

# PRIMJER ISPITA

Zara Horvat  
Mia Jurković  
Tina Kovačić  
Petra Rečić  
Tonka Šičić  
2.e

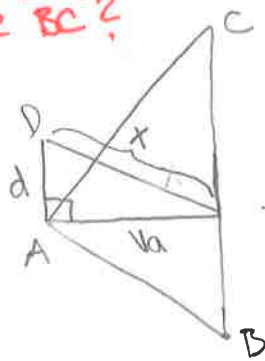
## Geometrija prostora, Poliedri i rotacijska tijela

20 bodova

17. zad, str. 70

1. Dane su duljine stranica trokuta ABC,  $|AB| = 13 \text{ cm}$ ,  $|BC| = 14 \text{ cm}$ ,  $|AC| = 15 \text{ cm}$ . Na okomnici (okomitost) podignutoj u vrhu A na ravninu trokuta nalazi se točka D te je  $|AD| = 5 \text{ cm}$ . Kolika je udaljenost točke D od stranice  $\overline{BC}$ ? (3 boda)

$$\begin{aligned} |AB| &= c = 13 \text{ cm} \\ |BC| &= a = 14 \text{ cm} \\ |AC| &= b = 15 \text{ cm} \\ |AD| &= 5 \text{ cm} = d \end{aligned}$$



(+1) (skica)



↳ pitagora!

= ?

$$P = \frac{a \cdot Va}{2}$$

$$\Rightarrow P = \frac{a \cdot Va}{2} \rightarrow Va = \frac{2P}{a} = 12$$

$$P = \sqrt{s(s-a)(s-b)(s-c)} = 84$$

(+1)

$$x = \sqrt{d^2 + Va^2}$$

$$\hookrightarrow s = \frac{a+b+c}{2} = 21$$

$$x = 13$$

(+1)

12. zad, str. 114.

2. Duljine osnovnih bridova uspravne trostrane piramide u omjeru su 9:10:17. Duljina bočnog brida je 16 cm, a površina pobočja  $1152 \text{ cm}^2$ . Koliki je obujam piramide? (3 boda)

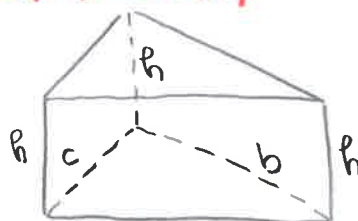
$$a:b:c = 9:10:17 \rightarrow a = 17k$$

$$h = 16$$

$$O = 1152$$

$$b = 10k$$

$$c = 9k$$



$$\begin{aligned} O &= P = P_1 + P_2 + P_3 \\ P &= 1152 \end{aligned}$$

$$\begin{cases} a = 17k \\ b = 10k \\ c = 9k \end{cases}$$

V = ?

$$P_1 = a \cdot h = 272k$$

$$P_2 = b \cdot h = 160k$$

$$P_3 = c \cdot h = 144k$$

$$\left. \begin{matrix} a \\ b \\ c \end{matrix} \right\} P = 576k$$

→

$$P = 0$$

$$L \rightarrow 0 = 2592 = 576k$$

$$2592 = 576k \quad | : 576$$

$$\underline{k = 2}$$

$$\rightarrow a = 17k = 34$$

$$\textcircled{+1} \quad b = 10k = 20$$

$$c = 9k = 18$$

$$V = B \cdot h$$

$\textcircled{+1}$

$$B = \sqrt{s(s-a)(s-b)(s-c)}$$

$$s = \frac{a+b+c}{2} = 36$$

$$\rightarrow B = 144$$

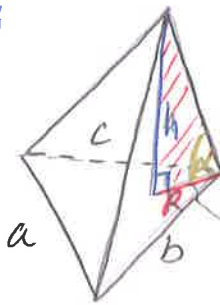
$$V = B \cdot h = 2304 \quad \textcircled{+1}$$

19. zad, str 125

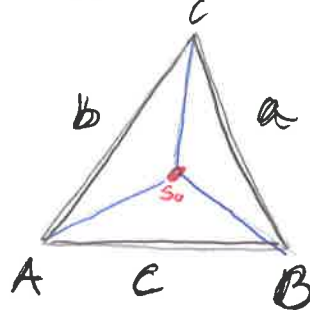
3. Koliki je obujam trostrane piramide kojoj je osnovica trokut sa stranicama dužina 13, 14, 15 cm, a svi su bočni bridovi prema osnovici priklonjeni pod  $70^\circ$ ? (4 boda)

$$\begin{aligned} a &= 13 \\ b &= 14 \\ c &= 15 \\ \varphi &= 70^\circ \\ \hline V &= ? \end{aligned}$$

skica:



BAZA:



$$V = \frac{B \cdot h}{3}$$

$$B = \sqrt{s \cdot (s-a) \cdot (s-b) \cdot (s-c)} = 84 \text{ cm}^2 \quad (+1)$$

$$s = \frac{13 + 14 + 15}{2} = 21$$

$$B = \frac{a \cdot b \cdot c}{4R} \quad | \cdot R$$

$$R = \frac{a \cdot b \cdot c}{4B}$$

$$R = \frac{13 \cdot 14 \cdot 15}{4 \cdot 84}$$

$$R = 8.125 \text{ cm} = \frac{65}{8} \text{ cm} \quad (+1)$$

$$\rightarrow \operatorname{tg} \alpha = \frac{h}{R}$$

$$\frac{h}{R} = \operatorname{tg} \alpha \quad | \cdot R$$

$$h = \operatorname{tg} \alpha \cdot R$$

$$h \approx 22.3233 \text{ cm} \\ = 22.32325403 \text{ cm} \quad (+1)$$

$$V = \frac{B \cdot h}{3}$$

$$(\cancel{V = 84 \cdot 22.2332})^{hl}$$

$$V = \frac{84 \cdot 22.32325403}{3}$$

$$V \approx 625.05 \text{ cm}^3 \quad (+1)$$



5 zadatka, 134 str

(4)

Plast valjka ima površinu  $72\pi\text{cm}^2$ , a opseg osnovke je  $12\pi\text{cm}$ . Odredite oplošje i obujam tog valjka.

$P_p$  - površina plasta     $O_{os}$  - opseg osnovke

(2 boda)

$$P_p = 72\pi\text{cm}^2$$

$$O_{os} = 12\pi\text{cm}$$

O?    V?

$$P_p = 2r\pi h$$

$$O_{os} = 2r\pi$$

$$72\pi = 2r\pi h$$

$$r = \frac{O}{2\pi} = \frac{12\pi}{2\pi} = 6$$

$$12h = 72 / : 12$$

$$h = 6$$

$$O = 2r\pi(r+h)$$

$$V = B \cdot h$$

$$O = 144\pi \quad (+1)$$

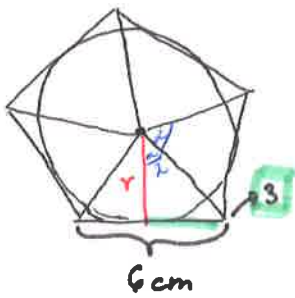
$$V = r^2\pi \cdot h$$

$$V = 216\pi \quad (+1)$$

$$B = r^2\pi$$

Gradivo .hr → Valjak - matematika 2 ~~(dodatni zadatke)~~

Valjak je upisan u uspravnu pravilnu peterostranu prizmu kojoj su osnovni bridovi dužine 6cm, a visina 8cm. Koliki je volumen valjka? (2 boda)



$$\alpha = \frac{360^\circ}{5} = 72^\circ$$

$$h = 8\text{cm}$$

$$\text{tg}\left(\frac{\alpha}{2}\right) = \frac{3}{r} / \cdot r$$

$$V = B \cdot h$$

$$V = r^2\pi \cdot h$$

$$r \cdot \text{tg}\left(\frac{\alpha}{2}\right) = 3$$

$$V = 4.1291^2 \cdot \pi \cdot 8$$

$$V = 136.40\pi \quad (+1)$$

$$r = \frac{3}{\text{tg}\left(\frac{\alpha}{2}\right)}$$

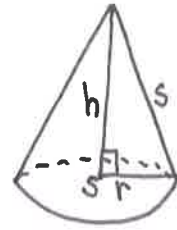
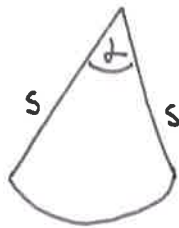
$$r = \frac{3}{\text{tg}\left(\frac{72^\circ}{2}\right)} = 4.1291 \quad (+1)$$



11. zad., str. 143.

5. Površina plašta uspravnog stožca iznosi  $20 \text{ cm}^2$ , a nakon razvijanja plašta u ravninu dobije se kružni isječak sa središnjim kutom  $72^\circ$ . Koliko je oplošje tog stožca? (3 boda)

$$\begin{aligned} P_k &= 20 \text{ cm}^2 \\ \alpha &= 72^\circ \\ O &=? \end{aligned}$$



$$O = r\pi(r+s)$$

$$\rightarrow O = B + P$$

$$O = r^2\pi + r\pi s$$

$$O = r\pi(r+s)$$

$$O = 1,13\pi(1,13 + 5,64)$$

$$\underline{O = 24 \text{ cm}^2} \quad (+1)$$

$$P_k = \frac{s^2 \pi \cdot \alpha}{360^\circ} \cdot \frac{360^\circ}{\alpha}$$

$$s = \sqrt{\frac{360^\circ \cdot P_k}{\pi \cdot \alpha}}$$

$$s = 5,64 \quad (+1)$$

$$O = \frac{s\pi\alpha}{180^\circ}$$

$$2r\pi = \frac{s\pi\alpha}{180^\circ} \cdot \frac{1}{2\pi}$$

$$r = \frac{s\pi\alpha}{180^\circ \cdot 2 \cdot \pi}$$

$$r = 1,13 \quad (+1)$$

51. zad., 116. str.

