Sección 3,2.

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$$\boxed{\begin{matrix} y(1)=6 & y'(1)=14 & y''(1)=22 \\ y_1=x & y_2=x^2 & y_3=x^3 \end{matrix}}$$

$$x^3y''' - 3x^2y'' + 6xy' - 6y = 0$$

$$y(x) = C_1y_1 + C_2y_2 + C_3y_3$$

$$y(x) = C_1x + C_2x^2 + C_3x^3$$

$$y(1) = C_1(1) + C_2(1)^2 + C_3(1)^3 \rightarrow 6 = C_1 + C_2 + C_3$$

$$y'(x) = C_1 + 2C_2x + 3C_3x^2$$

$$y'(1) = C_1 + 2C_2(1) + 3C_3(1)^2 \rightarrow 14 = C_1 + 2C_2 + 3C_3$$

$$y''(x) = 2C_2 + 6C_3x$$

$$y''(1) = 2C_2 + 6C_3(1) \rightarrow 22 = 2C_2 + 6C_3$$

$$\left|\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 14 \\ 0 & 2 & 6 & 22 \end{array}\right| \quad -1(f_1) + f_2 \rightarrow f_2 \quad \left|\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 8 \\ 0 & 2 & 6 & 22 \end{array}\right|$$

$-1f_2 + f_1 \rightarrow f_1$

$-2f_2 + f_3 \rightarrow f_3$

$$\left|\begin{array}{ccc|c} 1 & 0 & -1 & -2 \\ 0 & 1 & 2 & 8 \\ 0 & 0 & 2 & 6 \end{array}\right|$$

$-1f_3 + f_2 \rightarrow f_2$

$\frac{1}{2}f_3 \rightarrow f_3$

$1f_3 + f_1 \rightarrow f_1$

$$\left|\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{array}\right|$$

$c_1 = 1$

$c_2 = 2$

$c_3 = 3$

$y(x) = x + 2x^2 + 3x^3$

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$y'' - 2y' - 3y = 6$ $\qquad y(0) = 3 \quad y'(0) = 11$

$y_c = c_1e^{-x} + c_2e^{3x}$

$y(x) = y_c + y_p$

$y_p = -2$

$y(x) = c_1e^{x} + c_2e^{3x} - 2$

$y(0) = c_1\cancel{e^{-0}} + c_2\cancel{e^{3(0)}} - 2$

$3 = c_1 + c_2 - 2 \longrightarrow 5 = c_1 + c_2$

$y'(x) = -c_1e^{-x} + 3c_2e^{3x}$

$y'(0) = -c_1\cancel{e^{-0}} + 3c_2\cancel{e^{3(0)}}$

$11 = -c_1 + 3c_2 \longrightarrow c_1 = 3c_2 - 11$

$5 = (3c_2 - 11) + c_2 \longrightarrow 5 + 11 = 3c_2 + c_2 \longrightarrow$

$16 = 4c_2 \longrightarrow \frac{16}{4} = c_2 \longrightarrow 4 = c_2$

$5 = c_1 + (4)$

$1 = c_1$

$y(x) = e^{-x} + 4e^{3x} - 2$

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$$y'' + py' + qy = 0$$

b) $y'' - 2y' - 5y = 0$

$y(0) = 1$
$y'(0) = 0$
$y''(0) = C$

$r^2 - 2r - 5 = 0$

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$r_1 = 3,45$
$r_2 = -1,45$

$$y(x) = C_1 e^{3,45x} + C_2 e^{-1,45x}$$

$$y(0) = C_1 \cancel{e^{3,45(0)}} + C_2 \cancel{e^{-1,45(0)}}$$

$$1 = C_1 + C_2$$

$$y'(x) = 3,45 C_1 e^{3,45x} - 1,45 C_2 e^{-1,45x}$$

$$y'(0) = 3,45 C_1 \cancel{e^{3,45(0)}} - 1,45 C_2 \cancel{e^{-1,45(0)}}$$

$$0 = 3,45 C_1 - 1,45 C_2$$

$$y''(x) = 11,902 C_1 e^{3,45x} + 2,102 C_2 e^{-1,45x}$$

$$y''(0) = 11,902 C_1 \cancel{e^{3,45(0)}} + 2,102 C_2 \cancel{e^{-1,45(0)}}$$

$$C = 11,902 C_1 + 2,102 C_2$$

$$11,902 C_1 + 2,102 C_2 - C = 0$$

$$\left|\begin{array}{cc|c} 1 & 1 & 1 \\ 3{,}45 & -1{,}45 & 0 \\ 11{,}902 & 2{,}102 & c \end{array}\right| \quad \begin{array}{l} (-3{,}45)f_1 + f_2 \to f_2 \\ (-11{,}9)f_1 + f_3 \to f_3 \end{array}$$

