Tecnológico de Monterrey Preparatoria

Activity 5.2: Change of variable 2 – Double Substitution 100

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Solve the integrals

1) $\int 2x\sqrt{x+2}\,dx$ $\frac{4\sqrt{(x+2)^5}}{5} - \frac{8\sqrt{(x+2)^3}}{3} + C$

2) $\int \frac{8e^{2x}}{5-3e^{2x}}dx$ $-\frac{4}{3}\ln|5-3e^{2x}| + C$

3) $\int \frac{6Ln\sqrt{x}}{x}dx$ $6(\ln\sqrt{x})^2 + C$

4) $\int 15x^2(3x+2)^5\,dx$ $\frac{5}{72}(3x+2)^8 - \frac{20}{63}(3x+2)^7 + \frac{10}{27}(3x+2)^6 + C$

5) $\int \frac{x^2}{(5-3x)^4}dx$ $\frac{25-45x+27x^2}{81(5-3x)^3} + C$

6) $\int \frac{12x^2}{(4-x^3)^5}dx$ $\frac{1}{(4-x^3)^4} + C$

7) $\int \frac{4x}{1-2x}dx$ $1-2x - \ln|1-2x| + C$

8) $\int 8x^3(2-x^2)^9\,dx$ $-\frac{4}{55}(2-x^2)^{10}(1+5x^2) + C$

9) $\int_{-4}^{-2} \frac{x}{(2-5x)^3}dx$ $-\frac{23}{17424}$

10) $\int (2x+1)(2-x)^5\,dx$ $-\frac{5(2-x)^6}{6} + \frac{2(2x-7)^7}{7} + C$

11) $\int 6x^2\cdot\sqrt[3]{7+3x}\,dx$ $\frac{(7+3x)^{10/3}}{15} - \frac{4(7+3x)^{7/3}}{3} + \frac{49(7+3x)^{4/3}}{6}$

12) $\int_{-2}^{2} 3x\sqrt{2x+5}\,dx$ $\frac{38}{5}$

13) $\int_0^2 \frac{2x\,dx}{(3x+4)^3}$ $\frac{1}{100}$

14) $\int_1^2 (x-1)\sqrt{2-x}\,dx$ $\frac{4}{15}$

15) The acceleration of an object is given by $a(t)=12t\sqrt{2t+1}$ in m²/sec. Find the equation of velocity in m/sec if the initial velocity of the object (t = 0) is 20 m/sec $v(t) = \frac{6}{5}(2t+1)^{5/2} - 2(2t+1)^{3/2} + 20.8$

16) The equation of acceleration of an object is given by $a(t)=\frac{40t}{(1+2t)^3}$ in ft/min². Determine the equation of velocity if we know that after 5 min the velocity is 15 ft/min? $v(t) = -\frac{10}{1+2t} + \frac{5}{(1+2t)^2} + \frac{1920}{121}$

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ACTIVITY 5.2 - CHANGE OF VARIABLE 2

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1. $\int 2x\sqrt{x+2}\,dx = 2(u-2)u^{1/2}\,du = \int 2u^{1/2} - 4u^{1/2} = \int -2u^{1/2} = \frac{-2u^{3/2}}{3/2} = \frac{-4(x+2)^{3/2}}{3} + C$

$u = x+2 \quad x = u-2$

$du = 1dx \quad dx = du$

2. $\int \frac{8e^{2x}}{5-3e^{2x}}\,dx = \int -\frac{8}{6}\ln|5-3e^{2x}| = -\frac{4}{3}\ln|5-3e^{2x}| + C$

$u = 5-3e^{2x}$

$du = -6e^{2x}$

3. $\int \frac{6\ln\sqrt{x}}{x}\,dx = \int 6\ln\sqrt{x} = 6(\ln\sqrt{x})^2 + C$

$u = x$

$du = 1$

4. $\int 15x^2(3x+2)^5\,dx = \frac{5}{3}\int \left(\frac{u-2}{3}\right)^2 u^5\,du = \frac{5}{3}\int \frac{u^2-4u+4}{6}u^5\,du = \frac{5}{3}\int \frac{u^7-4u^6+4}{6}\,du =$

$u = 3x+2 \quad x = \frac{u-2}{3} \quad dx = \frac{du}{3} \quad \frac{5}{3}\left(\frac{u^8}{6(8)} - \frac{4u^7}{6(7)} + \frac{4u}{6}\right) = \frac{5}{3}\left[\frac{u^8}{48} - \frac{2u^7}{21} + \frac{2u}{3}\right] + C$

$du = 3dx$

5. $\int \frac{x^2}{(5-3x)^4}\,dx = \left(\frac{5-u}{3}\right)^2 u^{-4}\,\frac{-du}{3} = \frac{(5-u)^2}{9}(5-3x)^4 = \frac{-1}{3}\int \frac{(5-(5+3x))^2}{9}(u)^{-4} = \frac{-1}{3}\int \frac{(5-3x)^2}{9}\cdot\frac{1}{(u)^3}$