6. Koliki su površje i obujam pravilne šesterostrane prizme, ako je njen veći dijagonalni presjek kvadrat površine  $36\text{cm}^2$ ?

$$O = ? \quad V = ?$$

$$P_{\text{kvadrat}} = 36\text{cm}^2$$

$$P = a^2$$

$$d = \sqrt{P} = \sqrt{36}$$

$$d = 6$$

$$h = 6$$

$$a = \frac{d}{2} = \frac{6}{2}$$

$$a = 3$$

$$V = B \cdot h = \underline{81\sqrt{3}} \quad (+1)$$

$$P = \frac{a^2 \sqrt{3}}{4}$$

$$P = \frac{9\sqrt{3}}{4} \cdot 6$$

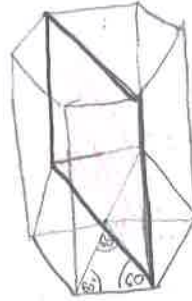
$$P = \frac{27\sqrt{3}}{2} = B \quad (+1)$$

$$O = P + 2B$$

$$O = 6 \cdot (3 \cdot 6) + \frac{27\sqrt{3}}{2}$$

$$O = 108 + 27\sqrt{3}$$

$$\underline{O = 157,77} \quad (+1)$$



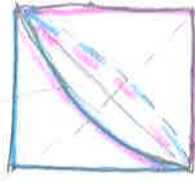


listić

7. Zadani je kvadrat i njegova stranica iznosi 6 cm. Kvadrat se rotira oko svoje dijagonale, izračunaj ovog lika? (3 boda)

$$\frac{a = 6 \text{ cm}}{V = ?}$$

skica:



(+1) (za prepoznavati bovu)

$a \rightarrow$  Imamo dva valjka

$\frac{\text{dijagonala}}{2}$

$$V_1 + V_2 = 2V$$

$$V = \frac{B \cdot h}{3}$$

$$= \frac{\left(\frac{a\sqrt{2}}{2}\right)^2 \pi \cdot \frac{a\sqrt{2}}{2}}{3}$$

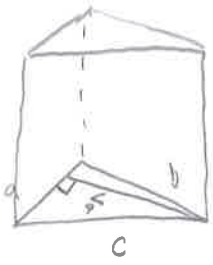
$$V = 2 \cdot V' \Rightarrow 36\sqrt{2}\pi$$

(+1)

$$V' = 18\sqrt{2}\pi$$



1. Osnovka uspravne prizme je trokut sa stranicama dužina 8, 9, 11. Visina prizme jednaka je najvećoj visini osnovke. Koliki je obujam prizme?



$$a = 8$$

$$b = 9$$

$$c = 11$$

$$h = V_a \quad (+1)$$

$$V = ?$$

$$B = \frac{a \cdot V_a}{2}$$

$$B = \frac{8 \cdot V_a}{2}$$

$$B = 4V_a$$

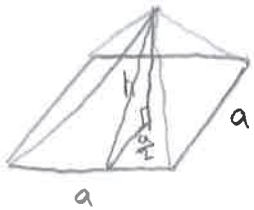
$$V_a = \frac{B}{4} = \frac{6\sqrt{35}}{4} = 8.87 \quad (+1)$$

$$B = \sqrt{s(s-a)(s-b)(s-c)} = 6\sqrt{35} \quad (+1)$$

$$V = B \cdot h$$

$$V = 6\sqrt{35} \cdot 8.87 = 319.47 \quad (+1)$$

2. Površina osnovke pravilne četverostrane piramide je  $484 \text{ cm}^2$ , a oplošje piramide iznosi  $2684 \text{ cm}^2$ . Izračunaj obujam piramide.



$$B = 484 \text{ cm}^2$$

$$O = 2684 \text{ cm}^2$$

$$V = ?$$

$$B = a^2$$

$$a = \sqrt{B} = 22$$

$$V = \frac{B \cdot h}{3}$$

$$P_d = \frac{O - B}{4} = 550 \quad (+1)$$

$$P_d = \frac{a \cdot V_a}{2}$$

$$V_a = \frac{2P_d}{a} = 50 \quad (+1)$$

$$h^2 = V_a^2 - \left(\frac{a}{2}\right)^2$$

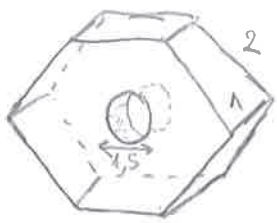
$$h = \sqrt{2500 - 121}$$

$$h = 48.77 \quad (+1)$$

$$V = \frac{484 \cdot 48.77}{3} = 7868.23 \quad (+1)$$



3. Izračunaj oplošje i obujam zadanog lika.



$$r = \frac{d}{2} = 0.75$$

$$a = 2$$

$$h = 1$$

$$V, O = ?$$

$$V = V_0 - V_1$$

$$V_0 = B \cdot h = 6\sqrt{3} \quad (+1)$$

$$B = 6 \cdot \sqrt{3}$$

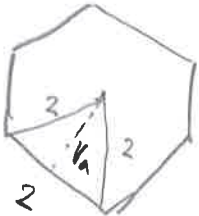
$$V_1 = r^2 \pi h = \frac{9}{16} \pi \quad (+1)$$

