CÁLCULO

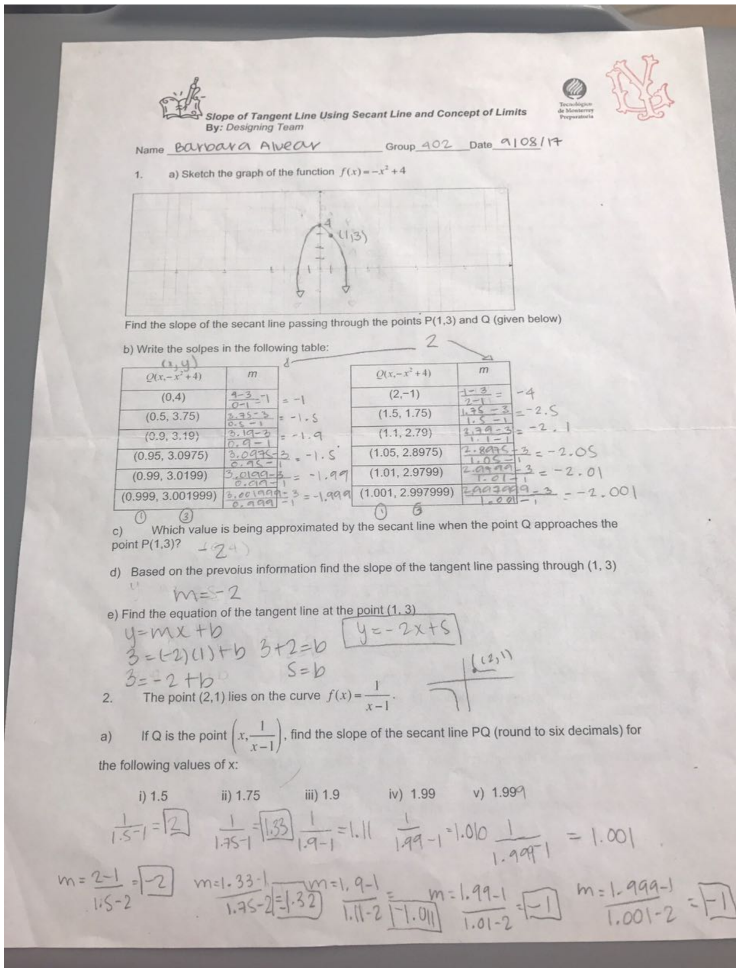
E-Portafolio

BARBARA ALVEAR A01570137

FIRST PARTIAL

CLASSWORKS

Slope of tangent line using secant line



Slope of Tangent Line Using Secant Line and Concept of Limits
By: Designing Team

Tecnológico de Monterrey Preparatoria

Name Barbara Alvear Group 402 Date 9/08/17

1. a) Sketch the graph of the function $f(x) = -x^2 + 4$

Find the slope of the secant line passing through the points P(1,3) and Q (given below)

b) Write the solpes in the following table:

$Q(x, -x^2+4)$	m
(0,4)	$\frac{4-3}{0-1} = -1$
(0.5, 3.75)	$\frac{3.75-3}{0.5-1} = -1.5$
(0.9, 3.19)	$\frac{3.19-3}{0.9-1} = -1.9$
(0.95, 3.0975)	$\frac{3.0975-3}{0.95-1} = -1.5$
(0.99, 3.0199)	$\frac{3.0199-3}{0.99-1} = -1.99$
(0.999, 3.001999)	$\frac{3.001999-3}{0.999-1} = -1.999$

$Q(x, -x^2+4)$	m
(2,−1)	$\frac{-1-3}{2-1} = -4$
(1.5, 1.75)	$\frac{1.75-3}{1.5-1} = -2.5$
(1.1, 2.79)	$\frac{2.79-3}{1.1-1} = -2.1$
(1.05, 2.8975)	$\frac{2.8975-3}{1.05-1} = -2.05$
(1.01, 2.9799)	$\frac{2.9799-3}{1.01-1} = -2.01$
(1.001, 2.997999)	$\frac{2.997999-3}{1.001-1} = -2.001$

c) Which value is being approximated by the secant line when the point Q approaches the point P(1,3)? −2

d) Based on the prevoius information find the slope of the tangent line passing through (1, 3)

$m = -2$

e) Find the equation of the tangent line at the point (1, 3)

$y = mx + b$
$3 = (-2)(1) + b$
$3 = -2 + b$
$3 + 2 = b$
$5 = b$
$\boxed{y = -2x + 5}$

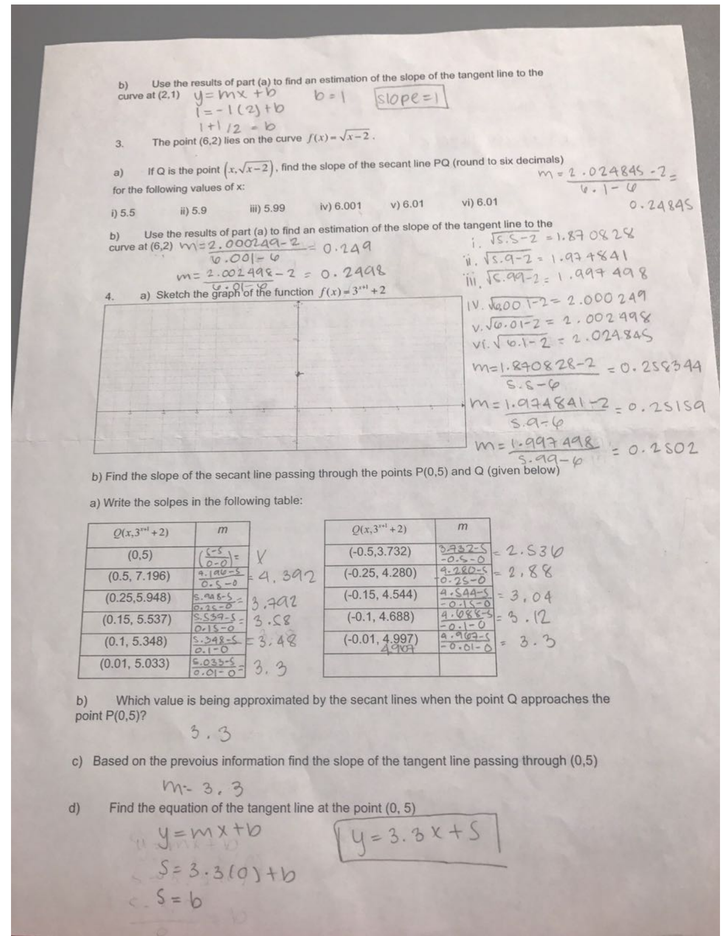
2. The point (2,1) lies on the curve $f(x) = \frac{1}{x-1}$.

a) If Q is the point $\left(x, \frac{1}{x-1}\right)$, find the slope of the secant line PQ (round to six decimals) for the following values of x:

i) 1.5 ii) 1.75 iii) 1.9 iv) 1.99 v) 1.999

$\frac{1}{1.5-1} = \boxed{2}$ $\frac{1}{1.75-1} = \boxed{1.33}$ $\frac{1}{1.9-1} = 1.11$ $\frac{1}{1.99-1} = 1.010$ $\frac{1}{1.999-1} = 1.001$

