PIRAMIDA

2.E
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IMENOVANJE: pravilna / nepravilna, uspravna / kosa, broj strana (kuta) PIRAMIDA

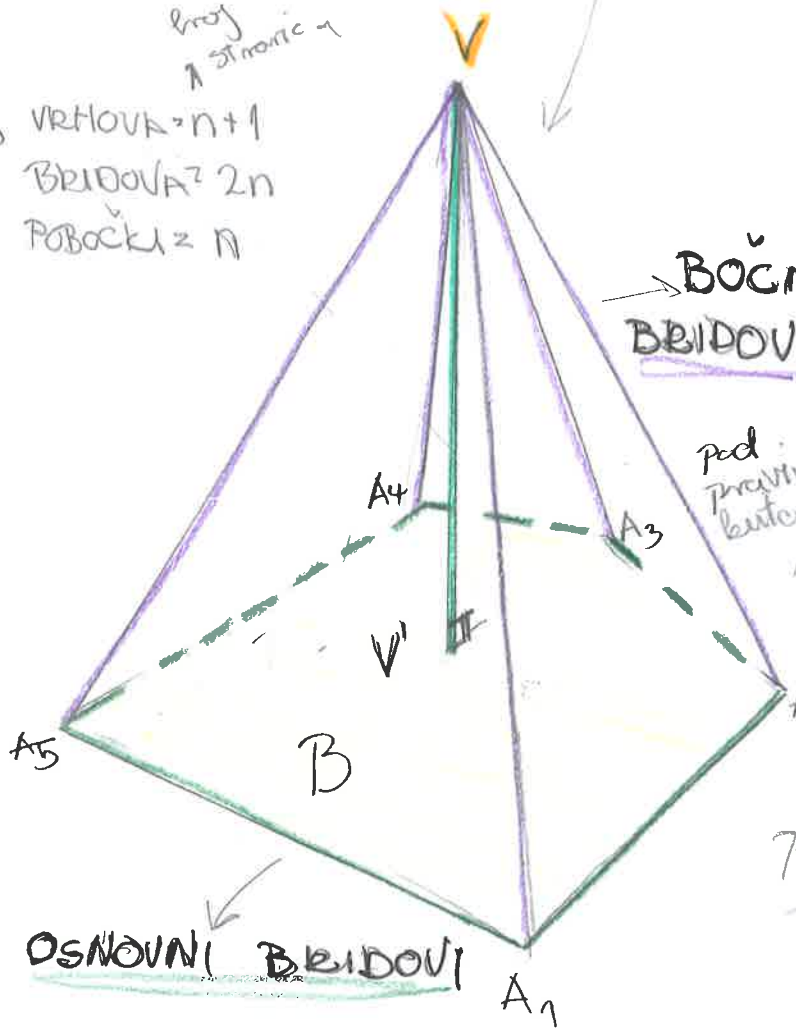
USPRAVNA NEPRAVILNA PETEROSTRANA PIRAMIDA

broj
n stranica

Broj vrhova $= n+1$

Broj bridova $= 2n$

Broj pobočki $= n$



BOČNI BRIDOVI

pod pravim kutom

OSNOVNI BRIDOVI

B = BAZA ili OSNOVKA

↳ nepravilni peterokut

V ⇒ VRH PIRAMIDE

VISINA PIRAMIDE

↳ udaljenost vrha V od ravnine baze V'