$$O = O_0 - 2B_0 + P_0 \quad (+1)$$

$$O = 2B + P - 2r^2 \pi + 2r \pi h$$

$$O = (2 \cdot 6\sqrt{3} + 6 \cdot 2) - (2r^2 \pi) + (2r \pi h)$$

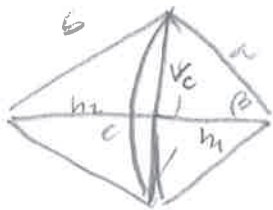
$$O = 33.96 \quad (+1)$$



$$2^2 = V_a^2 + 1^2$$

$$V_a = \sqrt{3}$$

4. Izračunaj volumen i oplošje rotirajućeg trokuta.



$$a = 15$$

$$b = 13$$

$$c = 14$$

$$\sin \beta = \frac{V_c}{a}$$

$$V_c = \sin \beta \cdot a$$

$$V_c = 12 \quad (+1)$$

$$h_1^2 = a^2 - V_c^2$$

$$h_1 = 9$$

$$h_2 = c - h_1$$

$$h_2 = 5$$

$$V = \frac{V_c^2 \cdot h_1 \cdot \pi}{3} + \frac{V_c^2 \cdot h_2 \cdot \pi}{3}$$

$$V = 672 \pi \quad (+1)$$

$$O = V_c \cdot \pi \cdot a + V_c \cdot \pi \cdot b$$

$$O = 336 \pi \quad (+1)$$

$$b^2 = a^2 + c^2 - 2ac \cdot \cos \beta$$

$$\cos \beta = \frac{b^2 - a^2 - c^2}{-2ac}$$

$$\beta = 53.13 \quad (+1)$$

2/17

Jan Marka, Šimun Dubravica,

Emanuel Boban, Marko Vujević, Luka Šutić z.e

Kvadraten piramida

1.  $h = 8 \text{ cm}$

~~$R = 10 \text{ cm}$~~

$O = B + P \quad B = a^2$

$(10 \text{ cm})^2 = (8 \text{ cm})^2 + \left(\frac{a}{2}\right)^2$

$a = \sqrt{2(10^2 - 8^2)}$

~~$a = 8$~~   ~~$a = 6 \text{ cm}$~~   $a = 12 \text{ cm}$

$B = a^2$

$B = 12^2$

$B = 144 \text{ cm}$

$O = 144 + 4 \frac{12 \cdot 10}{2} = 144 + 240$

$O = 384$

12

trastana piramida

2.

$a = 4$

$b = 13$

$c = 15$

$V = 240$

$O = ?$

~~$B =$~~   $s = \frac{4 + 13 + 15}{2} = 16$

$B = \sqrt{16(16-4)(16-13)(16-15)} = 24$

$V = B \cdot R \quad R = \frac{V}{B} = 10$

$O = 2B + R \cdot (a + R) + R \cdot (b + R) + R \cdot (c + R) = 368$

13

3. <sup>Rudder</sup>  
 $a_1 = 11$     $a_2 = 19$     $a_3 = 20$

~~$a^2 + b^2 = 11^2$~~   
 $b^2 + c^2 = 19^2$   
 $c^2 + a^2 = 20^2$

$$2a^2 + 2b^2 + 2c^2 = 11^2 + 19^2 + 20^2$$

$$D = \sqrt{a^2 + b^2 + c^2} = \sqrt{\frac{11^2 + 19^2 + 20^2}{2}} = 27$$

B

4. <sup>Rudder</sup>

$$P_1 = 20 \quad P_2 = 28 \quad P_3 = 35$$

$$V = ? \quad \begin{aligned} P_1 &= a \cdot b \\ P_2 &= b \cdot c \\ P_3 &= a \cdot c \end{aligned}$$

$$V = a \cdot b \cdot c$$

$$P_1 \cdot P_2 \cdot P_3 = (a \cdot b \cdot c)^2$$

$$V = \sqrt{P_1 \cdot P_2 \cdot P_3} = 140$$

13

Voljoh

5.

$$O = 112 \pi$$

$$r : R = 2 : 5$$

$$O = 2r\pi(r+R)$$

$$O' = 2 \cdot 2\pi(2+5) = 28\pi$$

$$R = \frac{O}{O'} = 4$$

$$r = 2 \cdot R = 8 \quad R = 5 \cdot R = 20$$

$$V = 8^2 \cdot 20\pi = 1280\pi$$

13





# Prijemni ispit: Poliedri i rotacijska tijela 16/16

Arhonas, Čunčić, Feljanić

1. Površina polučoka uspravne trostrane prizme iznose  $125 \text{ cm}^2$ ,  $400 \text{ cm}^2$  i  $945 \text{ cm}^2$ , a njena je visina  $25 \text{ cm}$ . Izračunaj oplošje i obujam prizme. [2]