$m = \frac{2-1}{1.5-2} = \boxed{-2}$ $m = \frac{1.33-1}{1.75-2} = \boxed{-1.32}$ $m = \frac{1.9-1}{1.11-2} = \boxed{-1.011}$ $m = \frac{1.99-1}{1.01-2} = \boxed{-1}$ $m = \frac{1.999-1}{1.001-2} = \boxed{-1}$



b) Use the results of part (a) to find an estimation of the slope of the tangent line to the curve at (2,1)

$y = mx + b$
$1 = -1(2) + b$
$1 + 1/2 = b$
$b = 1$
$\boxed{\text{slope} = 1}$

3. The point (6,2) lies on the curve $f(x) = \sqrt{x-2}$.

a) If Q is the point $(x, \sqrt{x-2})$, find the slope of the secant line PQ (round to six decimals) for the following values of x:

i) 5.5 ii) 5.9 iii) 5.99 iv) 6.001 v) 6.01 vi) 6.01

i. $\sqrt{5.5-2} = 1.870828$
ii. $\sqrt{5.9-2} = 1.974841$
iii. $\sqrt{5.99-2} = 1.997498$
iv. $\sqrt{6.001-2} = 2.000249$
v. $\sqrt{6.01-2} = 2.002498$
vi. $\sqrt{6.1-2} = 2.024845$

$m = \frac{1.870828-2}{5.5-6} = 0.258344$
$m = \frac{1.974841-2}{5.9-6} = 0.25159$
$m = \frac{1.997498}{5.99-6} = 0.2502$
$m = \frac{2.024845-2}{6.1-6} = 0.24845$

b) Use the results of part (a) to find an estimation of the slope of the tangent line to the curve at (6,2)

$m = \frac{2.000249-2}{6.001-6} = 0.249$
$m = \frac{2.002498-2}{6.01-6} = 0.2498$

4. a) Sketch the graph of the function $f(x) = 3^{x+1} + 2$

b) Find the slope of the secant line passing through the points P(0,5) and Q (given below)

a) Write the solpes in the following table:

$Q(x, 3^{x+1}+2)$	m
(0,5)	$\left(\frac{5-5}{0-0}\right) = X$
(0.5, 7.196)	$\frac{7.196-5}{0.5-0} = 4.392$
(0.25,5.948)	$\frac{5.948-5}{0.25-0} = 3.792$
(0.15, 5.537)	$\frac{5.537-5}{0.15-0} = 3.58$
(0.1, 5.348)	$\frac{5.348-5}{0.1-0} = 3.48$
(0.01, 5.033)	$\frac{5.033-5}{0.01-0} = 3.3$

$Q(x, 3^{x+1}+2)$	m
(-0.5,3.732)	$\frac{3.732-5}{-0.5-0} = 2.536$
(-0.25, 4.280)	$\frac{4.280-5}{-0.25-0} = 2.88$
(-0.15, 4.544)	$\frac{4.544-5}{-0.15-0} = 3.04$
(-0.1, 4.688)	$\frac{4.688-5}{-0.1-0} = 3.12$
(-0.01, 4.997) 4.967	$\frac{4.967-5}{-0.01-0} = 3.3$

b) Which value is being approximated by the secant lines when the point Q approaches the point P(0,5)?

3.3

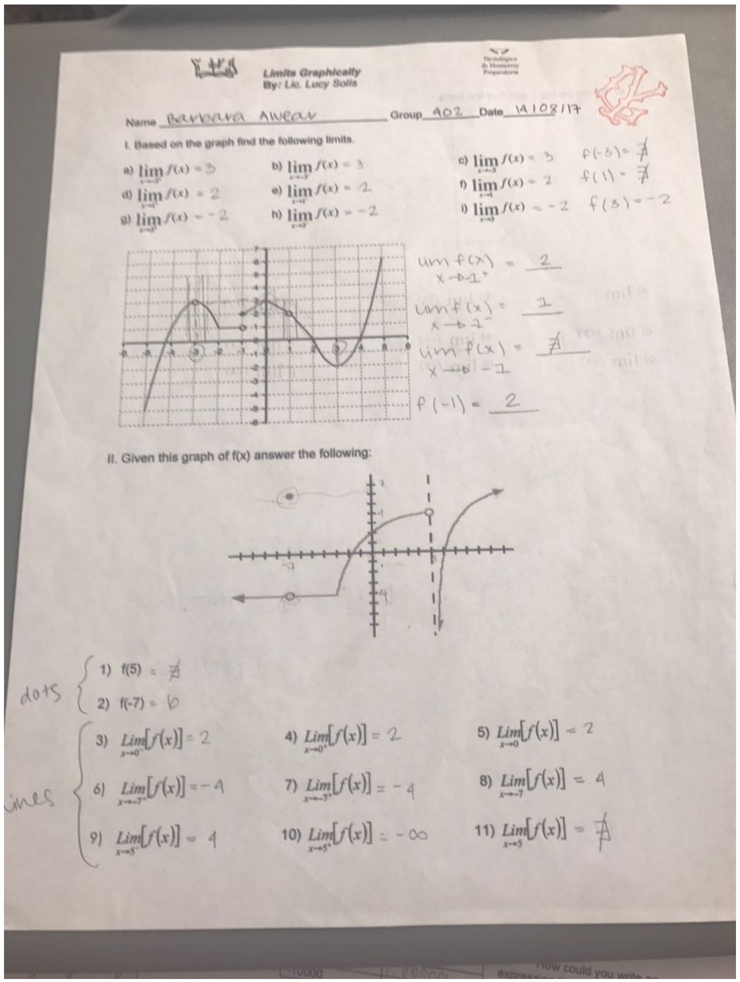
c) Based on the prevoius information find the slope of the tangent line passing through (0,5)

