

Secondary Education Examination
Model Question - 2078
Grade: 12

Time: 3 hrs **Mathematics (Mat. 402)** **F.M.:** 75

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Attempt **All** Questions.

Group 'A'

[11 × 1 = 11]

Rewrite the correct option in your answer sheet.

1. If ω is a complex cube root of unity, then the value of $(1 + \omega - \omega^2)(1 - \omega + \omega^2)$ is
A. ω B. ω^2 C. $1 + \omega$ D. 4
2. An equation $(m + 2)x^2 - 2(m + 4)x + (m + 7) = 0$ have equal roots. The value of m is
A. 2 B. -2 C. 7 D. -4
3. Solution of $\sin\left(2\sin^{-1}\frac{4}{5}\right)$ is
A. $\frac{4}{5}$ B. $\frac{24}{25}$ C. $\frac{25}{24}$ D. 1
4. If $\cos mx = \cos nx$, then the value of x is
A. $2n\pi$
B. $2n\pi \pm \frac{\pi}{3}$
C. $(4k - 1)\frac{\pi}{2(m \pm n)}, k = 0, \pm 1, \pm 2, \dots$
D. $\frac{2k\pi}{m \pm n}, k = 0, \pm 1, \pm 2, \dots$
5. The area of a parallelogram whose diagonals are the vectors $\vec{i} - 2\vec{k}$ and $4\vec{i} + 3\vec{j} + \vec{k}$ is
A. $5\sqrt{14}$ sq. units B. $\frac{3}{2}$ sq. units C. $\frac{3}{2}\sqrt{14}$ sq. units
D. $\sqrt{14}$ sq. units

6. The equation of a hyperbola in standard position satisfying transverse and conjugate axes are respectively 4 and 5 is
A. $\frac{x^2}{4} - \frac{4y^2}{25} = 1$ B. $4x^2 - 7y^2 = 36$
C. $4x^2 + 7y^2 = 36$ D. $\frac{x^2}{4} - \frac{y^2}{5} = 1$
7. Four unbiased coins are tossed successively. The mean and variance of the distribution differed by
A. 1 B. 2 C. 3 D. 4
8. The points on the curve $x^2 + y^2 - 2x - 3 = 0$ where the tangents are parallel to the X -axis are
A. $(1, 2), (1, -2)$ B. $(1, 2), (1, 2)$
C. $(-1, 2), (1, -2)$ D. $(1, 2), (1, 3)$
9. The order and degree of the differential equation $\left(\frac{dy}{dx}\right)^3 + 2y\left(\frac{d^2y}{dx^2}\right) = 0$ is
A. 2, 1 B. 1, 2 C. 1, 3 D. 3, 1
10. When Gauss forward elimination method is used for solving the equations

$$3x + 4y = 18 \dots (i)$$

$$3y - x = 7 \dots (ii)$$

we apply the operation

- A. $eq^n(i) + 4eq^n(ii)$ B. $eq^n(i) + 3eq^n(ii)$ C. $eq^n(i) + eq^n(ii)$
D. $eq^n(ii) + 3eq^n(i)$

11. If the resultant of two like parallel forces acting at a distance of 3 m is 80 N at a distance of 75 cm from one of the forces, then the force is
A. 20 N B. 9.8 N C. 60 N D. 40 N

OR

- A. 9 or 10 B. 1 or 10 C. 1 or 9 D. 4 or 5

Group 'B'

[8 × 5 = 40]

12. (a) If the numerical coefficients in the second, third and fourth terms of the expansion of $(x+a)^n$ are 30, 375 and 2500 respectively, find the value of n . Let a, b, c and x be elements of a group G . [3]

(b) Solve for x : $x^2 = a^2$ and $x^5 = e$. [2]

13. (a) If $Z = \cos \theta + i \sin \theta$, find the value of $z^n + \frac{1}{z^n}$ by using De Moivre's Theorem. [2]

(b) Solve the system of equations by the row - equivalent method:
 $x + y + z = 6, x - y + z = 2$ and $x + y - z = 0$. [3]

14. (a) If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, then show that: $x + y + z = xyz$. [3]

(b) Find the eccentricity and the foci of the ellipse $\frac{x^2}{9} + \frac{y^2}{16} = 1$. [2]

15. From the following data

Age in years (X)	5	15	30	45	50	60
Weight in kg (Y)	10	35	50	65	55	45

compute the

(a) correlation coefficient by Karl Pearson's method. [2]

(b) line of regression for estimating X on Y and estimate the most probable age of the weight 37 kg. [2]

16. Evaluate:

(a) $\int \frac{dx}{3 - 2x - x^2}$ [2]

(b) $\int \frac{x^2}{(x^2 + 9)(x^2 + 4)} dx$ [3]

17. Solve $\frac{dy}{dx} + \frac{\cos x \sin y}{\cos y} = 0$. An equation reacting to the stability of an aeroplane is $\frac{dy}{dt} = g \cos \alpha - kv$, where v is the velocity and g, α, k are constants. Find the expression for velocity, if $v = 0$, when $t = 0$. [5]

18. Maximize $P = 25x + 45y$ subject to $x + 3y \leq 21, 2x + 3y \leq 24, x, y \geq 0$ by using simplex method. [5]

19. (a) Two unlike parallel forces, the greater of which is 75N, have a resultant 25N. Find the ratio of the distances of the resultant from the component forces. [2]

- (b) A projectile thrown from a point in a horizontal plane comes back to the plane in 4 sec. at a distance of 60 m in front of the point of projection. Find the velocity of projection. ($g = 10m/s^2$). [3]

OR

State the Hawkins-Simon conditions for the viability of the system. The demand and supply curves for an item are given by $P_d = 20 - 3Q - Q^2$ and $P_s = Q - 1$ respectively. Find the difference between consumer and producer surplus at the equilibrium price. [1 + 4]

Group 'C' [3 × 8 = 24]

20. (a) In how many ways can the letters of the word "CALCULUS" be arranged so that the two L's do not come together? [3]

(b) Sum to n terms of the series $1^2 + 3^2 + 5^2 + \dots$ [3]

(c) The sum of the roots of a quadratic equation is 4 and the sum of their squares is 14. Find the equation. [2]

21. (a) Find the angle between the lines whose direction cosines are given by $l + m + n = 0$ and $2lm + 2ln - mn = 0$. [5]

(b) Prove by the vector method: $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ [3]

22. (a) Find the derivative of $\ln \sin x$ by using first principle. [4]

(b) State the mean value theorem. Use it to verify for the function $f(x) = \sqrt{x^2 - 4}$ in $[2, 4]$. [1+3]

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