$$P_1 = 125 \text{ cm}^2$$

$$P_2 = 400 \text{ cm}^2$$

$$P_3 = 945 \text{ cm}^2$$

$$h = 25 \text{ cm}$$

$$O, V = ?$$

$$P = P_1 + P_2 + P_3$$
$$= 2100 \text{ cm}^2$$

$$O = 2B + P$$
$$= 2520 \text{ cm}^2$$

$$P_1 = a \cdot h$$

$$125 = a \cdot 25 / 25$$

$$a = 14 \text{ cm}$$

$$P_2 = b \cdot h$$

$$400 = b \cdot 25 / 25$$

$$b = 28 \text{ cm}$$

$$B = \sqrt{s(s-a)(s-b)(s-c)} \quad (1)$$
$$= 210 \text{ cm}^2$$

$$V = B \cdot h = 5250 \text{ cm}^3 \quad (1)$$

$$P_3 = c \cdot h$$

$$945 = c \cdot 25 / 25$$

$$c = 39 \text{ cm}$$

$$s = \frac{a+b+c}{2} = 42 \text{ cm}$$

2. Zračnica je visine valjka koja iznosi  $8 \text{ cm}$  i prostorna dijagonala koja iznosi  $10 \text{ cm}$ . Izračunaj oplošje i volumen tijela. [3]

$$h = 8 \text{ cm}$$

$$D = 10 \text{ cm}$$

$$O, V = ?$$

$$D^2 = h^2 + d^2$$

$$100 = 64 + d^2$$

$$d^2 = 36 / \sqrt{\quad}$$

$$d = 6 \text{ cm}$$

$$d = 2r$$

$$6 = 2r / : 2$$

$$r = 3 \text{ cm} \quad (1)$$

$$B = \pi r^2$$
$$= 3^2 \pi$$
$$= 9\pi \text{ cm}^2$$

$$O = 2B + P$$
$$= 2 \cdot 9\pi + 48\pi$$
$$= 66\pi \text{ cm}^2 \quad (1)$$

$$P = 2\pi r \cdot h$$
$$= 2 \cdot 3\pi \cdot 8$$
$$= 48\pi \text{ cm}^2$$

$$V = B \cdot h$$
$$= 9\pi \cdot 8$$
$$= 72\pi \text{ cm}^3 \quad (1)$$

3. Volumen pravidelne čtverástrane (prizma) pyramidě je 3560 cm<sup>3</sup>. Radius kružnice, opisane baze je 15 cm. Povrch kryje pyramidě je polovicí baze pyramidě. [3]

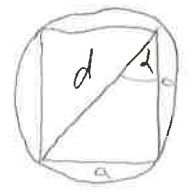
$$V = 3560 \text{ cm}^3$$

$$r = 15 \text{ cm}$$

$$B_R = \frac{B}{2}$$


---


$$V_R = ?$$



$$\sin(\alpha) = \frac{a}{d}$$

$$a = \sin(\alpha) \cdot d$$

$$a = \sin(45^\circ) \cdot 2r$$

$$a = 15\sqrt{2}$$

$$B = a^2$$

$$B = 450 \text{ cm}^2 \quad (1)$$

$$B_R = \frac{B}{2}$$

$$B_R = 225 \text{ cm}^2$$

$$V = \frac{B \cdot h}{3}$$

$$V_{KR} = \frac{h}{3} (B + \sqrt{0 \cdot B} + B)$$

$$h = \frac{3V}{B} = 23.73 \quad (1)$$

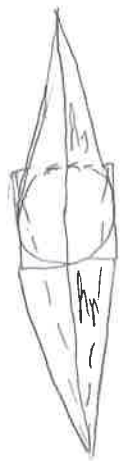
$$V_{KR} = 6284 \text{ cm}^3 \quad (1)$$

4. Upravní stájec i upravní čtverástrane (prizma) pyramidě imají jedake volumere. Baza stáje je upravní baze pyramidě, a výška výška je 18. Volumere su im jedake. Zpracuj im výšku. [3]

$$h_p + h_s = 18$$

$$V_p = V_s$$

$$h_p \cdot h_s = 1$$



$$a = 2r$$

$$h_s = 18 - h_p$$

$$h_p = 10.082 \quad (1)$$

$$V_p = V_s$$

$$\frac{a^2 \cdot h_p}{3} = \frac{r^2 \pi h_s}{3} \quad (1)$$

$$(2r)^2 h_p = r^2 \pi h_s$$

$$4r^2 \cdot h_p = r^2 \pi h_s \quad /: r^2$$

$$4h_p = \pi(18 - h_p)$$

$$4h_p = 18\pi - \pi h_p$$

$$4h_p + \pi h_p = 18\pi$$

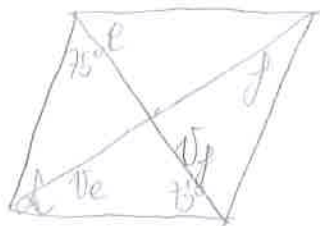
$$h_p(4 + \pi) = 18\pi \quad /: (4 + \pi)$$

$$h_p = \frac{18\pi}{4 + \pi} \approx 7.92 \quad (1)$$

5. Romb stranice  $a$  i silvastog kuta od  $30^\circ$  vrti se oko jedne pa oko druge dijagonale. Koliki je omjer obujama nastalih rotacijskih tijela?