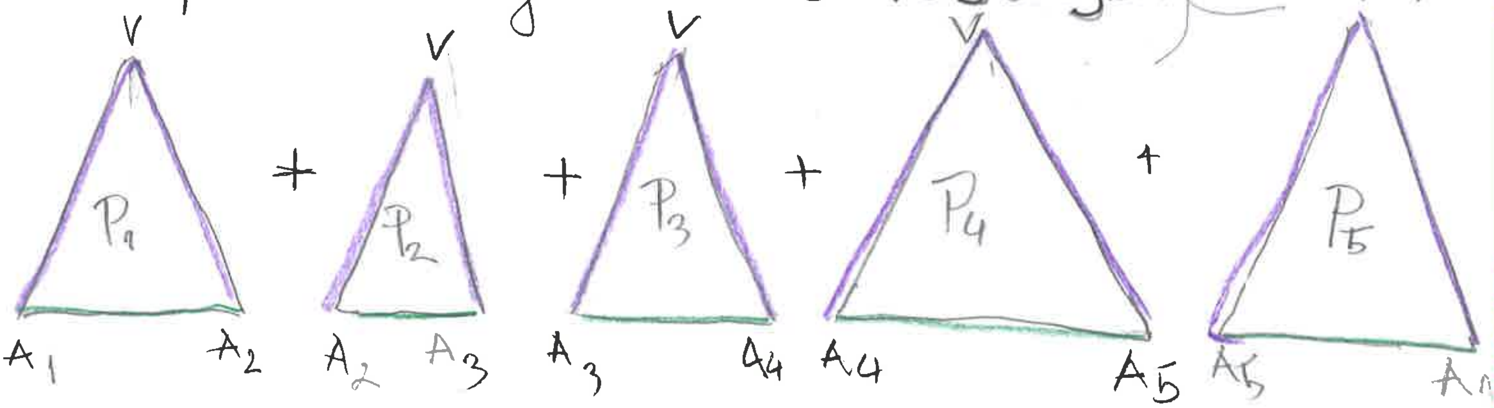
V' ⇒ ortogonalna projekcija vrha V na ravninu baze

samo jedna

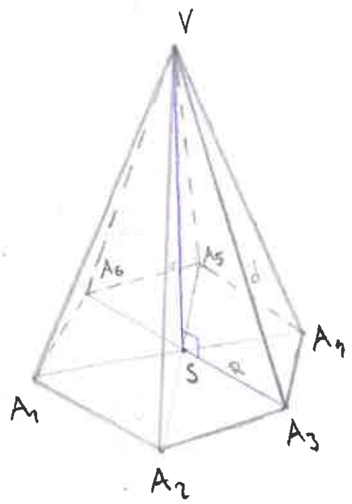
TROKUTI (A_1VA_2, A_2VA_3...) ⇒ zovu se strane (POBOČKA)

↳ sve pobočke zajedno čine POBOČJE

→ svih pet zajedno



PRAVILNA USPRAVNA 6strana PIRAMIDA



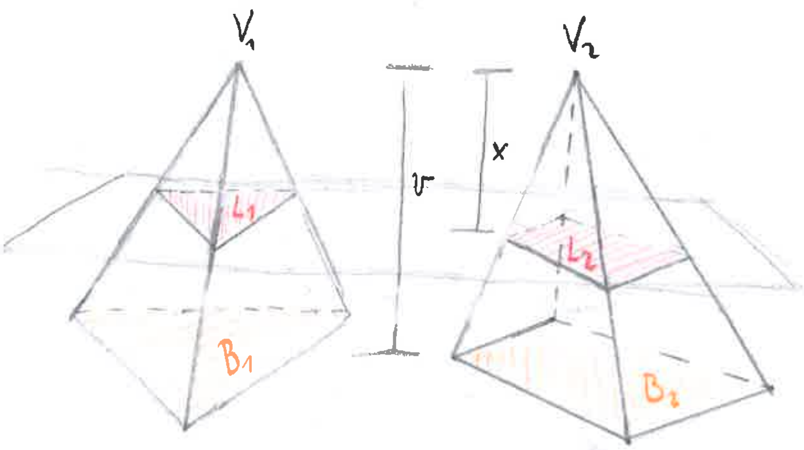
Piramida je PRAVILNA ako je njena baza pravilan mnogokut.

Za nju vrijedi:

- svi bočni bridovi su jednake duljine
  $d = \sqrt{R^2 + v^2}$

R je radijus opisane kružnice → „Euzebije"

OBUJAM PIRAMIDE (Cavalierijev princip)



