Recovering Motivation in Mathematics: Teaching with Original Sources

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A commonly recognized feature of undergraduate mathematics instruction, as well as that in high schools and graduate schools, is the lack of motivation we provide for abstract concepts. We often introduce an idea by way of another equally unmotivated abstract concept, or with some practical, often trivial, "real world" application as justification for what we are going to inflict upon our students. As a result, most students view mathematics as a game with arbitrary rules, set by us, that is unconnected to anything.

Even if we could convince students that the things we teach are indeed useful, why should that make it interesting to them? We have sold mathematics short by presenting it only as "the language of science", since the mere utility of a subject does not necessarily generate any excitement for it. For many the excitement comes from the creative, artistic aspect of the subject, and its intellectual fascination.

Another serious drawback to our present approach is that it deprives students of the sense that mathematics is a process. On a small scale, modern textbooks and typical methods of instruction fail to illustrate the way mathematicians actually think about and work on problems. On a larger scale, we deprive students of the longterm process by which a mathematical theory emerges from struggling with one or more central problems, often over many centuries. These processes, however, are the very things that we want them to understand. No wonder that so many people, even those who have had a fair amount of mathematics in high school or college, are incredulous when one tells them that, yes indeed, there is a lot of research going on in present day mathematics, and that this is what really makes mathematicians tick. They have only seen a seemingly petrified structure, with no remaining trace of its creative human origins, and all at least 100 years old.

Mathematicians know, of course, that we owe much to the work of our predecessors, not just in the obvious way, but as an ongoing source of inspiration for our own research: There are still many insights to be found in the work of Gauss, Weierstrass and Hilbert, and even as far back as Euler. For a novelist, poet, painter or philosopher such observations would be old news, since their disciplines have long recognized the importance of studying the original work, techniques and perspectives of classical masters. And in so doing, they are never removed from an understanding of how people have struggled, and have created works of art. Young artists thus see themselves as part of a creative tradition. Unfortunately, we have lost this sense of tradition in our discipline, and, ironically, we can perhaps blame much of this loss on the dazzling explosion of mathematics in this century. It is time we step back from our accomplishments and recapture a historical perspective.

If it were just a matter of informing our students that "all this comes from somewhere", then the usual remedy of offering a course on the history of mathematics (and maybe making them take it) might at first thought seem enough. But such courses tend to marginalize and eviscerate the very subject matter they champion, generally talking *about* mathematics without actually *doing* mathematics. In addition, many history of mathematics courses, as well as books on the history of mathematics, spend an inordinate amount of time on pre-Renaissance mathematics, what Gian-Carlo Rota calls paleontology, and hardly any time at all on late nineteenth and twentieth century mathematics. While, for instance, Babylonian mathematics is very interesting, it is hardly relevant to the things most students learn in their courses.

Neither will it suffice simply to add historical biography or commentary to the mathematics courses we teach, since, while such add-ons may provide a human dimension to the subject matter, they shed very little light on the mathematics. Instead, we contend it is necessary to integrate firmly the study of original sources into all our courses, presenting these sources to motivate the modern theories they have spawned. Study of original writings of the past is essential in order to understand where the subject came from, how it is currently evolving, and where it might go. In the words of Abel, "It appears to me that if one wants to make progress in mathematics, one should study the masters and not the pupils."

During the past seven years, we have used original sources with a wide variety of students: undergraduates at all levels, both in courses already in the curriculum and in specially designed courses based entirely on original sources, and with exceptional high school students in summer workshops. The results have been extraordinary. Students begin to view mathematics in a new way, and also see themselves differently in relation to it. Mathematics is no longer a collection of arcana, unrelated within and unconnected to anything without, but becomes a whole, an artform.

So how do we use original sources in our teaching? Certainly almost every mathematical idea is built upon a succession of preceding ideas. And as one goes back along this chain, the motivation for a problem which started the journey becomes ever clearer, with several works in the chain often standing out as milestones on the road toward our present knowledge. By working through these original sources which discuss and solve, or attempt to solve, antecedent problems, students discover the roots of modern problems, ideas, and concepts, even whole subjects. They also see the obstacles that earlier thinkers had to clear in order to move ahead, and thereby gain insight into current problems and how to approach them.