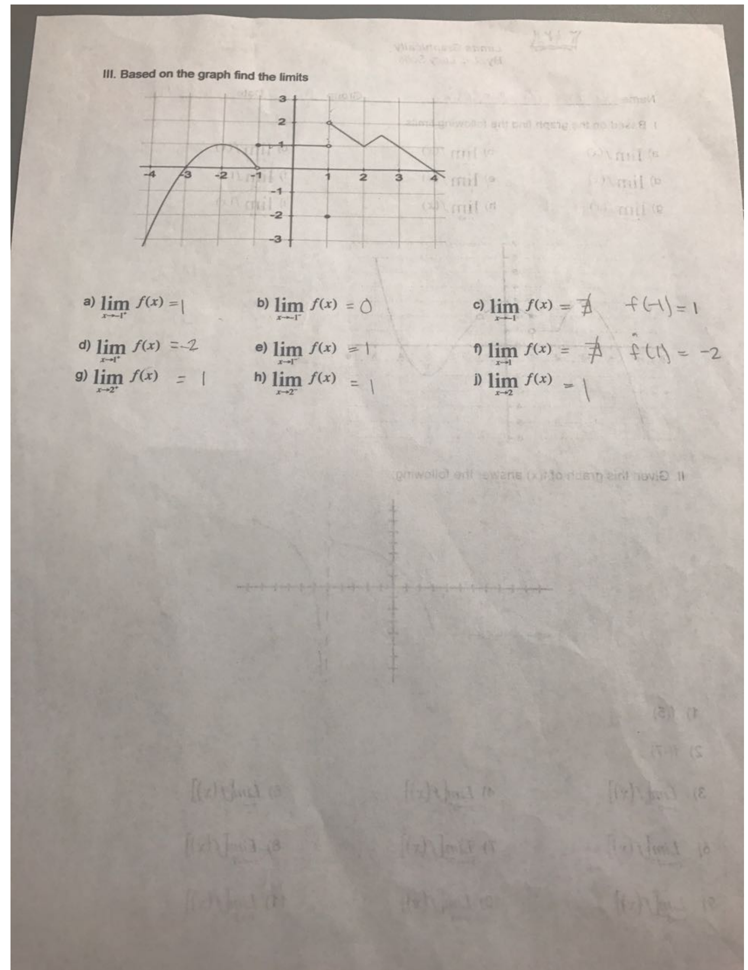
$m = 3.3$

d) Find the equation of the tangent line at the point (0, 5)

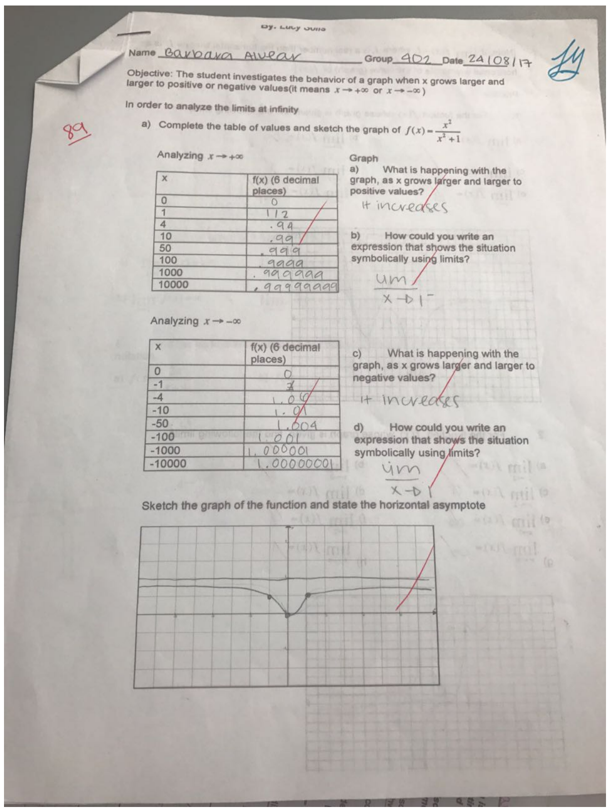
$y = mx + b$
$5 = 3.3(0) + b$
$5 = b$
$\boxed{y = 3.3x + 5}$

Limits graphically





Limits at infinity: horizontal asymptote



Name Barbara Alvear Group 402 Date 24/08/17

Objective: The student investigates the behavior of a graph when x grows larger and larger to positive or negative values(it means $x \to +\infty$ or $x \to -\infty$)

In order to analyze the limits at infinity

a) Complete the table of values and sketch the graph of $f(x) = \frac{x^2}{x^2+1}$

Analyzing $x \to +\infty$

x	f(x) (6 decimal places)
0	0
1	1/2
4	.94
10	.99
50	.999
100	.9999
1000	.999999
10000	.99999999

Graph

a) What is happening with the graph, as x grows larger and larger to positive values?

It increases

b) How could you write an expression that shows the situation symbolically using limits?

Analyzing $x \to -\infty$

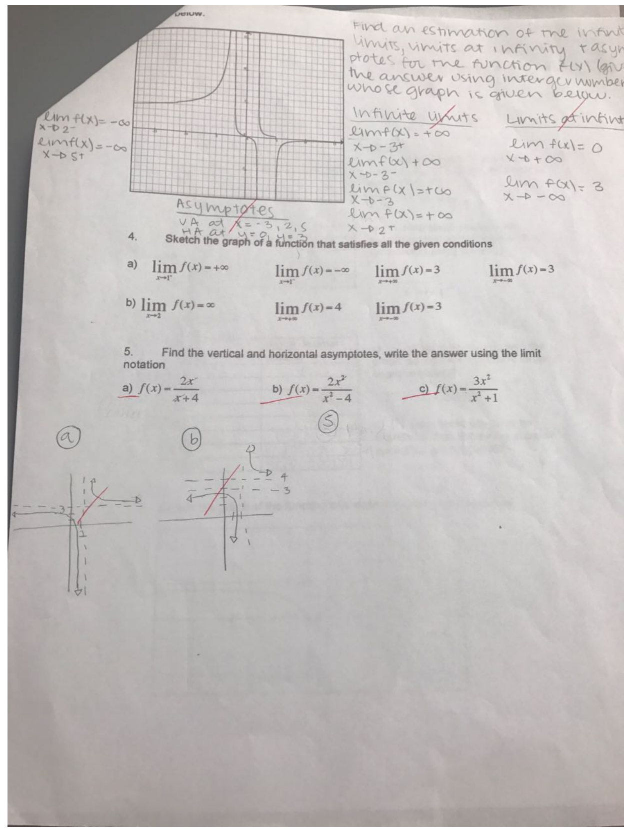
x	f(x) (6 decimal places)
0	0
-1	
-4	1.04
-10	1.01
-50	1.004
-100	1.001
-1000	1.000001
-10000	1.00000001

c) What is happening with the graph, as x grows larger and larger to negative values?

It increases

d) How could you write an expression that shows the situation symbolically using limits?

Sketch the graph of the function and state the horizontal asymptote



Find an estimation of the infinite limits, limits at infinity & asymptotes for the function f(x) (give the answer using integer numbers) whose graph is given below.