$$\left|\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -4{,}9 & -3{,}45 \\ 0 & -9{,}8 & c-11{,}9 \end{array}\right| \quad \begin{array}{l} -2f_2 + f_3 \to f_3 \\ -\frac{10}{49} f_2 \to f_2 \\ -1f_2 + f_1 \to f_1 \end{array}$$

$$\left|\begin{array}{cc|c} 1 & 0 & 0{,}3 \\ 0 & 1 & 0{,}7 \\ 0 & 0 & c-5 \end{array}\right| \quad \begin{array}{l} C_1 = 0{,}3 \\ C_2 = 0{,}7 \end{array} \quad C = 5$$

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$$w = \begin{vmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{vmatrix}$$

$f_1 = f_2$ $\qquad$ $f_2 = f_3$

$$\frac{dw}{dx} = \underbrace{\begin{bmatrix} y_1' & y_2' & y_3' \\ y_1' & y_2' & y_3' \\ y_1'' & y_2'' & y_3'' \end{bmatrix}}_{\text{Determinante} = 0} + \underbrace{\begin{bmatrix} y_1 & y_2 & y_3 \\ y_1'' & y_2'' & y_3'' \\ y_1'' & y_2'' & y_3'' \end{bmatrix}}_{\text{Determinante} = 0} + \begin{bmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1''' & y_2''' & y_3''' \end{bmatrix}$$

$$\frac{dw}{dx} = \begin{bmatrix} y_1 & y_2 & y_3 \\ y_1' & y_2' & y_3' \\ y_1''' & y_2''' & y_3''' \end{bmatrix}$$

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$$y_2(x) = v(x)\,y_1(x)$$

$$y_2'(x) = v'(x)\,y_1(x) + v(x)\,y_1'(x)$$

$$y_2''(x) = v''(x)y_1(x) + v'(x)y_1'(x) + v'(x)y_1'(x) + v(x)\,y_1''(x)$$

$$= v''(x)\,y_1(x) + 2v'(x)\,y_1'(x) + v(x)\,y_1''(x)$$

Entonces $y'' + py' + qy = 0$

- $(v''(x)y_1(x) + 2v'(x)y_1'(x) + v(x)y_1''(x)) + q\,(v(x)\,y_1(x))$
$+ p\,(v'(x)y_1(x) + v(x)\,y_1'(x)) = 0$

- $v(x)\underbrace{(y_1''(x) + py_1'(x) + qy_1(x))}_{0} + v'(x)\,(2y_1'(x) + py_1(x))$

$+ v''(x)(y_1(x)) = 0$

$$v''(x)\,(y_1(x)) + v'(x)\,(2y_1'(x) + py_1(x)) = 0$$

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$$\frac{(1-x^2)}{(1-x^2)} y'' + \frac{2xy'}{(1-x^2)} - \frac{2y}{(1-x^2)} = 0 \qquad (-1 < x < 1)$$

$$y'' + \frac{2x}{1-x^2} y' - \frac{2}{1-x^2} y = 0$$

$y_1(x) = x$

$y_2(x) = x\,v(x)$

$y_2'(x) = v(x) + xv'$

$y_2''(x) = v' + v' + xv''$
$= 2v' + xv''$

$$(2v' + xv'') + \frac{2x}{1-x^2}(v + xv') - \frac{2}{1-x^2}(xv) = 0$$

$$2v' + xv'' + \cancel{\frac{2xv}{1-x^2}} + \frac{2x^2v'}{1-x^2} - \cancel{\frac{2xv}{1-x^2}} = 0$$

$$2u + xu' + \frac{2x^2}{1-x^2} u = 0$$

$$xu' + u\left(2 + \frac{2x^2}{1-x^2}\right) = 0$$

$$x\frac{du}{dx} = -u\left(2 + \frac{2x^2}{1-x^2}\right)$$

$$\int \frac{du}{u} = \int -\frac{2}{x} - \frac{2x}{1-x^2}\, dx$$

$$e^{\ln|u|} = e^{-2\ln x + \ln|1-x^2|}$$

$$u = x^{-2}(1-x^2)$$

$$u = (x^{-2} - 1)$$

$$\int \cancel{dx} \frac{dv}{\cancel{dx}} = \int x^{-2} - 1\, dx$$

$$v = \frac{x^{-1}}{-1} - x$$

$$y_2(x) = x(x^{-1} + x)$$

$y_2(x) = 1 + x^2$