Homework: Linear Transformations from Geometry, Part II

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Similarly to Part I of this homework, we are finding a single matrix that combines two transformations (in a particular order). First, it is to reflect the vector over the line $y = 3x$; then, it is to reflect the resulting vector over the x-axis.

For the first reflection, we can use the general form of a reflection matrix (to reflect a vector over a line of slope m) from Part I:

$$M_{ref} = \frac{1}{1+m^2}\begin{pmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{pmatrix} \qquad \text{(derived in Part I)}$$

For the second reflection, we can make one simple modification to the identity matrix:

$$M_{ref-x} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

This inverts the y component of the vector, thus reflecting over the x-axis.

So, the combined transformation matrix will be:

$$
\begin{aligned}
M_{combined} &= M_{ref-x} M_{ref} \\
M_{combined} &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \frac{1}{1+m^2} \begin{pmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{pmatrix} \\
M_{combined} &= \frac{1}{1+m^2} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 1-m^2 & 2m \\ 2m & m^2-1 \end{pmatrix} \\
M_{combined} &= \frac{1}{1+m^2} \begin{pmatrix} 1-m^2 & 2m \\ -2m & 1-m^2 \end{pmatrix}
\end{aligned}
$$

Where, in this case, $m = 3$.