Infinite limits

Limits at infinity

Asymptotes

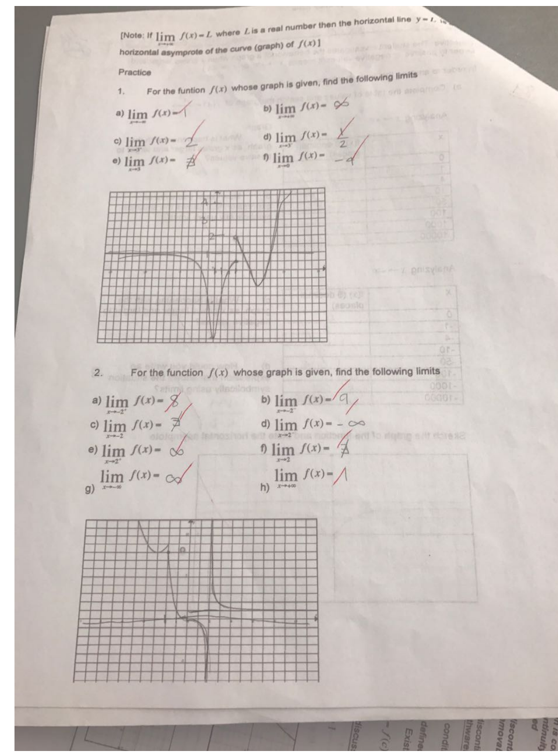
4. Sketch the graph of a function that satisfies all the given conditions

a) $\lim_{x\to 1^+} f(x) = +\infty$ $\lim_{x\to 1^-} f(x) = -\infty$ $\lim_{x\to +\infty} f(x) = 3$ $\lim_{x\to -\infty} f(x) = 3$

b) $\lim_{x\to 2} f(x) = \infty$ $\lim_{x\to +\infty} f(x) = 4$ $\lim_{x\to -\infty} f(x) = 3$

5. Find the vertical and horizontal asymptotes, write the answer using the limit notation

a) $f(x) = \frac{2x}{x+4}$ b) $f(x) = \frac{2x^2}{x^2-4}$ c) $f(x) = \frac{3x^2}{x^2+1}$



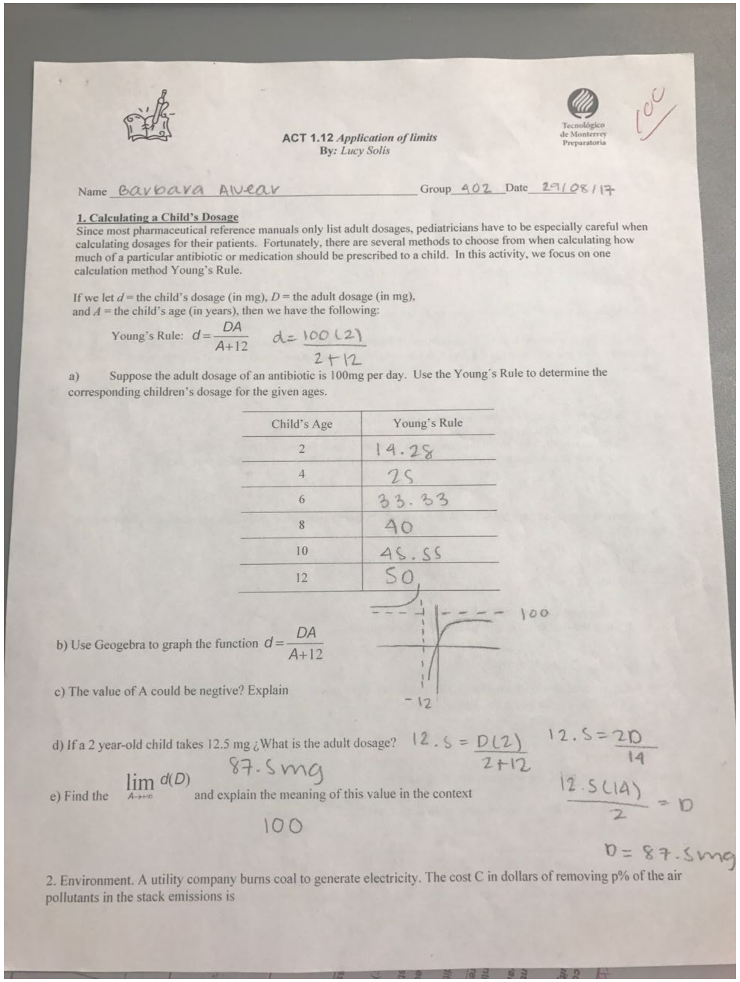
[Note: If $\lim_{x\to\infty} f(x) = L$ where L is a real number then the horizontal line y = L is horizontal asymptote of the curve (graph) of f(x)]

Practice

1. For the funtion f(x) whose graph is given, find the following limits

2. For the function f(x) whose graph is given, find the following limits

Application of limits



ACT 1.12 *Application of limits*
By: *Lucy Solis*

Tecnológico de Monterrey Preparatoria

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Name Barbara Alvear Group 402 Date 29/08/17

1. Calculating a Child's Dosage

Since most pharmaceutical reference manuals only list adult dosages, pediatricians have to be especially careful when calculating dosages for their patients. Fortunately, there are several methods to choose from when calculating how much of a particular antibiotic or medication should be prescribed to a child. In this activity, we focus on one calculation method Young's Rule.

If we let d = the child's dosage (in mg), D = the adult dosage (in mg), and A = the child's age (in years), then we have the following:

Young's Rule: $d=\frac{DA}{A+12}$ $d=\frac{100(2)}{2+12}$

a) Suppose the adult dosage of an antibiotic is 100mg per day. Use the Young´s Rule to determine the corresponding children's dosage for the given ages.

Child's Age	Young's Rule
2	14.28
4	25
6	33.33
8	40
10	45.55
12	50

100

b) Use Geogebra to graph the function $d=\frac{DA}{A+12}$

c) The value of A could be negtive? Explain

-12

d) If a 2 year-old child takes 12.5 mg ¿What is the adult dosage? $12.5=\frac{D(2)}{2+12}$ $12.5=\frac{2D}{14}$

87.5mg

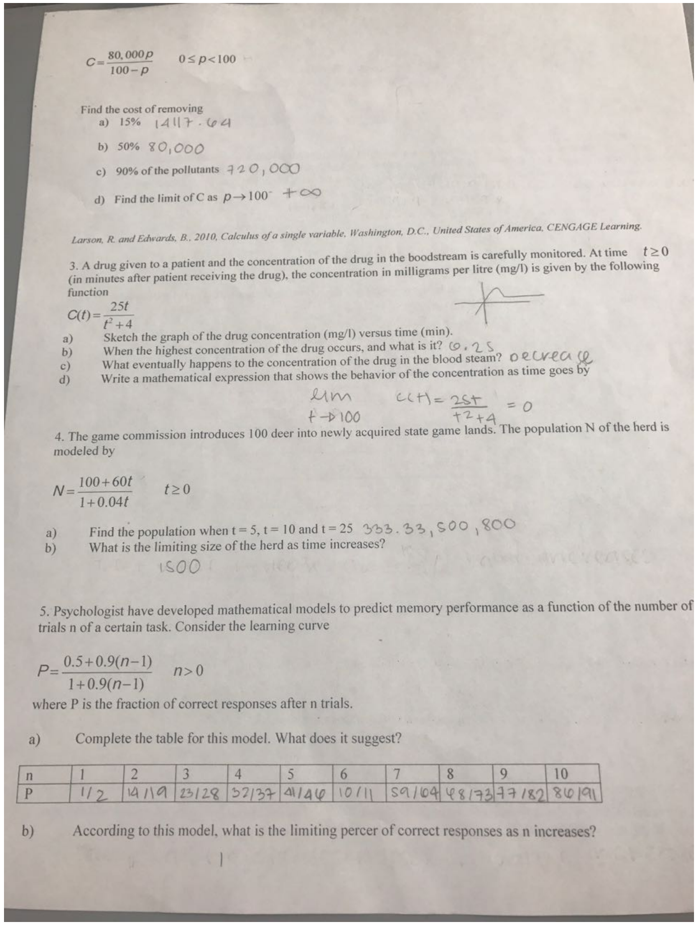
e) Find the $\lim_{A\to+\infty} d(D)$ and explain the meaning of this value in the context

$\frac{12.5(14)}{2}=D$

100

$D=87.5mg$

2. Environment. A utility company burns coal to generate electricity. The cost C in dollars of removing p% of the air pollutants in the stack emissions is



$C=\frac{80,000p}{100-p}$ $0\le p<100$

Find the cost of removing

a) 15% 14117.64

b) 50% 80,000

c) 90% of the pollutants 720,000

d) Find the limit of C as $p\to100^-$ $+\infty$

Larson, R. and Edwards, B., 2010, Calculus of a single variable, Washington, D.C., United States of America, CENGAGE Learning.