$$\alpha = 30^\circ$$

$$\frac{V_e}{V_f} = ?$$



$$\tan 75^\circ = \frac{V_e}{\frac{e}{2}}$$

$$\tan 75^\circ = \frac{2 V_e}{e}$$

$$V_e = \frac{e \tan 75^\circ}{2} \quad (1)$$

$$V_e = \frac{\left(\frac{e}{2}\right)^2 \pi \cdot V_e}{3} \cdot 2$$

$$V_e = \frac{\frac{e^2}{4} \pi \cdot \frac{e \tan 75^\circ}{2}}{3} \cdot 2$$

$$V_e = \frac{\frac{e^3 \tan 75^\circ \pi}{8}}{3} \cdot 2$$

$$V_e = \frac{e^3 \tan 75^\circ \pi}{24 \pi} \cdot 2 \quad (1)$$

$$V_e = \frac{e^3 \tan 75^\circ \pi}{12} \quad (1)$$

$$V_f = \frac{\left(\frac{f}{2}\right)^2 \pi \cdot V_f}{3} \cdot 2$$

$$V_f = \frac{e}{2}$$

$$V_f = \frac{\frac{e^2 \tan^2 75^\circ \pi}{4} \cdot \frac{e}{2}}{3} \cdot 2$$

$$\frac{f}{2} = V_e = \frac{e \tan 75^\circ}{2} \quad (1)$$

$$V_f = \frac{\frac{e^3 \tan^2 75^\circ \pi}{8}}{3} \cdot 2$$

$$V_f = \frac{e^3 \tan^2 75^\circ \pi}{24 \pi} \cdot 2 \quad (1)$$

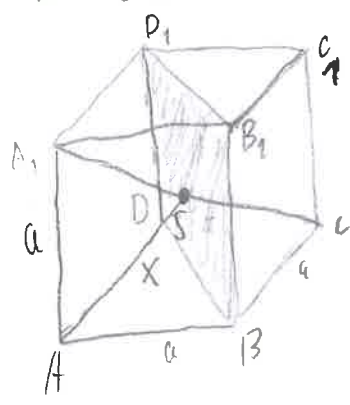
$$V_f = \frac{e^3 \tan^2 75^\circ \pi}{12} \quad (1)$$

$$\frac{V_e}{V_f} = \frac{\frac{e^3 \tan 75^\circ \pi}{12}}{\frac{e^3 \tan^2 75^\circ \pi}{12}} = \frac{1}{\tan 75^\circ} = 2 - \sqrt{3} \quad (1)$$



1.

Dana je kocka  $ABCD A_1 B_1 C_1 D_1$ . Odredi tačku u kojoj prostorna dijagonala  $A_1 C$  probada ravninu  $BB_1 D_1$  i izračunaj koliko je ta tačka udaljena od tačke  $A$  ako je duljina stranice kocke  $a$ .



ravnina probada u tački S (1)

$$|AC|^2 = a^2 + a^2$$

$$|AC| = a\sqrt{2}$$

$$|AS| = |A_1S| = x$$

$$x = ?$$

$$x = \frac{\sqrt{a^2 + (a\sqrt{2})^2}}{2}$$

$$x = \frac{a\sqrt{3}}{2} \quad (1)$$

/2

2. Iz kružnice poluprečnika 15 iznesen je kružni isječak sa središnjim kutom  $120^\circ$ . Izračunajte površinu slične plošte.

$s = 15$   
 $\alpha = 120^\circ$

$O, V = ?$

$O = r\pi(n + s)$

$r = 15$

$l = \frac{s\pi\alpha}{180^\circ} = 10\pi \quad (1)$



$l = O_B \quad (1)$

$O_B = 2r\pi$

$10\pi = 2r\pi$

$r = 5$

$O = 100\pi \quad (1)$

/3

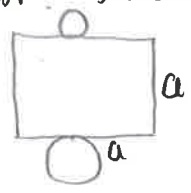
3. Plošt valjka načinjen je od kvadrata duljine stranice  $a$ . Koliki je poluprečnik cijevi u koju se savije taj kvadrat? Kolika je duljina te cijevi?

$h = a \quad (1)$

$a = O_B$

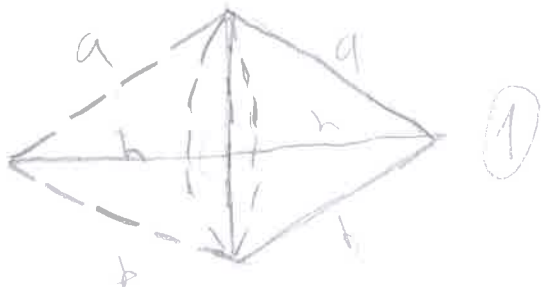
$a = 2r\pi$

$r = \frac{a}{2\pi} \quad (1)$



/2

4. Trokut sa stranicama dužine  $a=15$ ,  $b=13$ ;  $c=14$  se rotira oko stranice  $c$ . Izračunaj površinu nastalog tijela. Nacrtaj skicu!



$$0 = P_1 + P_2 \quad (1)$$

$$0 = h \pi a + h \pi b$$

$$\left. \begin{array}{l} a=15 \\ b=13 \\ c=14 \end{array} \right\} s=21$$

$$\frac{c \cdot h}{2} = \sqrt{s(s-a)(s-b)(s-c)} \quad (1)$$

$$\frac{14h}{2} = 84$$

$$7h = 84 / :7$$

$$\boxed{h=12} \quad (1)$$

$$0 = h \pi a + h \pi b$$

$$0 = h \pi (a+b)$$

$$\boxed{0=336\pi} \quad (1)$$

/5

5/ Kalibra se metara bakrena žica promjera 1 mm može dati od jednog kg bakra (gustota bakra je  $8.9 \text{ g/cm}^3$ )?

