Basic Mathematics XI

SET - I

Group A **[5×3×2=30]**

1. a. Construct the truth table for $\sim(p \vee q) \wedge q$ and hence draw a conclusion from the truth table.
   b. If A = {1, 2, 3}, find a relation on A determined by the condition $x + y > 4$.
   c. Test periodicity and symmetricity of the function $y = \tan 2x$.
2. a. Solve: $\sqrt{3}\ sinx - cosx = 1$ for $0 \le x \le 2\pi$
   b. Using principle of mathematical induction, prove that : $1 + 3 + 5 + \ldots + (2n-1) = n^2$.
   c. If $\begin{pmatrix} -3 & -2 \\ 5 & 3 \end{pmatrix}$ and $\begin{pmatrix} x & y \\ -5 & -3 \end{pmatrix}$ are inverse matrices of each other, find x and y.
3. a. Using Cramer's rule, solve $2x + y = 8,\ x - 2y = -1$
   b. If $\alpha = \dfrac{-1 + \sqrt{-3}}{2}$ and $\beta = \dfrac{-1 - \sqrt{-3}}{2}$, prove that $\alpha^4 + \alpha^2\beta^2 + \beta^4 = 0$.
   c. For what value of a will the equation $x^2 + (3a - 1)x - 2(a^2 - 1) = 0$ has equal roots.
4. a. Find the distance between two parallel lines $3x + 4y = 17$ and $6x + 8y + 1 = 0$
   b. Find the equation of a circle concentric with $x^2 + y^2 - 4x + 6y + 7 = 0$ and passing through a point (–2, 0).
   c. Evaluate: $\lim\limits_{\theta \to \pi/4} \dfrac{\cos\theta - \sin\theta}{\theta - \pi/4}$
5. a. Find $\dfrac{dy}{dx}$ if $x = t + \dfrac{1}{t}$ and $y = t - \dfrac{1}{t}$
   b. Evaluate: $\int ln\, x^2\ dx$
   c. The side of a square is increasing at the rate of 2 cm/min. At what rate is the area increasing when the side is 5 cm long?

Group B[5×2×4 = 40]

6. a. Define the complement of a set. State and prove De-Morgan's Laws.

   OR

   Define absolute value of a real number. For any two real numbers x and y prove that: $|x + y| \le |x| + |y|$
   b. Draw the graph of the function $y = x^2 - 4x + 3$ giving its different characteristics.
7. a. In any triangle ABC

   If $\dfrac{1}{a + c} + \dfrac{1}{b + c} = \dfrac{3}{(a + b + c)}$ show that $\angle C = 60°$.

   OR

   If $\cos^{-1} x + \cos^{-1} y + \cos^{-1} z = \pi$, show that $xy + yx + zx = 1$
   b. If a, b, c are non-zero and $\begin{vmatrix} a & a^2 & a^3 - 1 \\ b & b^2 & b^3 - 1 \\ c & c^2 & c^3 - 1 \end{vmatrix} = 0$

   then show that, abc = 1.
8. a. Using row equivalent method **or** inverse matrix method, solve the system of linear equations:

   $x + 2y - z = 8$

   $2x + 3y + z = 5$

   $3x + y + 2z = -1$
   b. Under what condition will quadratic equation: $ax^2 + bx + c = 0$ has,

i. reciprocal roots

ii. roots equal in magnitude but opposite in sign

9. a. Obtain the condition that $lx + my + n = 0$ may be a tangent to the circle $x^2 + y^2 + 2gx + 2fy + c = 0$.

b. Evaluate: $\lim_{x\to\theta} \frac{x \sin\theta - \theta \sin x}{x - \theta}$

OR

Show that the function $f(x) = \begin{cases} x & 0 \le x < \frac{1}{2} \\ 1 & x = \frac{1}{2} \\ 1-x & \frac{1}{2} < x < 1 \end{cases}$

is discontinuous at $x = \frac{1}{2}$. Also, write how it could be made continuous?

10. a. Find from the definition, the derivative of $\sin^2 3x$.

b. Using the method of integration, find the area bounded by the curve $\frac{x^2}{16} + \frac{y^2}{25} = 1$

Group C[5×6 = 30]

11. Define one to one function and onto function with suitable example. Determine whether the function f : R → R given by $f(x) = x^2 - 1$ is one to one or onto or both or neither.