3. A drug given to a patient and the concentration of the drug in the boodstream is carefully monitored. At time $t\ge0$ (in minutes after patient receiving the drug), the concentration in milligrams per litre (mg/l) is given by the following function

$C(t)=\frac{25t}{t^2+4}$

a) Sketch the graph of the drug concentration (mg/l) versus time (min).
b) When the highest concentration of the drug occurs, and what is it? 6.25
c) What eventually happens to the concentration of the drug in the blood steam? decrease
d) Write a mathematical expression that shows the behavior of the concentration as time goes by

$\lim_{t\to100} c(t)=\frac{25t}{t^2+4}=0$

4. The game commission introduces 100 deer into newly acquired state game lands. The population N of the herd is modeled by

$N=\frac{100+60t}{1+0.04t}$ $t\ge0$

a) Find the population when t = 5, t = 10 and t = 25 333.33, 500, 800
b) What is the limiting size of the herd as time increases?

1500

5. Psychologist have developed mathematical models to predict memory performance as a function of the number of trials n of a certain task. Consider the learning curve

$P=\frac{0.5+0.9(n-1)}{1+0.9(n-1)}$ $n>0$

where P is the fraction of correct responses after n trials.

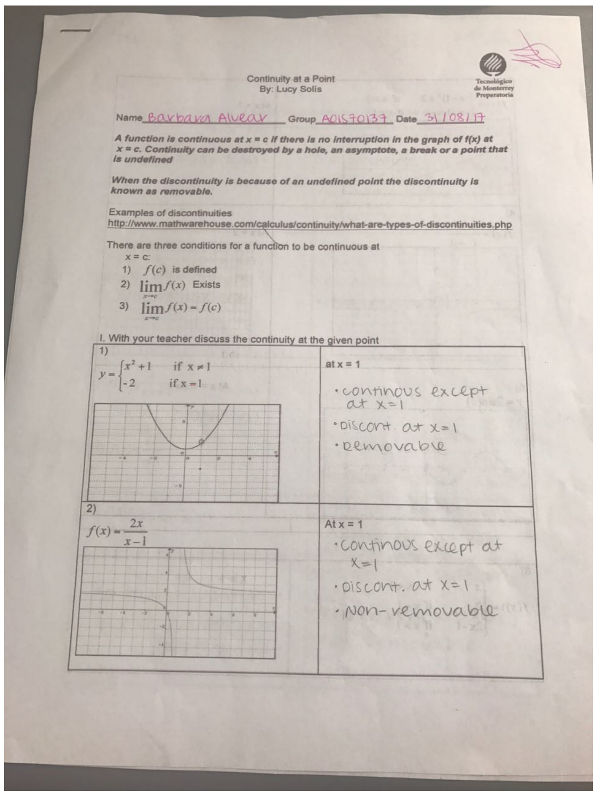
a) Complete the table for this model. What does it suggest?

n	1	2	3	4	5	6	7	8	9	10
P	1/2	14/19	23/28	32/37	41/46	10/11	59/64	68/73	77/82	86/91

b) According to this model, what is the limiting percer of correct responses as n increases?

1

Continuity at a point



Continuity at a Point
By: Lucy Solis

Tecnológico de Monterrey Preparatoria

Name Barbara Alvear Group A01570137 Date 31/08/17

A function is continuous at x = c if there is no interruption in the graph of f(x) at x = c. Continuity can be destroyed by a hole, an asymptote, a break or a point that is undefined

When the discontinuity is because of an undefined point the discontinuity is known as removable.

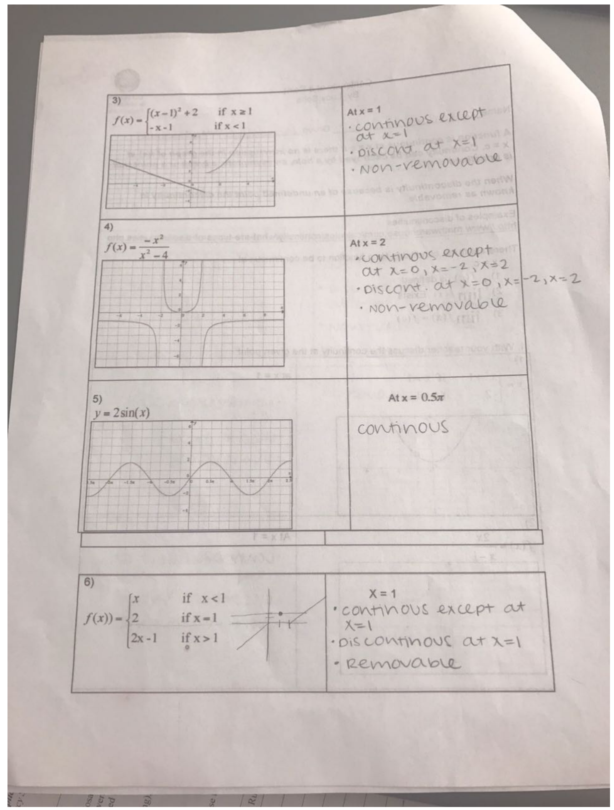
Examples of discontinuities
http://www.mathwarehouse.com/calculus/continuity/what-are-types-of-discontinuities.php

There are three conditions for a function to be continuous at x = c:

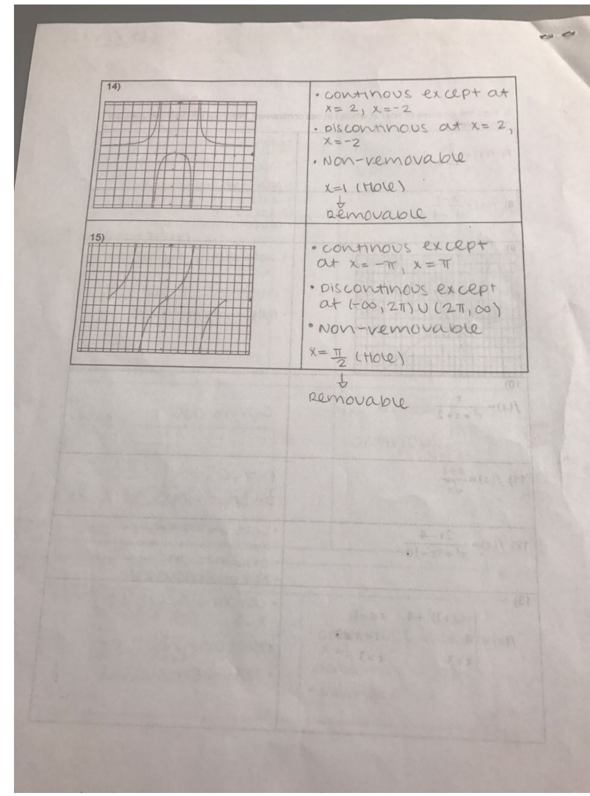
1) $f(c)$ is defined
2) $\lim_{x \to c} f(x)$ Exists
3) $\lim_{x \to c} f(x) = f(c)$

I. With your teacher discuss the continuity at the given point

1) $y = \begin{cases} x^2+1 & \text{if } x \neq 1 \\ -2 & \text{if } x = 1 \end{cases}$	at x = 1 · continous except at x=1 · Discont. at x=1 · Removable
2) $f(x) = \frac{2x}{x-1}$	At x = 1 · continous except at x=1 · Discont. at x=1 · Non-removable



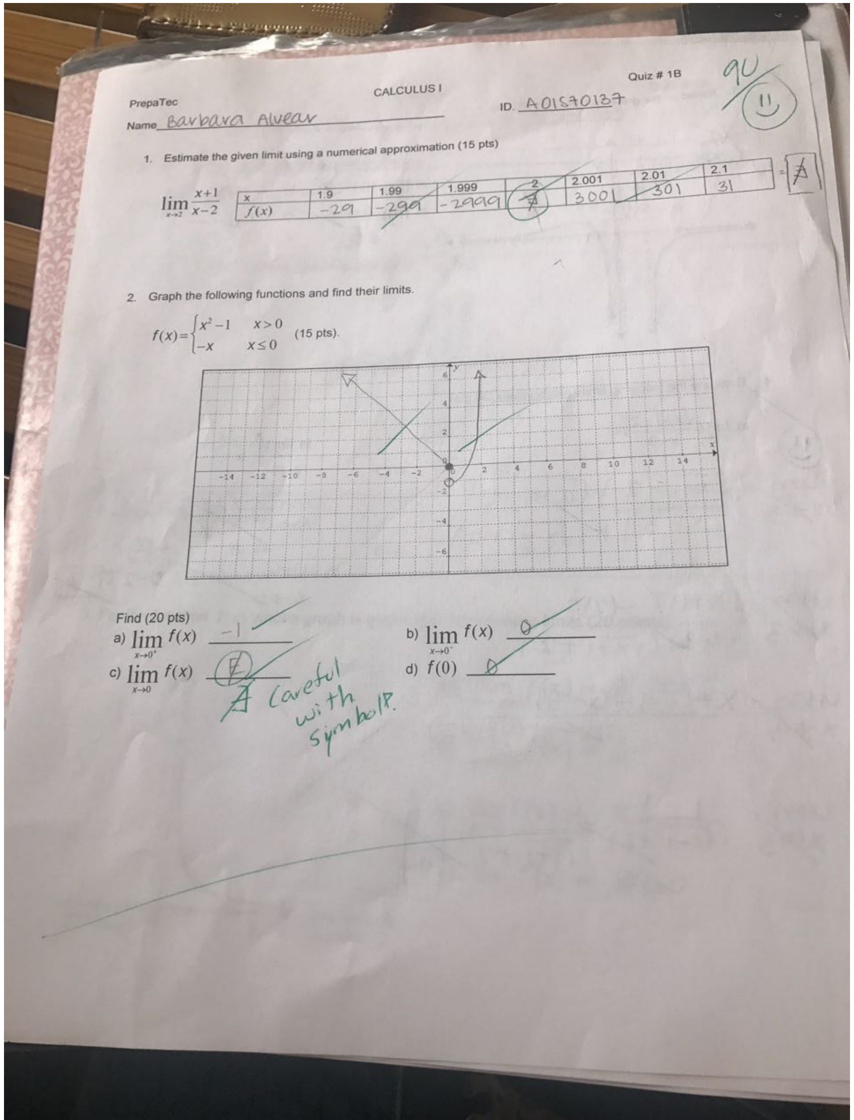
3) $f(x) = \begin{cases} (x-1)^2+2 & \text{if } x \geq 1 \\ -x-1 & \text{if } x < 1 \end{cases}$	At x = 1 · continous except at x=1 · Discont. at x=1 · Non-removable
4) $f(x) = \frac{-x^2}{x^2-4}$	At x = 2 · continous except at x=0, x=-2, x=2 · Discont. at x=0, x=-2, x=2 · Non-removable
5) $y = 2\sin(x)$	At x = 0.5π continous
6) $f(x) = \begin{cases} x & \text{if } x < 1 \\ 2 & \text{if } x = 1 \\ 2x-1 & \text{if } x > 1 \end{cases}$	X = 1 · continous except at X=1 · Discontinous at x=1 · Removable



14)	· continous except at x= 2, x=-2 · Discontinous at x= 2, x=-2 · Non-removable x=1 (Hole) ↓ Removable
15)	· continous except at x= -π, x = π · Discontinous except at (-∞, 2π) ∪ (2π, ∞) · Non-removable X= π/2 (Hole) ↓ Removable

QUIZZES

QUIZ 1



PrepaTec CALCULUS I Quiz # 1B 90

Name Barbara Alvear ID. A01570137

1. Estimate the given limit using a numerical approximation (15 pts)

$\lim_{x\to 2} \frac{x+1}{x-2}$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$	−29	−299	−2999	∄	3001	301	31

$= \boxed{\nexists}$

2. Graph the following functions and find their limits.

$f(x)=\begin{cases} x^2-1 & x>0 \\ -x & x\le 0 \end{cases}$ (15 pts).