$B_1 = B_2$

$L_1 = L_2$

2 zadana lika imaju jednaku visinu i površine baza

Povucimo raninu paralelnu s bazama koja je likove presjekla u L_1 i L_2

Za likove L_1 i B_1 kažemo da su HOMOTETIČNI

Središte homotetije je V_1, a koeficijent je $\left(\frac{x}{v}\right)^2$

$$\frac{L_1}{B_1} = \left(\frac{x}{v}\right)^2 = \frac{L_2}{B_2}$$

volumen ovisi samo o visini i bazi

* DVIJE PIRAMIDE ČIJE BAZE IMAJU JEDNAKU POVRŠINU I IMAJU JEDNAKE VISINE ⇒ IMAJU ISTI VOLUMEN

→ oblik baze ne utječe na volumen

FORMULA ZA OBUJAM I OPLOŠJE PIRAMIDE

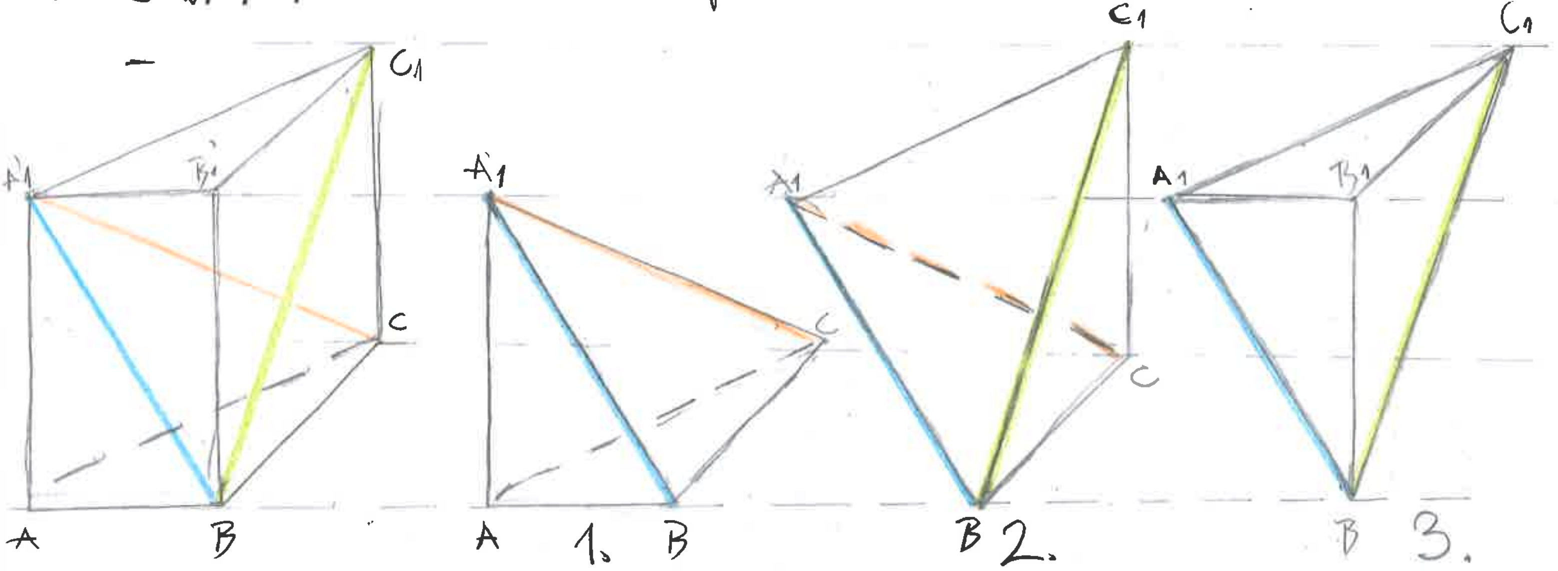
Oplošje je površina svih likova koji ograđuju piramidu.

↳ Računa se kao zbroj površina svih strana piramide

$O = B + P$ → zbroj svih pobočaka $(P_1 + P_2 + P_3 \ldots)$

B – površina donjeg lika

VOLUMEN - koliki dio prostora zauzima ⇒ volumen = obujam



⇒ PLOŠNE DIJAGONALE ⇒ nazad se može složiti prateći boje dijagonala

↳ različite duljine

dijagonale po pobočkama

- Rastavili smo trostranu prizmu na tri piramide. jednaki volumen

$ABCA_1$ i $A_1B_1C_1B$ ⇒ imaju iste baze i iste visine $|AA'| = |BB'|$

BCC_1A_1 i $A_1B_1C_1B$ ⇒ zamislimo da je A_1 vrh, a baze su

BCC_1 i B_1C_1B → obje su polovice paralelograma pa su jednake P

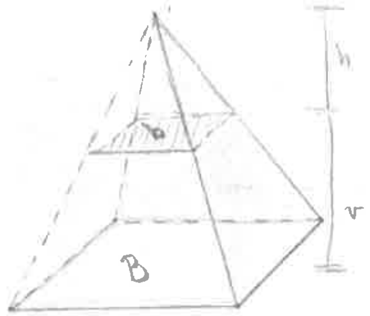
- a visine se podudaraju ⇒ $V_1 = V_2 = V_3$ → visina