12. If one A.M. 'A' and two G.M.'s G_1 and G_2 are inserted between two positive numbers, prove that $\frac{G_1^2}{G_2} + \frac{G_2^2}{G_1} = 2A$.

Also prove that , A.M., G.M. and H.M. between two unequal positive quantities satisfy
A.M. > G.M. > H.M.

13. The origin is a corner of the square and two of it's sides are $y + 2x = 0$ and $y + 2x = 3$. Find the equations of other two sides. Also prove that the distance between two parallel lines $y = mx + c_1$ and $y = mx + c_2$ is $\frac{|c_2 - c_1|}{\sqrt{1 + m^2}}$.

OR

Prove that the product of the perpendicular from (α, β) to the lines given by $ax^2 + 2hxy + by^2 = 0$ is $\frac{a\alpha^2 + 2h\alpha\beta + b\beta^2}{\sqrt{(a-b)^2 + 4h^2}}$. Also find the acute angle of bisector between the lines $4x + 3y - 7 = 0$ and $24x + 7y - 31 = 0$.

14. Define absolute value of complex number and solve by using De-Moivre's theorem : $Z^6 = 1$.

15. For a function, write the conditions so that its graph has concave upward and concave downward. Find the interval where given function $f(x) = x^4 - 8x^3 + 18x - 24$ is concave upward and concave downward. Also find the point of inflection.

OR

Two concentric circles are expanding in such a way that the radius of the inner circle is increasing at the rate 8 cm/sec. and that of the outer circle at the rate of 5 cm/sec. At a certain instant the radii of the inner and outer circles are respectively 24 cm and 30 cm. At what rate does the area between the two circles changes.

SET - II

Group A[5×3×2=30]

1. a. Write inverse and converse of the statement "If two triangles are similar then their corresponding sides are proportional."

b. If A = {1, 2} and B = {2, 3, 4}, find a relation from A on B defined by the condition $y = 2x$.

c. Examine whether the function $f(x) = 3x^2 + \cos x$ is even or odd. Also test for symmetricity.

2. a. Solve: $\tan\theta + \cot\theta = 2$ $(0 \le \theta \le \pi)$

b. Using principle of mathematical induction, prove that "$x^n - y^n$ is divisible by $x - y$."

c. If $A = \begin{pmatrix} 2 & -3 \\ 4 & 0 \end{pmatrix}$ and $B = \begin{pmatrix} 3 & 1 \\ 0 & -1 \end{pmatrix}$, find $(AB)^T$.

3. a. Using inverse matrix method, solve

$x + 2y = 5$; $3x - y = 1$

b. If w is complex cube root of unity, find the value of: $(1 - w + w^2)^4(1 + w - w^2)^4$.

c. Find the value of k so that the equation $(3k + 1)x^2 + 2(k + 1)x + k = 0$ has reciprocal roots.

4. a. Find the equation of a straight line passing through origin and perpendicular to the line $3x - 5y = 7$.

b. Find equation of a circle whose two of the diameters are $2x + y = 10$ and $x - y + 1 = 0$ and passing through origin.

c. Evaluate: $\lim_{x\to\infty} \sqrt{x}\left(\sqrt{x} - \sqrt{x - a}\right)$

5. a. Find $\frac{dy}{dx}$ if $x^3 + y^3 - 3xy = 0$.

b. Evaluate: $\int \cot x\,(\ln \sin x)^3\,dx$

c. Find the intervals in which $f(x) = x^2 - 2x + 10$ is increasing or decreasing

Group B[5×2×4 = 40]

6. a. If A, B and C be three non-empty sets, prove that,

$A - (B \cup C) = (A - B) \cap (A - C)$.

OR

Define absolute value of real number. Rewrite the given relation without using absolute value sign $|2x - 1| \le 5$. Also, draw the graph of the inequality.

b. Sketch the graph of $f(x) = (x - 4)^2 - 8$ indicating its characteristics.

7. a. If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \pi$, show that $x + y + z = xyz$.

