Cálculo vectorial

Ejercicios unidad 13.3 Derivadas Parciales

En los ejercicios 9-40, hallar las dos derivadas parciales de primer orden.

15) $Z = x^2 - 4xy + 3y^2$

$$\frac{\partial f}{\partial x} = 2x - 4y$$

$$\frac{\partial f}{\partial y} = -4x + 6y$$

23) $Z = \ln(x^2 + y^2)$

$$\frac{\partial Z}{\partial x} = \frac{2x}{x^2+y^2} \quad ; \quad \frac{\partial Z}{\partial y} = \frac{2y}{x^2+y^2}$$

27) $h(x,y) = e^{-(x^2+y^2)}$

$$\frac{\partial h}{\partial x} = e^{-(x^2+y^2)} \cdot -(2x) \quad ; \quad \frac{\partial h}{\partial y} = e^{-(x^2+y^2)} \cdot -(2y)$$

$$= -2x\, e^{-(x^2+y^2)} \qquad\qquad = -2y\, e^{-(x^2+y^2)}$$

35) $z = e^y \operatorname{sen} xy$

$$\frac{\partial z}{\partial x} = e^y \cos(xy) \cdot (x \cdot 0 + y(1)) + \operatorname{sen} xy\, e^y \cdot (0)$$

$$= y e^y \cos(xy)$$

$$\frac{\partial z}{\partial y} = e^y \cos(xy) \cdot (x(1) + y(0)) + \operatorname{sen} xy\, e^y \cdot 1$$

$$= x e^y \cos(xy) + e^y \operatorname{sen} xy$$

37) $z = \operatorname{senh}(2x + 3y)$

$$\frac{\partial z}{\partial x} = \cosh(2x+3y) \cdot (2(1) + 3(0))$$

$$= \cosh(2x+3y) \cdot 2$$

$$= 2\cosh(2x+3y)$$

$$\frac{\partial z}{\partial y} = \cosh(2x+3y) \cdot (2(0) + 3(1))$$

$$= 3\cosh(2x+3y)$$

En los ejercicios 45 a 52, evaluar f_x y f_y en el Punto dado

47) $f(x,y) = \cos(2x-y)$, $\left(\frac{\pi}{4}, \frac{\pi}{3}\right)$

$$\frac{\partial f}{\partial x} = -\operatorname{sen}(2x-y)\cdot(2)$$
$$= -2\operatorname{sen}(2x-y)$$

$$\left.\frac{\partial f}{\partial x}\right|_{(\pi/4,\pi/3)} = -2\operatorname{sen}\left(2(\pi/4)-\pi/3\right)$$
$$= -2\operatorname{sen}\left(\pi/2-\pi/3\right)$$
$$= -2\operatorname{sen}\left(\frac{3\pi-2\pi}{6}\right) = -2\operatorname{sen}\left(\pi/6\right)$$
$$= -2\cdot 1/2 = \boxed{-1}$$

$$\frac{\partial f}{\partial y} = -\operatorname{sen}(2x-y)\cdot(-1)$$
$$= \operatorname{sen}(2x-y)$$

$$\left.\frac{\partial f}{\partial y}\right|_{(\pi/4,\pi/3)} = \operatorname{sen}\left(2(\pi/4)-\pi/3\right)$$
$$= \operatorname{sen}\left(\pi/2-\pi/3\right)$$
$$= \operatorname{sen}\left(\pi/6\right)$$
$$= 1/2$$

Calcular las Pendientes de la superficie en las direcciones de x y de y en el Punto dado.

53) $g(x,y) = 4 - x^2 - y^2 \quad , \quad (1,1,2)$

$$\frac{\partial g}{\partial x} = -2x \Big|_{(1,1)} = -2(1) = \boxed{-2}$$

$$\frac{\partial g}{\partial y} = -2y \Big|_{(1,1)} = -2(1) = \boxed{-2}$$

En los ejercicios 59 a 64, Calcular las derivadas Parciales de Primer orden con respecto a $x, y, y\ z$.