Volumen piramide ⇒ $V = \frac{B \cdot h}{3}$ ← $\frac{1}{3}$ od prizme

↳ neovisno o bazi uvijek je $\frac{1}{3}$

B – površina donjeg lika

KRNJA USPRAVNA PIRAMIDA



Presjecimo ravninu paralelno njezinoj bazi i dobit ćemo 2 dijela: dopunjak (GORNJI) i krnju piramidu (DONJI)

h - visina dopunjka

v - visina krnje piramide

$h+v$ - visina cijele piramide

b - baza dopunjka

B - baza krnje piramide

Volumen krnje piramide:

- dobije se oduzimanjem volumena dopunka od volumena cijele piramide (visine $h+v$, baze B)

- $V = \frac{v}{3}\left(B + \sqrt{Bb} + b\right)$

PIRAMIDE - zadatci

8.3

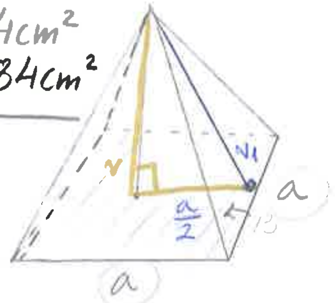
21. travnja 2024.

14.

$B = 484\,cm^2$

$O = 2684\,cm^2$

$V = ?$



B - kvadrat

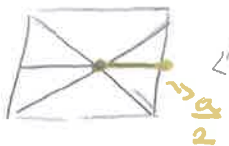
$B = a^2$

$a^2 = B \,/\sqrt{}$

$a = \sqrt{484}$

$a = 22\,cm$

BAZA:



→ pobočje = 4 pobočke

$O = B + P$

$P = O - B$

$P = 2684 - 484$

$P = 2200\,cm^2$

$P = 4 \cdot \frac{a \cdot v_1}{2}$

$2200 = 2 \cdot a \cdot v_1 \,/:2$

$1100 = a \cdot v_1$

$v_1 = \frac{1100}{a}$

$v_1 = \frac{1100}{22}$

$v_1 = 50\,cm$

$V = B \cdot h$

$v^2 = v_1^2 - \left(\frac{a}{2}\right)^2$

$v^2 = 50^2 - \left(\frac{22}{2}\right)^2$

$v^2 = 2500 - 121$

$v^2 = 2379 \,/\sqrt{}$

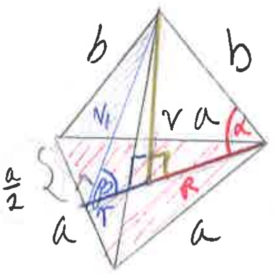
$v = \sqrt{2379}$

$v \approx 48.77\,cm$

$V = \frac{484 \cdot 48.77}{3}$

$V = 7868\,cm^3$

17.



$a = 12\,cm$

$b = 13\,cm$

$\alpha = ?$

$\beta = ?$

JEDNAKOSTRANIČAN TROKUT

$P = \frac{a^2\sqrt{3}}{4}$

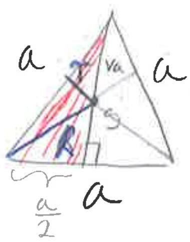
$P = \frac{a \cdot v}{2} = \frac{a \cdot \frac{\sqrt{3}a}{2}}{2} = \frac{\sqrt{3}a^2}{4}$

$\boxed{P = \frac{abc}{4R} = \frac{a^3}{4R}}$

$P = r \cdot s$

$s = \frac{a+b+c}{2}$

BAZA: → jednakostranični trokut



$v^2 = a^2 - \left(\frac{a}{2}\right)^2 = a^2 - \frac{a^2}{4}$

$= \frac{4a^2 - a^2}{4} = \frac{3a^2}{4}$

$v^2 = \frac{3a^2}{4} \,/\sqrt{}$

$v = \frac{\sqrt{3} \cdot a}{2}$

$\frac{a^2\sqrt{3}}{4} = \frac{a^3}{4R}$

$R \cdot a^2\sqrt{3} = a^3 \,/: a^2\sqrt{3}$

$R = \frac{a^3}{a^2\sqrt{3}}$

$R = \frac{a}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$R = \frac{a\sqrt{3}}{3}$