OR

Solve the triangle if $a = \sqrt{6}$, $b = 2$ and $c = \sqrt{3} - 1$.

b. Prove that:

$$\begin{vmatrix} a & b & ax + by \\ b & c & bx + cy \\ ax + by & bx + cy & 0 \end{vmatrix} = (b^2 - ac)(ax^2 + 2bxy + cy^2)$$

8. a. Applying row equivalent matrix method **or** inverse matrix method, solve the following system of linear equations:

$$x + y + z = 1$$
$$x + 2y + 3z = 4$$
$$x + 3y + 7z = 13$$

b. Find the condition for two given quadratic equations $p_1x^2 + q_1x + r_1 = 0$ and $p_2x^2 + q_2x + r_2 = 0$ may have one root common.

9. a. Find the equation of the line through (1, –1) which cuts off a chord of length $4\sqrt{3}$ from the circle $x^2 + y^2 - 6x + 4y - 3 = 0$.

b. Evaluate: $\lim_{x \to 0} \frac{(a+x)\sec(a+x) - a\sec a}{x}$

OR

A function f(x) is defined as follows:

$$f(x) = \begin{cases} 2x+3 & for \quad x<1 \\ 3 & for \quad x=1 \\ 6x-1 & for \quad x>1 \end{cases}$$

Is the function continuous at x = 1 ? If not how can you make it continuous?

10. a. Find the derivative of $\sqrt{sin2x}$ by the definition.

b. Using method of integration, find the area of the circle $x^2 + y^2 = r^2$

Group C[5×6 = 30]

11. Let a function f: A → B be defined by $f(x) = \frac{x+1}{2x-1}$ with A = {–1, 0, 1, 2, 3, 4} and B = {–1, 0, $\frac{4}{5}$, $\frac{5}{7}$, 1, 2, 3}. Find the range of f. Is the function *f* one-one and onto both? If not, how can the function be made one-one and onto both?

12. Define sequence and series. Find the n^{th} term and then sum of first n term of the given series
6 + 13 + 24 + 29 + ……

13. Derive the formula for the length of the perpendicular from a point (x_1, y_1) to the line $x\ cos\alpha + y\ sin\alpha = p$. Also find, the distance between the parallel lines $3x + 5y = 11$ and $3x + 5y = -23$

OR

Find the condition that the general equation of second degree may represent a pair of lines. For what value of k will the given equation represent a pair of straight lines.
$x^2 - kxy + 4y^2 + x + 2y - 2 = 0$

14. State De-Moivre's theorem for any complex number solve by using De-Moivre's theorem : $z^6 = -1$.

15. Write the conditions for a function to have local maximum and local minimum value of a function. Find the maxima or minima for the given function $4x^3 - 15x^2 + 12x + 7$. Also, find the point of inflection.

OR

Water is running into a conical reservoir, 10 cm deep and 5 cm in radius at the rate of 1.5 cm^3 /min.

i. At what rate is the water level rising when the water is 4 cm deep.

ii. At what rate is the area of water surface of the reservoir increasing when the water is 6 cm deep.

SET - III

Group A[5×3×2=30]

1.a. Write inverse and converse of the statement "if 3 is an odd number then 6 is not an odd number"

b. Check whether the function f:[-2,3] → R given by $f(x) = x^2$ is one to one, onto or both.

c. What are the odd and even functions? Determine whether the function
$f(x) = 2^x + 2^{-x}$ is even, odd or neither.

2.a. Solve: $tanx + tan2x = tan3x$.

b. If H is the harmonic mean between a and b, prove that : $(H-2a)(H-2b)=H^2$

c. For the given,matrices A=$\begin{bmatrix} -1 & 2 \\ 3 & 1 \end{bmatrix}$ and B=$\begin{bmatrix} 2 & 3 \\ 0 & -1 \end{bmatrix}$. Show that $(A+B)^T = A^T + B^T$.

3.a. Can you solve the system $3x + 4y = 10, 6x + 8y = 24$ by the inverse matrix method, if not why?

b. Prove that $\frac{a+bw+cw^2}{b+cw+aw^2}$ =w where w,$w^2$are complex cube roots of unity.

c. If one root of the equation $ax^2 + bx + c = 0$ be twice the other show that $2b^2 = 9ac$.