Find (20 pts)

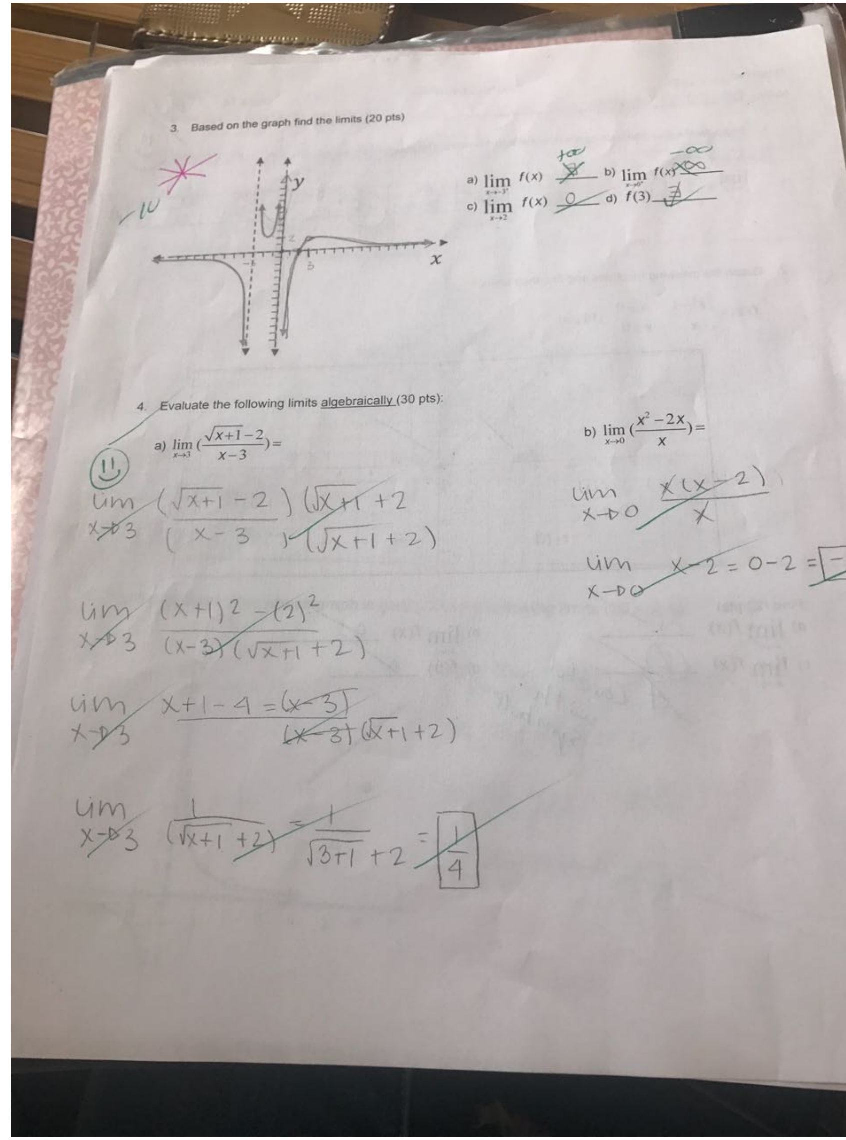
a) $\lim_{x\to 0^+} f(x)$ −1

b) $\lim_{x\to 0^-} f(x)$ 0

c) $\lim_{x\to 0} f(x)$ ∄

d) $f(0)$ 0

∄ Careful with symbol.



3. Based on the graph find the limits (20 pts)

a) $\lim_{x\to -3^+} f(x)$ $+\infty$

b) $\lim_{x\to 0^+} f(x)$ $-\infty$

c) $\lim_{x\to 2} f(x)$ 0

d) $f(3)$ ∄

−10

4. Evaluate the following limits algebraically (30 pts):

a) $\lim_{x\to 3} \left(\frac{\sqrt{x+1}-2}{x-3}\right) =$

$\lim_{x\to 3} \frac{(\sqrt{x+1}-2)(\sqrt{x+1}+2)}{(x-3)(\sqrt{x+1}+2)}$

$\lim_{x\to 3} \frac{(x+1)^2-(2)^2}{(x-3)(\sqrt{x+1}+2)}$

$\lim_{x\to 3} \frac{x+1-4=(x-3)}{(x-3)(\sqrt{x+1}+2)}$

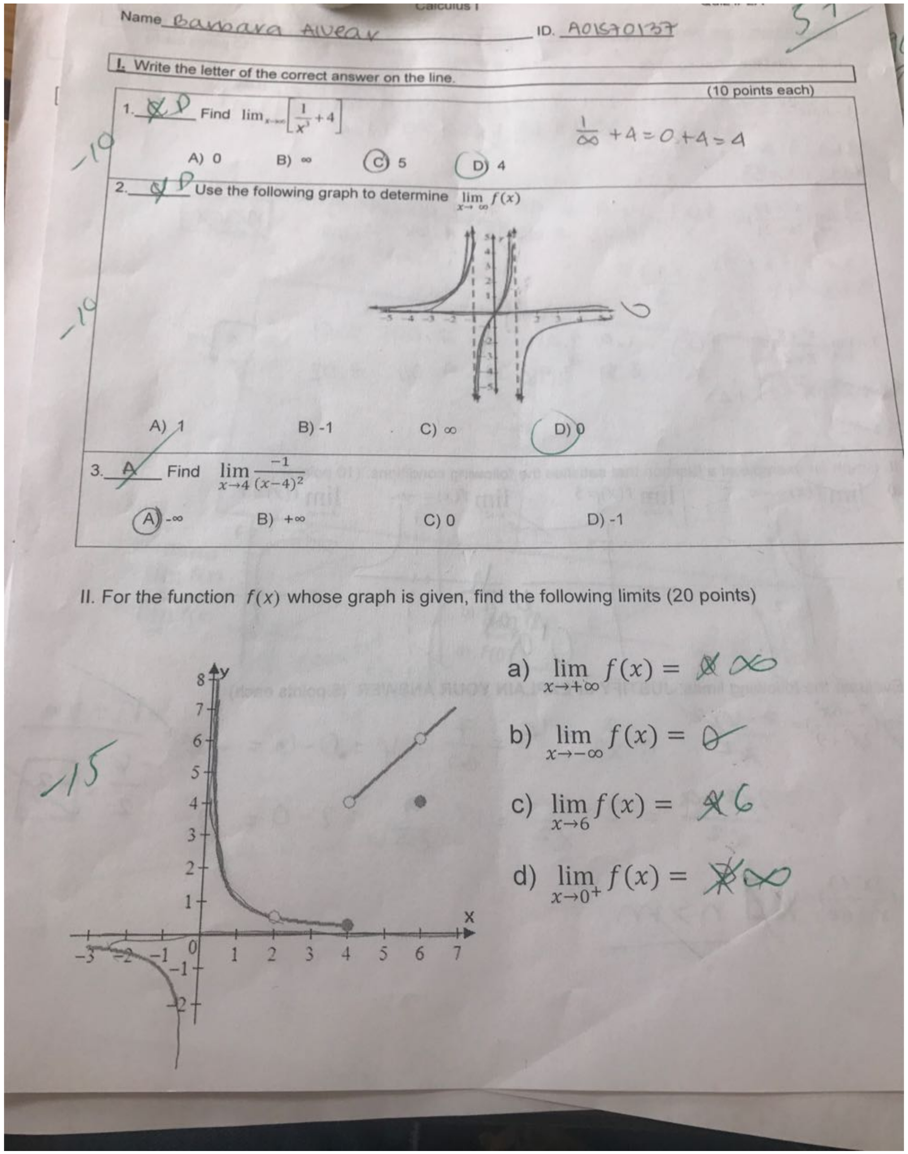
$\lim_{x\to 3} \frac{1}{(\sqrt{x+1}+2)} = \frac{1}{\sqrt{3+1}+2} = \boxed{\frac{1}{4}}$

b) $\lim_{x\to 0} \left(\frac{x^2-2x}{x}\right) =$

$\lim_{x\to 0} \frac{x(x-2)}{x}$

$\lim_{x\to 0} x-2 = 0-2 = \boxed{-2}$

QUIZ 2



Name Barbara Alvear   ID. A01570137   57

I. Write the letter of the correct answer on the line. (10 points each)

1. ~~C~~ D Find $\lim_{x\to\infty}\left[\frac{1}{x^3}+4\right]$ $\frac{1}{\infty}+4=0+4=4$