$R = \frac{12\sqrt{3}}{3}$

$R = 4\sqrt{3}\,cm$

$\cos\alpha = \frac{R}{b}$

$\cos\alpha = \frac{4\sqrt{3}}{13}$

$\cos\alpha = 0.53294$

$\alpha = 57°47'45''$

$v_1^2 = b^2 - \left(\frac{a}{2}\right)^2$

$v_1^2 = 13^2 - 6^2$

$v_1^2 = 169 - 36$

$v_1^2 = 133 \,/\sqrt{}$

$v_1 \approx 11.53\,cm$

$P = r \cdot s$

$P = r \cdot \frac{3a}{2}$

$\frac{a^2\sqrt{3}}{4} = \frac{3ar}{2} \,/\cdot 4$

$a^2\sqrt{3} = 6ar \,/: 6a$

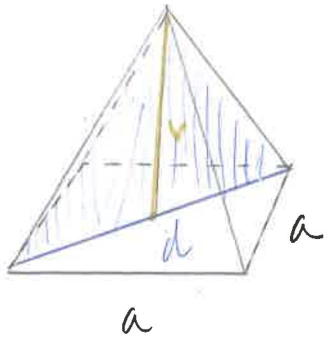
$r = \frac{a\sqrt{3}}{6} = \frac{12\sqrt{3}}{6} = 2\sqrt{3}\,cm$

$\cos\beta = \frac{r}{v_1}$

$\cos\beta = \frac{2\sqrt{3}}{\sqrt{133}}$

$\beta = 72°31'11''$

24.



$v = 18cm$
$P_D = 378cm^2$

$V = ?$

$P_D = \frac{d \cdot v}{2}$

$378 = \frac{d \cdot \cancel{18}^{9}}{\cancel{2}}$

$9d = 378 / :9$

$d = 42cm$

BAZA:

$d = a\sqrt{2}$

$a = \frac{d\sqrt{2}}{2}$

$a = \frac{42\sqrt{2}}{2}$

$a = 21\sqrt{2}cm$

$B = a^2$

$V = \frac{B \cdot v}{3}$

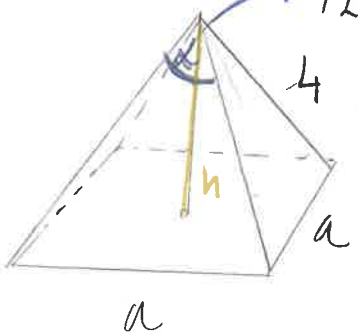
$= \frac{a^2 \cdot v}{3}$

$= \frac{(21\sqrt{2})^2 \cdot \cancel{18}^{6}}{\cancel{3}}$

$V = 441 \cdot 2 \cdot 6$

$V = 5292 cm^3$

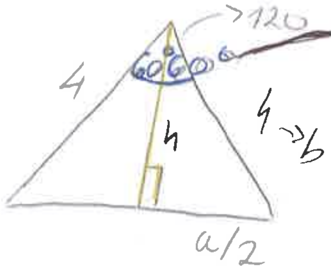
27.



$b = 4cm$
$\alpha = 120°$

$P = ?$

$P = P_1 \cdot 4 \to 16\sqrt{3}$



posto je jednakokračni

$P_1 = h \cdot \frac{a}{2} \to$

$P_1 = 2 \cdot 2\sqrt{3}$

$P_1 = 4\sqrt{3}$

$\sin(60°) = \frac{\frac{a}{2}}{4}$

$\frac{a}{2} = \sin(60°) \cdot 4$

$\frac{a}{2} = 2\sqrt{3} / \cdot 2$

$a = 4\sqrt{3} cm$

$\cos(60°) = \frac{h}{4}$

$h = \cos(60°) \cdot 4$

$h = 2cm$

$P = P_1 \cdot 4$

$P = 4\sqrt{3} \cdot 4$

$P = 16\sqrt{3} cm^2$