4.a. Find the angle between the pairs of line represented by $7x^2 + 8xy + y^2 = 0$

b. Find the equation of the circle which has the points (0,-1)and (2,3) as ends of a diameter.

c. Evaluate $\lim_{x \to 0} \frac{1-\cos qx}{1-\cos px}$

5. a. Find the derivative of $\frac{1}{\sqrt[3]{3x^2+x-5}}$

b. Show that the function $y = x^3 - 3x^2 + 6x + 3$ has neither maximum nor minimum value.

c. Evaluate $\int \sec x \, dx$

Group B[5×2×4 = 40]

6. a. What is contradiction? Prepare the truth table for the compound statement $(p \wedge \sim q) \wedge (\sim p \vee q)$.

Or

Define absolute value of any real number.Solve the inequality; $x(x-2)(x+3) \leq 0.$

b. Sketch the graph of the function $y = -x^2 - 2x + 5$ indicating its different characteristics.

7. a. In any ΔABC if $\frac{\sin A + \cos A}{\cos B} = \sqrt{2}$ prove that $\angle C$ =135^0.

Or

If $\tan^{-1} x + \tan^{-1} y + \tan^{-1} z = \frac{\pi}{2}$, provethat $xy + yz + zx = 1$

b. Write the any two properties of determinant? Hence prove that $\begin{vmatrix} a & b & c \\ a^2 & b^2 & c^2 \\ b+c & c+a & a+b \end{vmatrix} = (b-c)(c-a)(a-b)(a+b+c)$

8. a. Solve; $3x + 4y + 5z = 18$; $2x - y + 8z = 13$; $5x - 2y + 7z = 20$ by using Row –equivalent matrix method or inverse matrix method.

b. Prove that the quadratic equation $ax^2 + bx + c = 0$ $(a \neq 0)$ cannot have more than two roots.

9. a. Find the equations of tangents drawn from the point (4,-2) to the circle $x^2 + y^2 = 10$ .Also show that they are at right angle.

b. Evaluate: $\lim_{y \to x} \frac{y \sec y - x \sec x}{y - x}$.

10. a. Find from the first principle the derivative of $\frac{1}{\sqrt{4-3x}}$

b. Evaluate: $\int e^x \cos x \, dx$ or

Find the area of ellipse $\frac{x^2}{16}+\frac{y^2}{9}=1$ by using integration.

Group C[5×6 = 30]

11. Define domain and range of a function. Find the domain and range of the function $y=\sqrt{6-x-x^2}$

12. Using the principle of mathematical induction prove that $1^2+2^2+3^2+\ldots+n^2=\frac{n(n+1)(2n+1)}{6}$.

13. State Demoivre's theorem. Use it find the fourth roots of 'i'.

14. Derive the formula for the length of the perpendicular form a point (x_1,y_1) to a line $ax+by+c=0$.Also find the distance between the parallel lines $5x+12y+8=0$ and $10x+24y-3=0$

OR

Prove that the bisectors of the angle between the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ is given by

$$\frac{x^2-y^2}{xy}=\frac{a-b}{h}.$$

15. A closed cylindrical can is to be made so that its volume is 52 cm^3. Find its dimensions if the surface is to be a minimum.

OR

The volume of a spherical balloon is increasing at the rate of 25 cubic cm/sec. Find the rate of change of its surface at the instant when its radius is 5 cm.

SET - IV

Group A[5×3×2=30]

1.a. Write the truth table of the statement $p\wedge(p\Rightarrow q)\Rightarrow q$ .Draw a conclusion from a table.

b. Examine the function $y = cosx$ for symmetry and even or odd in nature.

c. Prove by the principle of mathematical induction:

$$2+4+6+\cdots \ldots+2n = n(n+1)$$

2.a. If $x=\sqrt{3}$,and $y=\sqrt{3}$,test the validity of:

$$\tan^{-1}x+\tan^{-1}y=\tan^{-1}\frac{x+y}{1-xy}$$

b. If A=$\begin{bmatrix}3 & -1\\ 5 & -2\end{bmatrix}$ and B=$\begin{bmatrix}1 & 2\\ 3 & -2\end{bmatrix}$ find $(AB)^T$.

c. Using Cramer's rule, solve: $x-2y = 0; 3x + 7y = 5$.