A) 0   B) ∞ C) 5   D) 4

-10

2. ~~C~~ D Use the following graph to determine $\lim_{x\to\infty} f(x)$

A) 1   B) -1   C) ∞ D) 0

-10

3. A Find $\lim_{x\to 4}\frac{-1}{(x-4)^2}$

A) $-\infty$ B) $+\infty$ C) 0   D) -1

II. For the function $f(x)$ whose graph is given, find the following limits (20 points)

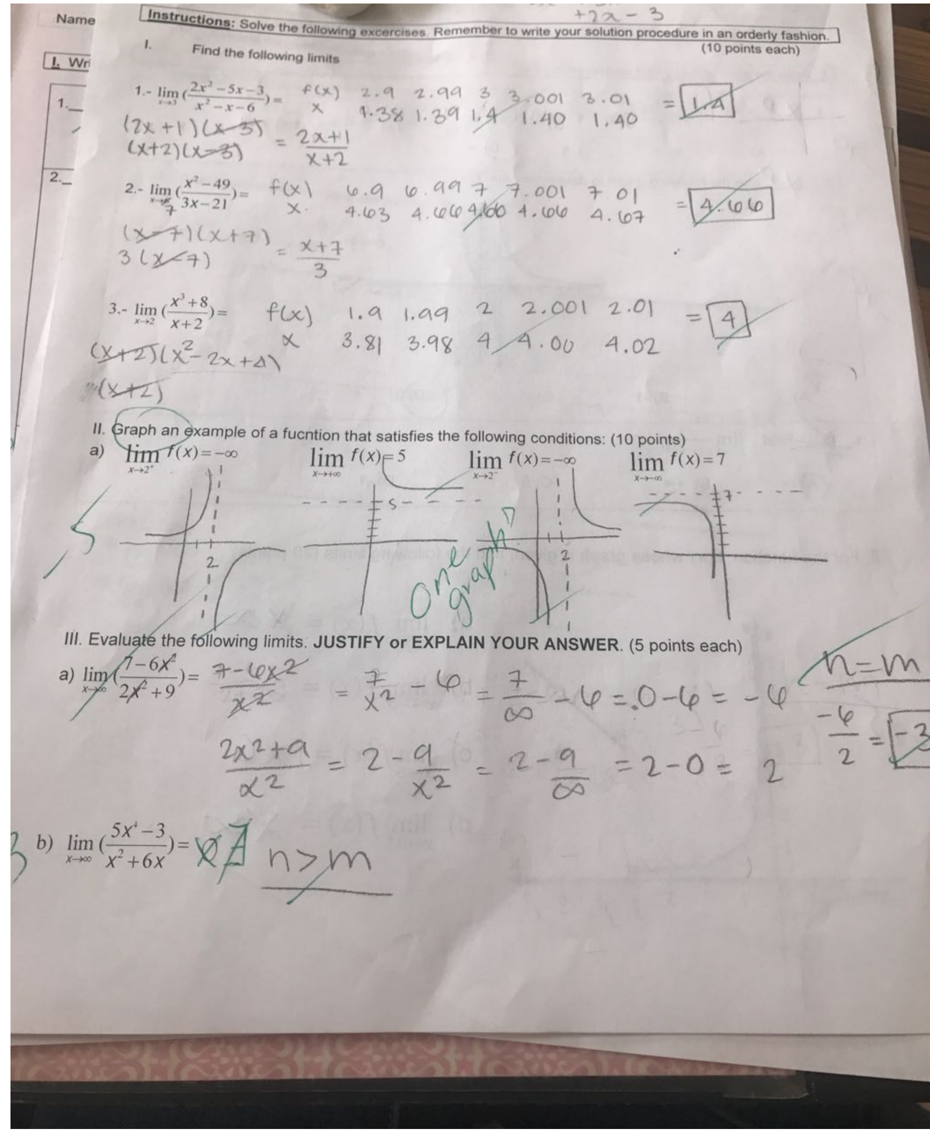
a) $\lim_{x\to+\infty} f(x) =$ ~~X~~ ∞

b) $\lim_{x\to-\infty} f(x) =$ 0

c) $\lim_{x\to 6} f(x) =$ ~~4~~ 6

d) $\lim_{x\to 0^+} f(x) =$ ~~X~~ ∞

-15



Name $+2x-3$

Instructions: Solve the following excercises. Remember to write your solution procedure in an orderly fashion. (10 points each)

I. Find the following limits

1.- $\lim_{x\to3}\left(\frac{2x^2-5x-3}{x^2-x-6}\right)=$

f(x)	2.9	2.99	3	3.001	3.01
x	1.38	1.39	1.4	1.40	1.40

$= \boxed{1.4}$

$\frac{(2x+1)\cancel{(x-3)}}{(x+2)\cancel{(x-3)}} = \frac{2x+1}{x+2}$

2.- $\lim_{x\to7}\left(\frac{x^2-49}{3x-21}\right)=$

f(x)	6.9	6.99	7	7.001	7.01
x	4.63	4.66	4.66	4.66	4.67

$= \boxed{4.66}$

$\frac{\cancel{(x-7)}(x+7)}{3\cancel{(x-7)}} = \frac{x+7}{3}$

3.- $\lim_{x\to-2}\left(\frac{x^3+8}{x+2}\right)=$

f(x)	1.9	1.99	2	2.001	2.01
x	3.81	3.98	4	4.00	4.02

$= \boxed{4}$

$\frac{\cancel{(x+2)}(x^2-2x+4)}{\cancel{(x+2)}}$

II. Graph an example of a fucntion that satisfies the following conditions: (10 points)

a) $\lim_{x\to2^+} f(x)=-\infty$ $\lim_{x\to+\infty} f(x)=5$ $\lim_{x\to2^-} f(x)=-\infty$ $\lim_{x\to-\infty} f(x)=7$

5

One graph

III. Evaluate the following limits. **JUSTIFY or EXPLAIN YOUR ANSWER**. (5 points each)

a) $\lim_{x\to\infty}\left(\frac{7-6x^2}{2x^2+9}\right)= \frac{7-6x^2}{x^2} = \frac{7}{x^2}-6 = \frac{7}{\infty}-6 = 0-6=-6$

$\frac{2x^2+9}{x^2} = 2-\frac{9}{x^2} = 2-\frac{9}{\infty} = 2-0 = 2$

$n=m$ $\frac{-6}{2} = \boxed{-3}$

b) $\lim_{x\to\infty}\left(\frac{5x^4-3}{x^2+6x}\right)=$ ∅ ∄ $n>m$

3

Partial Project

https://www.youtube.com/watch?v=TYtomGavJy8&app=desktop