14.1 Ejercicios

- Evaluar la integral iterada

(13) $$\int_1^2\int_0^4 (x^2-2y^2)\,dx\,dy \rightarrow \int_1^2 \left[\frac{x^3}{3}-2xy^2\right]_0^4 dy = \boxed{\frac{8}{3}}$$

$$\int_0^4 (x^2-2y^2)\,dx = \left[\frac{x^3}{3}-2xy^2\right]_0^4 = \left(\frac{(4)^3}{3}-2(4)y^2\right)-(0)$$

$$= \boxed{\frac{64}{3}-8y^2}$$

$$\int_1^2 \left(\frac{x^3}{3}-2xy^2\right)_0^4 dy = \int_1^2 \left(\frac{64}{3}-8y^2\right) dy = \int_1^2 \frac{64}{3}\,dy - \int_1^2 8y^2\,dy$$

$$\left[\frac{64}{3}y-\frac{16y^3}{3}\right]_1^2 = \left[\frac{64(2)}{3}-\frac{16(2)}{3}\right]-\left[\frac{64}{3}-\frac{8}{3}\right]$$

$$=\left(\frac{128}{3}-\frac{64}{3}\right)-\left(\frac{64}{3}-\frac{8}{3}\right)=\frac{64}{3}-\frac{56}{3}$$

$$=\boxed{8/3}$$

(17) $$\int_0^\pi\int_0^{\operatorname{sen}x} (1+\cos x)\,dy\,dx.$$

$$\int_0^{\operatorname{sen}x}(1+\cos x)\,dy = \int_0^{\operatorname{sen}x} 1\,dy + \int_0^{\operatorname{sen}x}\cos x\,dy$$

$$=\left[y+y\cos x\right]_0^{\operatorname{sen}x}$$

$= [(\text{sen}\, x) + (\text{sen}\, x)(\cos x)] - [0 + (0)\cos x]$

$= \boxed{\text{sen}\, x + \text{sen}\, x \cos x}$

$$\int_0^{\pi} (\text{sen}\, x + \text{sen}\, x \cos x)\, dx = \int_0^{\pi} \text{sen}\, x\, dx + \int_0^{\pi} \text{sen}\, x \cos x\, dx$$

$$= \left[-\cos x\right]_0^{\pi} + \left[\frac{1}{2} \sin^2\right]_0^{\pi} = \left[-\cos x + \frac{1}{2} \sin^2\right]_0^{\pi}$$

$$= -\cos \pi + \frac{1 \sin^2 \pi}{2} = 1 + 1 = \boxed{2}$$

(25) $$\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{2}{\sqrt{4-y}}\, dx\, dy = \int_0^{\sqrt{4-y^2}} \frac{2}{\sqrt{4-y}}\, dx$$

$$= \left.\frac{2x}{\sqrt{4-y}}\right]_0^{\sqrt{4-y^2}} = \int_0^2 \left.\frac{2x}{\sqrt{4-y}}\right]_0^{\sqrt{4-y}} dy$$

$$\frac{2(\sqrt{4-y^2})}{\sqrt{4-y^2}} = \int_0^2 2\, dy = 2y\Big]_0^2 = 2(2) = \boxed{4}$$

$= [(\operatorname{sen} x) + (\operatorname{sen} x)(\cos x)] - [0 + (0)\cos x]$

$= \boxed{\operatorname{sen} x + \operatorname{sen} x \cos x}$

$\int_0^{\pi} (\operatorname{sen} x + \operatorname{sen} x \cos x)\, dx = \int_0^{\pi} \operatorname{sen} x\, dx + \int_0^{\pi} \operatorname{sen} x \cos x\, dx$

$= \left[-\cos x\right]_0^{\pi} + \left[\frac{1}{2}\sin^2\right]_0^{\pi} = \left[-\cos x + \frac{1}{2}\operatorname{sen}^2\right]_0^{\pi}$

$= -\cos \pi + \frac{1}{2}\sin^2 \pi = 1 + 1 = \boxed{2}$

25 $\int_0^2 \int_0^{\sqrt{4-y^2}} \frac{2}{\sqrt{4-y}}\, dx\, dy = \int_0^{\sqrt{4-y^2}} \frac{2}{\sqrt{4-y}}\, dx$