3.a. For what value of p will the equation $5x^2 - px + 45 = 0$ has equal roots.

b. If w be a complex cube root of unity, find the value of : $(1-w+w^2)^4(1+w-w^2)^4$.

c. Define one to one and onto functions with an example.

4.a. What are the points on x-axis whose perpendicular distance from the straight line $\frac{x}{a}+\frac{y}{b}=1$ is 'a'?

b. Find the equation of the circle concentric with the circle $x^2+y^2 - 8x + 12y + 15 = 0$ and passing through at the point (5, 4).

c. Prove that $\lim_{x\to 0}\frac{\ln(1+x)}{x}=1$

5.a. Find $\frac{dy}{dx}$ if $x^3+y^3 - 3axy = 0$.

b. The side of a square sheet is increasing at the rate of 5 cm/min. At what rate is the area increasing when the side is 12 cm. long?

c. Evaluate: $\int \cot x\,(\log \sin x)^3\,dx$

Group B[5×2×4 = 40]

6. a. Define symmetric difference of two sets. Prove that $A-(B\cup C)=(A-B)\cap(A-C)$

OR

Define the absolute value of real number. For any positive real number a, prove that $|x|\le a \Rightarrow -a\le x\le a$

b. Sketch the graph of the function $y = \sin x(-\pi\le x\le\pi)$ indicating its different characteristics.

7. a. Solve : $2\sin^2 x - 3\sin x\cos x + 3\cos^2 x = 1$

OR

State and prove that sine law of trigonometry.

b. Prove that

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix} = 2(a+b+c)^3$$

8. a. When does a matrix have its inverse? Using inverse matrix method or row equivalent matrix method solve: $x+y+z=6;\ x-y+z=2;\ x+y-z=0.$

b. Find the conditions that the two quadratic equations $a_1x^2+b_1x+c_1=0$ *and* $a_2x^2+b_2x+c_2=0$ have one root common and both roots common.

9. a. Find the equation of the line through the point (1,-1) which cuts off a chord of length $4\sqrt{3}$ from the circle $x^2 + y^2 - 6x + 4y - 3 = 0$.

b. Discuss the limit of the function $f(x)=\frac{|x-2|}{2-x}$ at $x=2$. Is f continuous at $x=2$?

10. a. Find from the first principle the derivative of $\sqrt{\sec x}$.

b. Evaluate; $\int_0^{\frac{\pi}{4}} \tan^3 x\,dx$.

OR

Find the area of the region between the curves. $y^2 = 16x$ and the line $y = 2x$.

Group C[5×6 = 30]

11. What is function? Find the domain and range of the function $y=\sqrt{36-x^2}$.

12. Find the n^{th} term and the sum of the n-term of the series: $1^2.2 + 2^2.3 + 3^2.4 + \ldots..$

13. Define the conjugate of a complex number. Find the square roots of $\frac{(8,-6)}{(1,1)}$.

14. Find the equation of the bisectors of the angle between the straight lines $3x+4y+2=0$ and $5x-12y-6=0$. Verify that bisectors are perpendicular to each other. Also identify the acute angle bisector.

OR

What is the homogenous equation of degree two? Prove that the two lines represented by

$(x^2+y^2)\sin^2\alpha=(x\cos\theta-y\sin\theta)^2$ include an angle 2α

15. A door is in the form of a rectangle surmounted by a semi-circle. If total perimeter is 9m, find the radius of semi-circle for the greatest door area.

OR

Water is running into a conical reservoir, 10cm deep and 5cm in radius at the rate of 1.5 cm^3/min.At what rate is the level rising when the water is 4 cm deep?

SET - V

Group A(5×3×2 =30)

1. a. If M={1,3,5,7}, N={2,4,6,8}, and U={1,2,3,4,5,6,7,8,9}, find $\overline{M\cap N}$ and $\overline{M-N}$

b. If A={1,2,3}, find the relation A satisfy the condition $x+y<4$. Is this relation a function? Give a reason.

c. Evaluate $\lim_{x\to 0}\frac{1-\cos x}{x^2}$

2. a. Rewrite $-1\le x\le 5$ using absolute value sign.

b. Does the limit of the function $f(x)=\begin{cases} x & when\, x>0 \\ -x & when\, x<0\end{cases}$ exist at $x=0$? Justify your answer.

c. Find $\frac{dy}{dx}$ if $x=e^{\cos 2t}$ and $y=e^{\sin 2t}$

3. a. Show that the function $f(x)=2-3x+3x^2-x^3$ is increasing on R.

b. If $A=\begin{pmatrix}-1 & -1\\ 1 & 0\end{pmatrix}$, find the value of A^3.