$= \frac{25-45x+27x^2}{81(5-3x)^3} + C$

$u = 5-3x \quad x = \frac{5-u}{3} \quad dx = -\frac{du}{3}$

$du = -3dx$

6. $\int \frac{12x^2}{(4-x^3)^5}\,dx = -\frac{1}{4}\,\frac{(4-x^3)^{-4}}{-4} = \frac{1}{16(4-x^3)^4} + C$

$u = 4-x^3$

$du = -3x^2$

7. $\int \frac{4x}{1-2x}\,dx = 4\left(\frac{1-u}{2}\right)u^{-1}\,\frac{du}{-2} = -\frac{1}{2}\int (2-2u)u^{-1}\,du = \frac{-1}{2}\int 2u^{-1} - 2\,du = \frac{-1}{2}[2(\ln|u| - 2u] + C$

$= \frac{1}{2}[2(\ln|1-2x| - 2(1-2x)] + C$

$u = 1-2x \quad x = \frac{1-u}{2} \quad dx = \frac{du}{-2}$

$du = -2$

8. $\int 8x^3(2-x^2)^9\,dx = 8(\sqrt{2-u})^3 u^9\,\frac{du}{-2} = -\frac{1}{2}\int 8(2-u)^{3/2}u^9\,du = -\frac{4}{55}(2-x^2)^{10}(1+5x^2)+C$

$u = 2-x^2 \quad x = \sqrt{2-u}$

$du = -2x \quad dx = du/-2$

9. $\int_4^3 \frac{x}{(2-5x)^5}\,dx = \left[-\frac{1}{5}(2-5x)^{-4}/4\right] = \frac{1}{20(2-5x)^4} + C = \frac{1}{419,720} - \frac{1}{4,685,120} = \frac{23}{192424}$

$u = 2-5x$

$du = -5$

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10. $\int (2x+1)(2-x)^5\,dx = (2(2-u)+1)(u)^5(-du) = -1\int (5-2u)u^5 = \frac{-5(2-x)^6}{6} + \frac{2(2-x)^7}{7} + C$

$u = 2-x \quad x = -u+2$

$du = -1 \quad dx = -du$

11. $\int 6x^2 \cdot \sqrt[3]{7+3x}\,dx = 6(7+3x)^2 \cdot u^{3/2}\,\frac{du}{3} = \frac{(7+3x)^{10/3}}{15} - \frac{4(7+3x)^{7/3}}{3} + \frac{49(7+3x)^{4/3}}{6} + C$

$u = 7+3x \quad x = \frac{u-7}{3} \quad dx = \frac{du}{3}$

$du = 3$

12. $\int_{-2}^{2} 3x\sqrt{2x+5}\,dx = 3\left(\frac{u-5}{2}\right)u^{1/2}\,\frac{du}{2} = \frac{1}{2}\int \frac{3u}{2} - \frac{15}{2}u^{1/2} = \frac{38}{5}$

$u = 2x+5 \quad x = \frac{u-5}{2} \quad dx = \frac{du}{2}$

$du = 2$

13. $\int_{0}^{2} \frac{2x\,dx}{(3x+4)^3}\,dx = 2\left(\frac{u-4}{3}\right)u^{-3}\,\frac{du}{3} = \frac{1}{3}\int \frac{2u}{3} - \frac{8}{3}u^{-3} = \frac{1}{100}$

$u = 3x+4 \quad x = \frac{u-4}{3} \quad dx = \frac{du}{3}$

$du = 3$

14. $\int_{1}^{2} (x-1)\sqrt{2-x}\,dx = (-u+2-1)(u)^{1/2}(-du) = -1\int (-u+1)u^{1/2} = -1\int -u^{1/2} + u^{1/2} = \frac{4}{15}$

$u = 2-x \quad x = -u+2$

$du = -1 \quad dx = -du$

15. $\int 12t\sqrt{2t+1}\,dx = \int 6(2t+1)^{3/2}/3/2 = \frac{12(2t+1)^{3/2}}{3} = v(t) = \frac{6}{5}(2t+1)^{5/2} - 2(2t+1)^{3/2} + 20.8$

$u = 2t+1$

$du = 2$

16. $\int \frac{40t}{(1+2t)^3}\,dt = 40\left(\frac{u-1}{2}\right)(1+2t)^{-3}\,\frac{du}{2} = \frac{1}{2}\int (20u-20)u^{-3} = \frac{1}{2}\int 20u^{-2} - 20u^{-3} = \frac{1}{2}\int \frac{20}{(u)^{-1}} - \frac{20}{u^{-2}}$

$= v(t) = \frac{-10}{1+2t} + \frac{5}{(1+2t)^2} + \frac{1970}{121}$

$u = 1+2t \quad t = \frac{u-1}{2} \quad dt = \frac{du}{2}$

$du = 2$