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TECNOLÓGICO DE MONTERREY – PREPA TEC – CAMPUS CUMBRES

CALCULUS I – THIRD PARTIAL QUIZ

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1. Consider the function $f(x) = 2e^{-3x}$. Find $\frac{d^3f}{dx^3}$. **[20 points]**

$f(x) = 2e^{-3x}$

① $f'(x) = -6e^{-3x}$

② $f''(x) = 18e^{-3x}$

③ $f'''(x) = -54e^{-3x}$

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2. The height of a particle moving on a vertical path is given by the function $h(t) = 20 + 35t - 5t^2$, where the height h is in meters and the time t is in seconds.

a) Find the function of velocity $v(t)$. **[5 points]**

$h(t) = 20 + 35t - 5t^2$

$v(t) = 35 - 10t$

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b) Find the function of acceleration $a(t)$. **[5 points]**

$v(t) = 35 - 10t$

$a(t) = -10$

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c) Find the maximum height reached by the particle. **[10 points]**

$0 = 35 - 10t$

$-10t = -35$

$t = -35/-10$

$t = 3.5$ sec

$h(3.5) = 20 + 35(3.5) - 5(3.5)^2$

$h(3.5) = 20 + 122.5 - 61.25$

$h(3.5) = 81.25$ m

$h = 81.25$ m

3. Find $\frac{dy}{dx}$ from the relation $x^2 - xy + \cos y = y^2$ **[20 points]**

$2x - (x \cdot y') + (1 \cdot y) - (y' \sin(y) \cdot y') = (2y \cdot y')$

$2x - xy' + y - y'^2 \sin(y) = 2yy'$

$-xy' - y'^2 \sin(y) - 2yy' = -2x - y$

$y'(-x - y'\sin(y) - 2y) = -2x - y$

$y' = \frac{-2x - y}{-x - y'\sin(y) - 2y}$

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4. Use logarithmic differentiation to find the derivative of each of the following functions:

a) $y = x^{2x+1}$ **[20 points]**

$\ln y = (2x+1)\ln(x)$

$\frac{y'}{y} = \frac{2x+1}{x} + 2\ln(x)$

$y' = \left(\frac{2x+1}{x} + 2\ln(x)\right)\left(x^{2x+1}\right)$

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b) $y = \frac{(x-4)^3}{\sqrt{2-3x}}$ $(2-3x)^{1/2}$ **[20 points]**

$\ln y = 3\ln(x-4) - \frac{1}{2}\ln(2-3x)$

$\frac{y'}{y} = \frac{3}{x-4} - \frac{-3}{2(2-3x)}$

$y' = \left(\frac{3}{x-4} + \frac{3}{2(2-3x)}\right)\left(\frac{(x-4)^3}{\sqrt{2-3x}}\right)$

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