Then why not read a modern text that lays out this grand scheme? Why study original writings? For two reasons. First, by reading original sources students are brought as close as possible to the experience of mathematical creation, without an intermediary interpreter. They see and feel the tenacity, the false starts and triumphs of its practitioners, the salient leaps which revolutionize fields and lead the way to the next cycle of tumult and passage.

The second reason is more subtle and perhaps derivative of the first, but profound nonetheless. When students read original sources, they are initiated into the way mathematics is practiced: through research, publication, and discussion. Mathematicians at the cutting edge of their field don't read textbooks; they read research papers. So our students too should read papers from their edge. Students can read the sources in a combination of individual work, small groups, and whole class exploration, after we preface their reading with an overview and alert them to particularly difficult parts. Discussion gradually spreads to the whole class which then reconstructs the argument, ponders the consequences of the result, and asks "Where do we go from here?". This emulates in large part the dynamic of research mathematics. Our students understand that we believe in them enough to ask them to confront the sources as we would, and their response to this faith is manifest in the heightened intensity of their motivation and study, and in the spirit that drives their work.

What relevance does this approach have to our current undergraduate curriculum? Our courses often have syllabi that need to be "covered" for use in the next course down the line. These requirements should not and, we claim, need not stand in the way of providing real intellectual motivation for the material at hand. When introducing graphs in a course on discrete mathematics, students may appreciate the power of a little bit of abstract language when they see a two-line resolution of the Königsberg Bridge Problem, after having slugged through Euler's lengthy solution; and along the way they get to see the first graph theory paper. Or, in a first course on group theory, we feel there can be no better introduction to the subject than Arthur Cayley's paper abstracting for the first time the concept of a "group" from numerous examples abounding in contemporary research of the day. Here too, comparison with a passage out of Gauss' *Disquisitiones Arithmeticae* might once again drive home the point that a little bit of abstraction goes a long way. In a beginning real analysis course, there truly is no more lucid and well-motivated insight into the true nature of the real numbers than in Dedekind's original tract on the subject, where he explains how his exploration of their essential qualities emerged directly from dissatisfaction with his lectures to calculus students. And in calculus, students can benefit in both understanding and motivation from seeing firsthand the struggles of Archimedes, Cavalieri, Torricelli, Leibniz, and Cauchy, as the subject moved from elaborate ad hoc exhaustion methods, through slippery but effective indivisibles, to Leibniz's original geometric proof with indivisibles of the fundamental theorem for computing areas, culminating in an almost modern view of the integral in the work of Cauchy.

There is in fact a vast supply of sources that illustrates and brings alive almost every concept taught to students at any level. Our experience is that, if carefully chosen, original sources can be accessible to and highly enriching for both students and instructors. Excellent original sources can be found in published collected works of many mathematicians from ancient times to the twentieth century, and in sourcebook collections edited by R. Calinger, J. Fauvel and J. Gray, H. O. Midonick, D. E. Smith, D. J. Struik, N. Biggs, G. Birkhoff, and J. Van Heijenoort.

For more detailed information on the particular sources we have used in our teaching, and on our pedagogical techniques, see the following articles:

- R. Laubenbacher and D. Pengelley, Great Problems of Mathematics: A Course Based on Original Sources, *American Mathematical Monthly* **99** (1992) 313-317.

- R. Laubenbacher and D. Pengelley, Mathematical Masterpieces: Teaching with Original Sources, in *Vita Mathematica: Historical Research and Integration with Teaching*, R. Calinger (ed.), MAA, Washington, DC, 1996, pp. 257-260.
- R. Laubenbacher and M. Siddoway, Great Problems of Mathematics: A Summer Workshop for High School Students, *College Mathematics Journal* **25** (1994), 112-114.

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