59) $H(x,y,z) = \text{sen}(x + 2y + 3z)$

$$\frac{\partial H}{\partial x} = \cos(x+2y+3z) \cdot (1)$$
$$= \cos(x+2y+3z)$$

$$\frac{\partial H}{\partial y} = \cos(x+2y+3z) \cdot (2)$$
$$= 2\cos(x+2y+3z)$$

$$\frac{\partial H}{\partial z} = \cos(x+2y+3z) \cdot (3)$$
$$= 3\cos(x+2y+3z)$$

61) $w = \sqrt{x^2+y^2+z^2} = (x^2+y^2+z^2)^{1/2}$

$$\frac{\partial w}{\partial x} = \frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot (2x)$$

$$= \frac{2x}{2\sqrt{x^2+y^2+z^2}} = \frac{x}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial w}{\partial y} = \frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot (2y)$$

$$= \frac{2y}{2\sqrt{x^2+y^2+z^2}} = \frac{y}{\sqrt{x^2+y^2+z^2}}$$

$$\frac{\partial w}{\partial z} = \frac{1}{2}(x^2+y^2+z^2)^{-1/2} \cdot (2z)$$

$$= \frac{2z}{2\sqrt{x^2+y^2+z^2}} = \frac{z}{\sqrt{x^2+y^2+z^2}}$$

En los ejercicios 65-70, evaluar f_x, f_y y f_z en el punto dado.

66) $f(x,y,z) = x^2y^3 + 2xyz - 3yz$, $(-2,1,2)$

$$\frac{\partial f}{\partial x} = [x^2 \cdot 0 + y^3 2x] + [2yz] - [3(y \cdot 0 + z \cdot 0)]$$

$$= y^3 2x + 2yz \; ; \; \left.\frac{\partial f}{\partial x}\right|_{(-2,1,2)} = (1)^3(-2)(2) + 2(1)(2) = -4+4 = \boxed{0}$$

$$\frac{\partial f}{\partial y} = x^2 3y^2 + 2xz - 3z$$

$$\left.\frac{\partial f}{\partial y}\right|_{(-2,1,2)} = (-2)^2 3(1)^2 + 2(-2)(2) - 3(2)$$

$$= 12 - 8 - 6$$

$$= \boxed{-2}$$

$$\frac{\partial f}{\partial z} = 2xy - 3y$$

$$= \left.\frac{\partial f}{\partial z}\right|_{(-2,1,2)} = 2(-2)(1) - 3(1)$$

$$= -4 - 3 = \boxed{-7}$$

70) $f(x,y,z) = \sqrt{3x^2+y^2-2z^2}, \quad (1,-2,1)$

$$\frac{\partial f}{\partial x} = \frac{1}{2}(3x^2+y^2-2z^2)^{-1/2} \cdot (6x)$$

$$= \frac{3x}{\sqrt{3x^2+y^2-2z^2}}$$

$$\left.\frac{\partial f}{\partial x}\right|_{(1,-2,1)} = \frac{3(1)}{\sqrt{3(1)^2+(-2)^2-2(1)^2}} = \frac{3}{\sqrt{3+4-2}} = \frac{3}{\sqrt{5}}$$

$$\frac{\partial f}{\partial y} = \frac{1}{2}(3x^2+y^2-2z^2)^{-1/2} \cdot (2y)$$

$$= \frac{y}{\sqrt{3x^2+y^2-2z^2}}$$

$$\left.\frac{\partial f}{\partial y}\right|_{(1,-2,1)} = \frac{-2}{\sqrt{3(1)^2+(-2)^2-2(1)^2}} = \frac{-2}{\sqrt{5}}$$

$$\frac{\partial f}{\partial z} = \frac{1}{2}\left(3x^2+y^2-2z^2\right)^{-1/2}\cdot(-4z)$$

$$= \frac{-2z}{\sqrt{3x^2+y^2-2z^2}}$$

$$\left.\frac{\partial f}{\partial z}\right|_{(1,-2,1)} = \frac{-2(1)}{\sqrt{5}} = \frac{-2}{\sqrt{5}}$$