$= \frac{2x}{\sqrt{4-y}}\Big]_0^{\sqrt{4-y^2}} = \int_0^2 \frac{2x}{\sqrt{4-y}}\Big]_0^{\sqrt{4-y}} dy$

$\frac{2(\sqrt{4-y^2})}{\sqrt{4-y^2}} = \int_0^2 2\, dy = 2y\Big]_0^2 = 2(2) = \boxed{4}$

$= [(\sin x) + (\sin x)(\cos x)] - [0 + (0)\cos x]$

$= \boxed{\sin x + \sin x \cos x}$

$$\int_0^{\pi} (\sin x + \sin x \cos x)\, dx = \int_0^{\pi} \sin x\, dx + \int_0^{\pi} \sin x \cos x\, dx$$

$$= \Big[-\cos x\Big]_0^{\pi} + \Big[\frac{1}{2}\sin^2\Big]_0^{\pi} = \Big[-\cos x + \frac{1}{2}\sin^2\Big]_0^{\pi}$$

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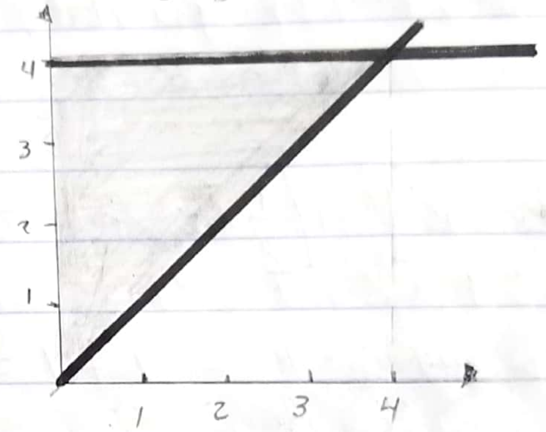
• Dibujar la región R de integración y cambiar el orden de integración.

(47) $\int_0^4 \int_0^y F(x,y)\,dx\,dy = \int_0^4 \int_x^4 F(x,y)\,dy\,dx$

$0 \le x \le y$

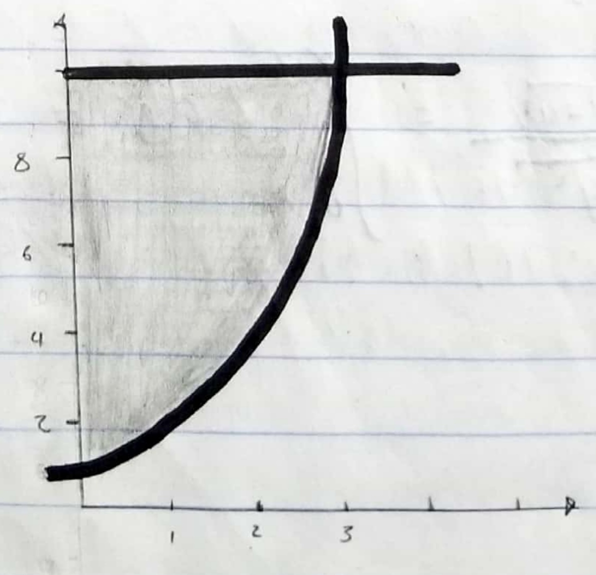
$0 \le y \le 4$



(51) $\int_1^{10} \int_0^{\ln y} F(x,y)\,dx\,dy = \int_0^{\ln 10} \int_{e^x}^{10} F(x,y)\,dy\,dx$

$0 \le x \le \ln y$

$1 \le y \le 10$



(67) Trazar la región de integración. Después evaluar la integración iterada.

