1080. a) $1+2+3+\ldots+n=\frac{n(n+1)}{2}$

***po indukcijskoj hipotezi**

(1) n=1

$S_1=a_1=1$

$F(1)=\frac{1\cdot 2}{2}=1$, T

(2) $S_n=F(n) \longrightarrow S_{n+1}=F(n+1)$

$$\underbrace{1+2+3+\ldots+n}_{S_n}=\underbrace{\frac{n(n+1)}{2}}_{F(n)} \longrightarrow \underbrace{1+2+3+\ldots+n+(n+1)}_{S_{n+1}}=\underbrace{\frac{(n+1)(n+2)}{2}}_{F(n+1)}$$

$$\frac{n(n+1)}{2}+(n+1)=(n+1)\left(\frac{n}{2}+1\right)=(n+2)\,\frac{n+1}{2}=\frac{(n+1)(n+2)}{2},\ T$$

1081. b) $\frac{3}{1\cdot 2}+\frac{7}{2\cdot 3}+\cdots+\frac{n^2+n+1}{n(n+1)}=\frac{n(n+2)}{n+1}$

***po indukcijskoj hipotezi**

(1) n=1

$S_1=a_1$

$S_1=\frac{1^2+1+1}{1(1+1)}$

$S_1=\frac{3}{2}$

$F(1)=\frac{1(1+2)}{2}$

$F(1)=\frac{3}{2}$, T

(2) $S_n=F(n) \longrightarrow S_{n+1}=F(n+1)$

$$\underbrace{\frac{3}{2\cdot 1}+\frac{7}{2\cdot 3}+\cdots+\frac{n^2+n+1}{n(n+1)}}_{S_n=F(n)}+\underbrace{\frac{(n+1)^2+(n+1)+1}{(n+1)(n+2)}}_{a_{n+1}}=\underbrace{\frac{(n+1)(n+3)}{n+2}}_{F(n+1)}$$

$$\frac{n(n+2)}{n+1}+\frac{n^2+3n+3}{(n+1)(n+2)}=\frac{n(n+2)^2+n^2+3n+3}{(n+1)(n+2)}=$$

$$\frac{n^3+4n^2+4n+n^2+3n+3}{(n+1)(n+2)}=\frac{\mathbf{n^3+5n^2+7n+3}}{(n+1)(n+2)}=\frac{(n+1)(n+3)}{n+2},\ T$$

$n^3+5n^2+7n+3:\mathbf{(n+1)}=\underline{n^2+4n+3}$

$-(n^3+n^2)$

$4n^2+7n+3$

$-(4n^2+4n)$

$3n+3$

$-(3n+3)$

0

$n^2+3n+n+3=n(n+1)+3(n+1)=$

$\mathbf{(n+3)(n+1)}$