{groups} R_{parallel}$$

sum over as many parallel groups as there are in the circuit

$$I_{total} = \frac{V}{R_{eff}}$$

TOTAL CURRENT IN CIRCUIT

$$P = I_{total} V \qquad P = \frac{V^2}{R_{eff}}$$

TOTAL POWER IN CIRCUIT

$$\Delta V = I_{total} R$$

VOLTAGE DROP (POTENTIAL DIFFERENCE) ACROSS RESISTOR(S)

(R is *effective* resistance of a parallel *group,* when appropriate; the voltage drop is the same for all resistors in that group)

$$I_j = I_{total} \frac{R_{parallel}}{R_j}$$

CURRENT FLOW THROUGH J-TH RESISTOR IN PARALLEL GROUP

$$I_i = I_{total}$$

CURRENT FLOW THROUGH I-TH RESISTOR IN SERIES

$$P_j = \frac{\left(I_{total} R_{parallel}\right)^2}{R_j}$$

POWER DISSIPATED IN J-TH RESISTOR IN A PARALLEL GROUP

$$P_i = \left(I_{total}\right)^2 R_i$$

POWER DISSIPATED IN I-TH RESISTOR IN SERIES