$$R = 1 \text{ mm}$$

$$r = 0.05 \text{ cm}$$

$$\rho = 8.9 \text{ g/cm}^3$$

$$m = 1 \text{ kg} = 1000 \text{ g}$$

$$h = ?$$

$$m = \rho V / : \rho$$

$$V = \frac{m}{\rho} = \frac{10000}{89} \text{ cm}^3 \quad (1)$$

$$V = r^2 \pi \cdot h / : r^2 \pi \quad (1)$$

$$h = \frac{V}{r^2 \pi} = 14306 \text{ cm} = 143 \text{ m}$$

(1)

/3

6/ Prizma i Piramida imaju osnovke jednake površine, a visine su im jednake  
duljina. Ako je obujam prizme  $63 \text{ cm}^3$ , koliki je obujam piramide.

$$V_p = 63$$

$$B_p = B_{pi}$$

$$V_p = B \cdot h \quad (1)$$

$$h_p = h_{pi}$$

$$V_{pi}$$

$$V_{pi} = \frac{B \cdot h}{3}$$

$$V_{pi} = \frac{V_p}{3} \quad (1)$$

$$V_{pi} = 21$$

Mikal Mikala, Bartol Baciwin, Benedikt Vrkavici, Petar Turkovici 2.0





Lucija Lončić  
 Delia Benedetti  
 Vica Peber

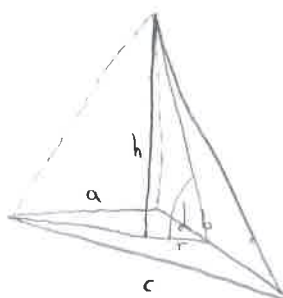
# PISANA PROVJERA

2.e - POLIEDRI I ROTACIJSKA TIJELA -

1. OBJASNI PETI AKSIOM. Kroz svaku točku može se povući točno 1 ravnilu s 1 zoolnim pravcem

2. Osnovka piramide je trokut čije su stranice dužine 13, 20 i 21 cm. Pobočke zatvaraju s ravninom osnovke kutove od  $30^\circ$ . Koliki je obujam piramide?

$a = 13$   
 $b = 20$   
 $c = 21$   
 $\alpha = 30^\circ$



$$B = P_{\triangle abc} = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= 126$$

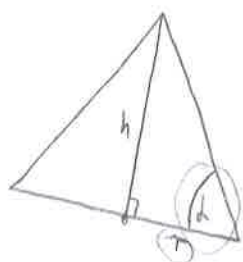
$$s = \frac{a+b+c}{2}$$

$$s = 27$$

$V = ?$

$V = B \cdot h$

$V = 196\sqrt{3}$



$\tan \alpha = \frac{h}{r}$

$h = \tan \alpha \cdot r = \frac{14\sqrt{3}}{3}$

$P_{\triangle abc} = r \cdot s$

$r = \frac{P_{\triangle abc}}{s} = \frac{126}{27} = \frac{14}{3}$

3. Opseg osnovke kvadra je  $O_k$ , dubina bočnog brida iznosi  $c$ , a prostorne dijagonale  $D$ . Izračunaj oplošje i obujam kvadra.