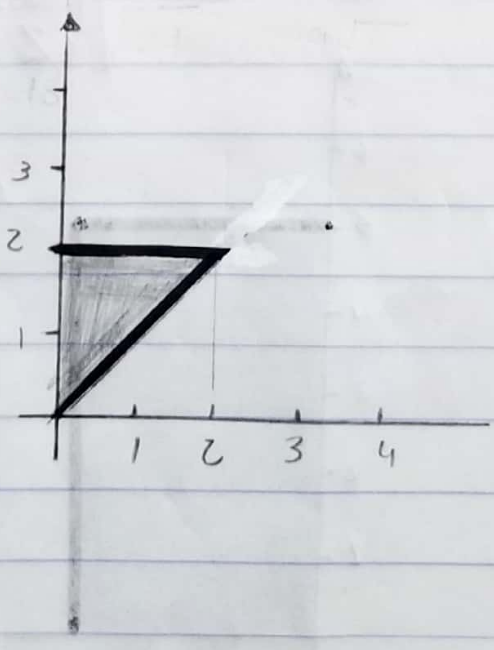
$$\int_0^2\int_x^2 x\sqrt{1+y^3}\,dy\,dx = \int_0^2\int_0^y x\sqrt{1+y^3}\,dx\,dy$$

$$= \int_0^y x\sqrt{1+y^3}\,dx = \left[\frac{x^2}{2}\sqrt{1+y^2}\right]_0^y = \frac{y^2}{2}\sqrt{1+y^3}$$

$$\int_0^2 \frac{y^2}{2}\sqrt{1+y^3}\,dy = \frac{1}{2}\int_0^2 y^2\sqrt{1+y^3}\,dy$$

$$\left[\frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)(1+y^3)^{3/2}\right]_0^2 = \frac{1}{2}\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)(1+2)^{3/2} - 0$$

$$\frac{1}{9}(27) - \frac{1}{9}(1) = \boxed{\frac{26}{9}}$$



- Dibujar la región R de integración y cambiar el orden de integración.

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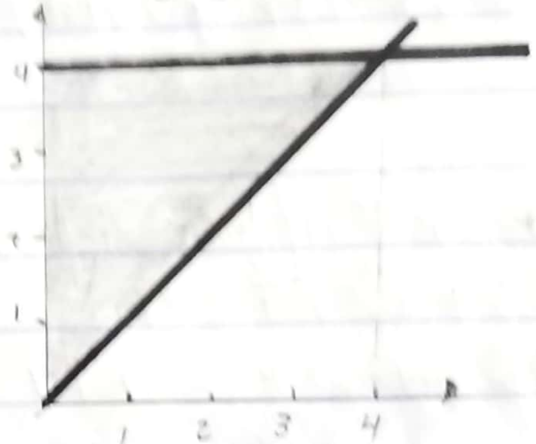

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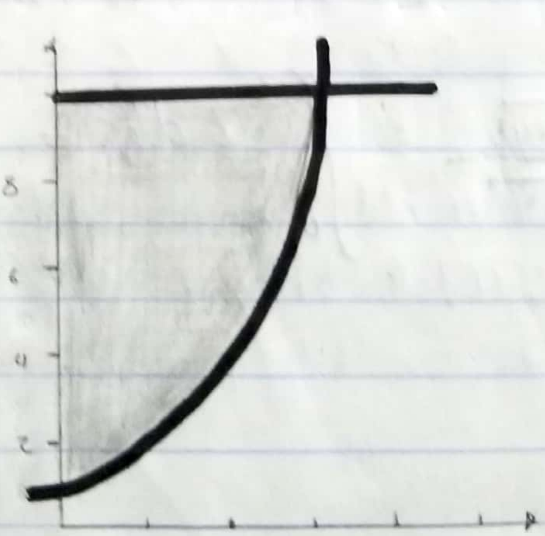

• Utilizar los multiplicadores de Lagrange para hallar todos los extremos de f indicados sujetos a dos restricción. Indicar x, y y z son no negativos.

(17) Maximizar $f(x,y,z) = xyz$
Restricción $x+y+z=32$, $x-y+z=0$.

$$\nabla f = \lambda \nabla g + \mu \nabla h$$
$$yz\,i + xz\,j + xy\,k = \lambda(i+j+k) + \mu(i-j+k).$$

$$\left.\begin{array}{l} yz = \lambda+\mu \\ xz = \lambda+\mu \\ xy = \lambda+\mu \end{array}\right\} yz = xy \to x = z.$$

$$\left.\begin{array}{l} x+y+z=32 \\ x-y+z=0 \end{array}\right\} 2x+2z=32$$

$$x = z = 8$$
$$y = 16$$

$P(8,8,16)$
$F(8,8,16) = 8(8)(16) = 1024.$