4. a. Using Cramer's rule, solve the system of equations.

$3x+4y=-2$

$5x-7y-24=0$

b. Without expanding the determinant prove that

$$\begin{vmatrix}1 & a(b+c) & bc\\ 1 & b(c+a) & ca\\ 1 & c(a+b) & ab\end{vmatrix}=0$$

c. If the equation $2x^2+7xy+3y^2-4x-7y+K=0$ represent a pair of line, find the value of K.

5. a. If $1, w, w^2$ are the cube roots of unity, prove that

$(1+w^2)^3-(1+w)^3=0$

b. Find $\frac{dy}{dx}$ if $y=\tan^{-1}(1+x+x^2)$

c. Express $-1-\sqrt{3}\,i$ into polar form.

Group B(5×4×2=40)

6. a. If A,B and C be any three non-empty sets, prove that

$A\cup(B\cap C)=(A\cup B)\cap(A\cup C)$

b. Evaluate $\lim_{x \to \frac{\pi}{4}} \frac{\sec^2 x - 2}{\tan x - 1}$

7. a. Define bi-conditional of two statements . Prepare the truth table of $(p \to q) \leftrightarrow (\sim pvq)$.

b. A function f(x) is defined as follows.

$$f(x) = \begin{cases} 2x - 3 & for\ x < 2 \\ 2 & for\ x = 2 \\ 2x - 5 & for\ x > 2 \end{cases}$$

Is f(x) continuous at x=2. If not, how can f(x) be made continuous at x=2?

8. a. If $x = \log_{2a} a$, $y = \log_{3a} 2a$ and $z = \log_{4a} 3a$. Prove that $xyz + 1 = 2yz$.

b. Find from the definition, the derivative of $\sqrt{\tan x}$

9. a. Prove that

$$\begin{vmatrix} x & y & z \\ x^2 & y^2 & z^2 \\ yz & xz & xy \end{vmatrix} = (y - z)(z - x)(x - y)(yz + zx + xy)$$

b. Solve the following system of linear equations by row equivalent matrix method ***OR*** inverse matrix method.
$2x + 6y = 23x - z = -82x - y + z = -3$

10. a. If $x + y = 2$ is the equation of the chord of the circle $x^2 + y^2 - 2y = 0$, find the equation of the circle so that thischord is a diameter.

b. Find $\frac{dy}{dx}$ when $x^2 y + xy^2 = a^3$

Group C(5×6=30)

11. Define bijective function with an example. Let R be the set of real numbers. Show that the function $f : R \to R$ define by $f(x) = 4x - 7$ is bijective and find the formula for f^{-1}
.

12. The AM, GM and HM between two unequal positive numbers satisfy the relations
i. AM × HM = $(GM)^2$ ii. AM > GM > HM

13. Prove that the straight lines joining the origin to the point of intersection of the line $\frac{x}{a} + \frac{y}{b} = 1$ and the curve $x^2 + y^2 = c^2$ are right angle if $\frac{1}{a^2} + \frac{1}{b^2} = \frac{2}{c^2}$.

OR

What are the three standard equations of straight lines? The origin is a corner of a square and two of its sides are y + 2x = 0 and y + 2x = 3 , find the equations of the other two sides.

14. State De-Moivre's theorem. Using De-Moivre's theorem, find the square roots of $4 + 4\sqrt{3}i$.

15. List the criteria for the function y=f(x) to have local maxima and local minima at a point. Find the local maxima and minima of $f(x) = 4x^3 - 6x^2 - 9x + 1$ on the interval (-1, 2), also find the point of inflection.

OR

A ladder 5 meters long rests against a vertical wall. If its top slides down wards at the rate of 10 cm/s, find the rate at which the foot of the ladder is sliding when the foot of the ladder is 4 meters away from the wall.