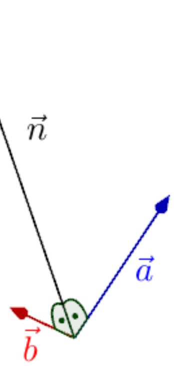
Kreuzprodukt



$$\vec{n} = \vec{a} \times \vec{b} \quad \text{mit} \quad \begin{pmatrix} n_1 \\ n_2 \\ n_3 \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} \times \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} = \begin{pmatrix} a_2b_3 - a_3b_2 \\ a_3b_1 - a_1b_3 \\ a_1b_2 - a_2b_1 \end{pmatrix}$$

<u>Herleitung:</u> Sämtliche Faktoren bei den Äquivalenzumformungen seien für die Herleitung o.B.a.A. $\neq 0$

$$\begin{aligned} \vec{a} \circ \vec{n} = 0 \\ \vec{b} \circ \vec{n} = 0 \end{aligned} \quad \Rightarrow \quad \begin{aligned} * \quad a_1n_1 + a_2n_2 + a_3n_3 &= 0 && | \cdot b_1 \\ b_1n_1 + b_2n_2 + b_3n_3 &= 0 && | \cdot (-a_1) \end{aligned}$$

$$\begin{aligned} a_1b_1n_1 + a_2b_1n_2 + a_3b_1n_3 &= 0 \\ -a_1b_1n_1 - a_1b_2n_2 - a_1b_3n_3 &= 0 && |+ \end{aligned}$$

$$\begin{aligned} (a_2b_1 - a_1b_2)n_2 + (a_3b_1 - a_1b_3)n_3 &= 0 && | - (a_2b_1 - a_1b_2)n_2 \\ (a_3b_1 - a_1b_3)n_3 &= (a_1b_2 - a_2b_1)n_2 && | : n_2 \quad | : (a_3b_1 - a_1b_3) \\ \frac{n_3}{n_2} &= \frac{a_1b_2 - a_2b_1}{a_3b_1 - a_1b_3} \end{aligned}$$

Der Komponentenvergleich bestätigt n_2 und n_3 aus obiger Formel.
Einsetzen in $*$ ergibt

$$\begin{aligned} a_1n_1 + a_2(a_3b_1 - a_1b_3) + a_3(a_1b_2 - a_2b_1) &= 0 \qquad | - a_2(a_3b_1 - a_1b_3) \; | - a_3(a_1b_2 - a_2b_1) \\ a_1n_1 &= a_2(a_1b_3 - a_3b_1) + a_3(a_2b_1 - a_1b_2) \\ a_1n_1 &= a_1a_2b_3 - a_2a_3b_1 + a_2a_3b_1 - a_1a_3b_2 \\ a_1n_1 &= a_1a_2b_3 - a_1a_3b_2 \quad | : a_1 \\ n_1 &= a_2b_3 - a_3b_2 \qquad \textit{q.e.d.} \end{aligned}$$

<u>Schema:</u>

$$\begin{array}{ccc} a_1 & — & b_1 \\ a_2 & & b_2 \\ & \times & \\ a_3 & & b_3 \\ & \times & \\ a_1 & & b_1 \\ & \times & \\ a_2 & & b_2 \\ a_3 & — & b_3 \end{array} \Rightarrow \begin{pmatrix} {\color{red}a_2b_3} - {\color{blue}a_3b_2} \\ {\color{red}a_3b_1} - {\color{blue}a_1b_3} \\ {\color{red}a_1b_2} - {\color{blue}a_2b_1} \end{pmatrix}$$

<u>Beispiel:</u>

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \times \begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} = \begin{pmatrix} {\color{red}2 \cdot 6} - {\color{blue}3 \cdot 5} \\ {\color{red}3 \cdot 4} - {\color{blue}1 \cdot 6} \\ {\color{red}1 \cdot 5} - {\color{blue}2 \cdot 4} \end{pmatrix} = \begin{pmatrix} -3 \\ 6 \\ -3 \end{pmatrix}$$

$$\begin{array}{ccc} 1 & — & 4 \\ 2 & & 5 \\ & \times & \\ 3 & & 6 \\ & \times & \\ 1 & & 4 \\ & \times & \\ 2 & & 5 \\ 3 & — & 6 \end{array}$$

Proben: $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \circ \begin{pmatrix} -3 \\ 6 \\ -3 \end{pmatrix} = 1 \cdot (-3) + 2 \cdot 6 + 3 \cdot (-3) = -3 + 12 - 9 = 0$ ✓

$\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix} \circ \begin{pmatrix} -3 \\ 6 \\ -3 \end{pmatrix} = 4 \cdot (-3) + 5 \cdot 6 + 6 \cdot (-3) = -12 + 30 - 18 = 0$ ✓

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