$O_k = 12$

$c = 3$

$D = 5.39$

$O, V = ?$

(1.)  $D = 5.39$

$\sqrt{a^2 + b^2 + c^2} = 5.39 / 2$

$a^2 + b^2 + c^2 = 29$

(2.)  $a^2 + b^2 + c^2 = 29$

$(b-6)^2 + b^2 + 3^2 = 29$

$b_1 = 2 \Rightarrow a_1 = 4$

$b_2 = 4 \Rightarrow a_2 = 2$

(2.)  $2(a+b) = 12$

$\Rightarrow a+b = 6 \Rightarrow a = 6-b$

$c = 3$

$a^2 + b^2 + c^2 = 29$

(4.)  $O = 2B + P$

$O = 2ab + O_k c$

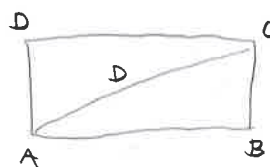
$O = 2 \cdot 4 \cdot 2 + 12 \cdot 3$

$O = 52$

$V = B \cdot h$

$V = a \cdot b \cdot h$

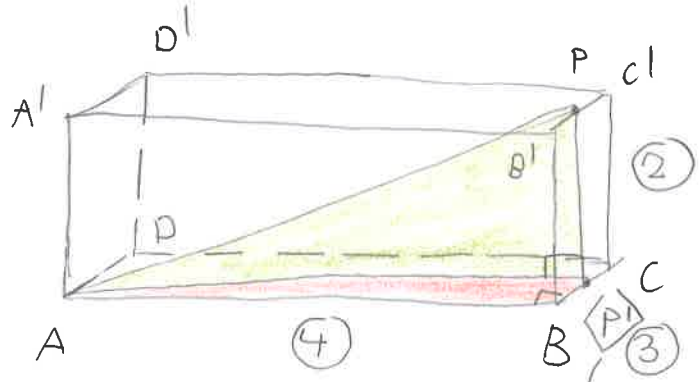
$V = 24$



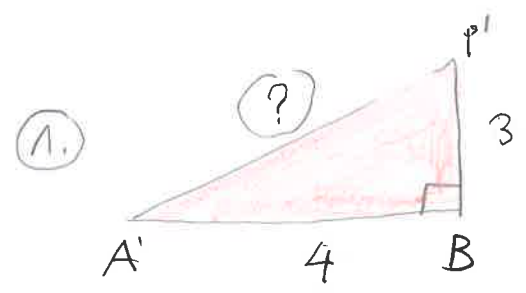
/4

4. Odredi dužinu  $\overline{AP}$  ako je P polovište dužine  $B'C'$  i  $|CC'| = 2$ ,  $|AB| = 4$  i  $|BC| = 3$ .

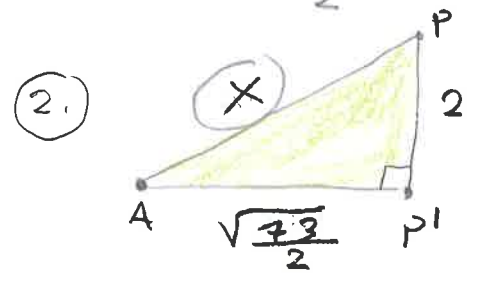
$|CC'| = 2$ ,  $|AB| = 4$ ,  $|BC| = 3$   
 $\overline{AP} = ?$



ORTOGONALNA PROJEKCIJA TOČKE P

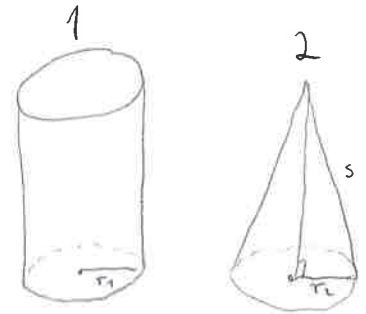


$AP' = \sqrt{4^2 + \left(\frac{3}{2}\right)^2}$   
 $AP' = \frac{\sqrt{73}}{2}$



$X = \sqrt{2^2 + \left(\frac{\sqrt{73}}{2}\right)^2} = \frac{\sqrt{89}}{2}$

5. Koje tijelo ima veće površine i za koliko ako je  $V_1 = V_2 = 180$  i  $h_1 = h_2 = 15$ ?



$O_1 = 2B + P$   
 $O_1 = 2r_1^2\pi + 2r_1\pi \cdot h$   
 $O_1 = 2r_1\pi(r_1 + h)$   
 $O_1 = 208.199$

$O_2 = B + P$   
 $O_2 = r_2^2\pi + r_2\pi s$   
 $O_2 = r_2\pi(r_2 + s)$   
 $O_2 = 199.52$

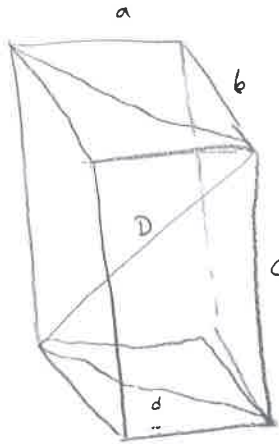
$s = \sqrt{h^2 + r_2^2} = 15.3772$

$O_1 > O_2$   
 $\Delta O = O_1 - O_2 = 8.679$

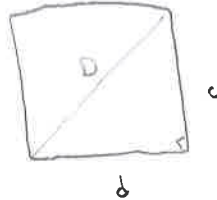
$V_1 = B \cdot h = r_1^2\pi \cdot h$   
 $r_1 = \sqrt{\frac{V}{h\pi}} = 1.9544$

$V_2 = \frac{B \cdot h}{3} = \frac{r_2^2\pi \cdot h}{3}$   
 $r_2 = \sqrt{\frac{3V}{h\pi}} = 3.385$

6.) Dijkagonalni presjek kvadra je kvadrat i ima površinu 400, a opseg baze 56. Koli je obujam, površje i prostorna dijagonala?



$$P_0 = 400$$



$$d = c = \sqrt{P_0} = 20$$

$$\sigma = 2b + 2a = 56$$

$$2(a+b) = 56$$

$$a+b = 28$$

$$b = 28 - a$$

$$d^2 = a^2 + b^2$$

$$d^2 = a^2 + (28-a)^2$$

$$d^2 = a^2 + 28^2 - 2 \cdot 28 \cdot a + a^2$$

$$d^2 = 2a^2 + 28^2 - 2 \cdot 28a$$

$$0 = 2a^2 - 56a + 28^2 - d^2$$

$$a_1 = 16 \quad a_2 = 12$$

$$b_1 = 12 \quad b_2 = 16$$

$$O = 2B + P$$

$$O = 2a \cdot b + 2b \cdot c + 2a \cdot c$$

$$O = 1504$$

$$V = a \cdot b \cdot c$$

$$V = 3840$$

$$D = \sqrt{d^2 + c^2}$$

$$D = \sqrt{2d^2}$$

$$D = 20\sqrt{2}$$

