Arithmetic operations

Jesús Omar Gómez Monteagudo
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Simplification

Guidelines to simplify *a/b*

- Find ***d*** **= gcf(a, b).**
- Divide both, ***a*** and ***b*** by ***d***.
- The simplified equivalent fraction is the ratio of the quotients obtained in the last step.

Example: Simplify the fraction ***30/54*** to its simplest form.

Solution: According to the guidelines:

- Find **gcf:**
  - **gcf(*30, 54*) = 6**.
- Divide numerator and denominator by **gcf:**
  - **30 = 6(5).**
  - **54 = 6(9).**
- **Simplified equivalent fraction: 30/54 = 5/9.**

Additon and subtraction between fractions

How to add / subtract fractions

- Find ***m* = lcm(a, b).**
- Produce equivalent fractions to each original term, having ***m*** as denominator.
- Add / subtract the new numerators.
- Simplify the resulting fraction (if possible).

Example: Simplify 4/15 - 4/5 + 1/2.

Solution: According to the guidelines:

- Find **lcm: lcm(*15, 5, 2*) = 60**.
- Produce equivalent fractions...:
  - **4/15 = 16/60.**
  - **- 4/5 = - 48/60.**
  - **1/2 = 30/60.**
- Add / subtract new numerators:
  - **16 - 48 + 30 = -2**
- **Simplified equivalent fraction:**
  - **4/15 - 4/5 + 1/2 = -2/30 = -1/15.**

Product and quotient between fractions

Product of fractions

- Given two fractions *a/b* and *c/d*, we define their ***product*** to be the fraction:

Factors → ***(a/b)(c/d) = (ab)/(cd).*** ← Product

- It is useful to express a part of something that was already divided.

Quotient of fractions

- Given two fractions *a/b* and *c/d*, where *c/d* **is not zero,** we define their ***product*** to be the fraction:

Factors → ***(a/b)/(c/d) = (ad)/(bc).*** ← Quotient

- It is useful to express a shares of something that was already divided.

Product and quotient between fractions

Example: Multiply *12/5* by *15/21* and simplify the product.

Solution: Accordng to the definition:

- *(12/5)(15/21)=(12 X 15)/(5 X 21) = 180/105.*
- *Now we simplify:*
  - *180/105 = (**15** X 12) / (**15** X 7) = 12/7.*

Conclusion: *(12/5)(15/21)=**12/7**.*

Comments: When performing multiplication to obtain the product, it is better to factor out each factor to simplify first **like factors**:

- Numerators:
  - *12 = 2^2X3.*
  - *15 = 3X5.*
- *Denominators:*
  - *5 = 5.*
  - *21 = 3X7.*
- *Product:*
  - *(2^2 X 3 X **3 X 5**)/(**5 x 3** X 7)=2^2X3/7=**12/7**.*

Product and quotient between fractions

Example: Divide *16/15* by *12/35* and simplify the quotient.

Solution: Accordng to the definition:

- *(16/15)/(12/35)=(16X35)/(15X12)=560/180.*
- *Now we simplify:*
  - *560/180 = (**20** X 28) / (**20** X 9) = 29/8.*

Conclusion: *(16/15)/(12/35)=**28/9**.*

Comments: As for the product, when performing division to obtain the quotient, it is better to factor out each factor to simplify first **like factors**:

- Numerators:
  - *16 = 2^4 = 2^2 X 2^2.*
  - *35 = 7X5.*
- *Denominators:*
  - *15 = 3X5.*
  - *12 = 2^2X3.*
- *Quotient:*
  - *(2^2X**2^2**X7X**5**)/(3X**5**X**2^2**X3)=2^2X7/3^2=**28/9**.*

Quotient: Intercalation (sandwich)

$$\frac{\frac{a}{b}}{\frac{c}{d}} = \frac{ad}{bc} \qquad \frac{a}{\frac{c}{d}} = \frac{\frac{a}{1}}{\frac{c}{d}} = \frac{ad}{c} \qquad \frac{\frac{a}{b}}{c} = \frac{\frac{a}{b}}{\frac{c}{1}} = \frac{a}{bc}$$