EXERCISES [MAI 2.18]

VORONOI DIAGRAM

SOLUTIONS

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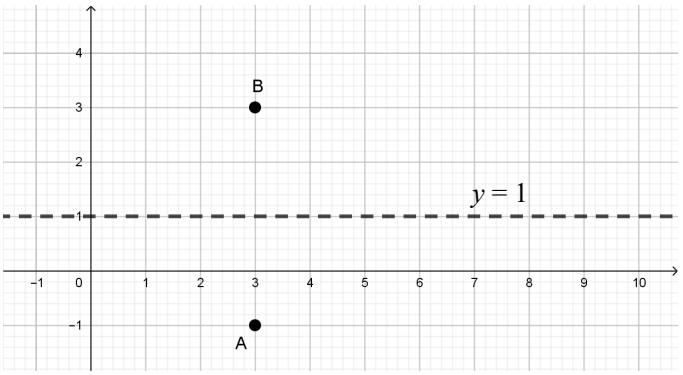
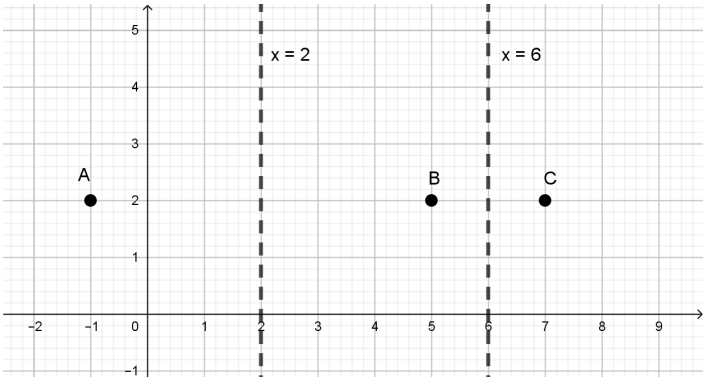
A. Paper 1 questions (SHORT)

1. (a)



(b)



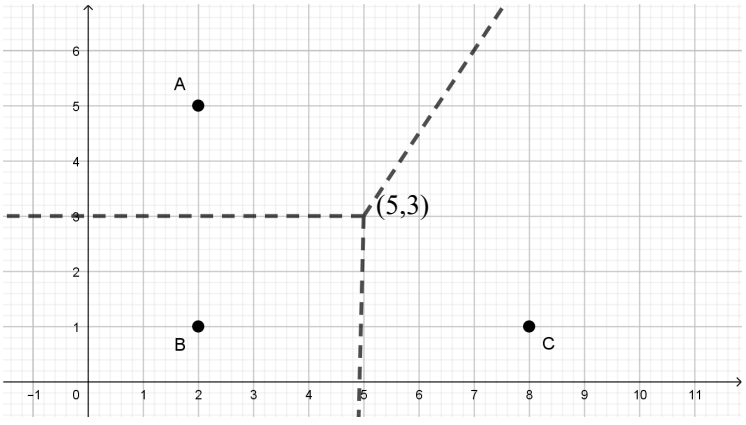
2. (a) (i) $y = 3$, (ii) $x = 5$

(b) $m_{AC} = \dfrac{1-5}{8-2} = \dfrac{-4}{6} = -\dfrac{2}{3}$, $m_{\perp} = \dfrac{3}{2}$

Midpoint of [AC]: M(5,3)

Perpendicular bisector: $y - 3 = \dfrac{3}{2}(x-5) \Rightarrow y = \dfrac{3}{2}x - \dfrac{9}{2}$

(c)



3. (a) $x = 5$

(b) $m_{AB} = \dfrac{1-5}{2-4} = \dfrac{-4}{-2} = 2, \quad m_{\perp} = -\dfrac{1}{2}$

Midpoint of [AB]: M(3,3)

Perpendicular bisector: $y-3 = -\dfrac{1}{2}(x-3) \Rightarrow 2y-6 = -(x-3) \Rightarrow x+2y-9 = 0$

(c) (5,2)

(d) $y = x + c \Rightarrow 2 = 5 + c \Rightarrow c = -3$

4. (a) $m_{AC} = \dfrac{2-6}{6-4} = \dfrac{-4}{2} = -2, \quad m_{\perp} = \dfrac{1}{2}$

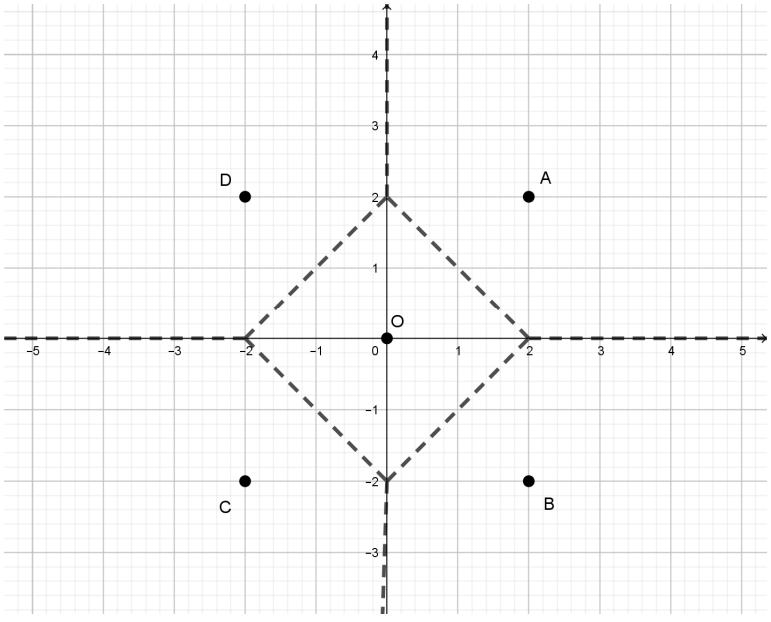
Midpoint of [AC]: M(5,4)

Perpendicular bisector: $y-4 = \dfrac{1}{2}(x-5) \Rightarrow y = \dfrac{1}{2}x + \dfrac{3}{2}$

(b)

Line	Equation
L_3	$y = 0.5x + 1.5$
L_1	$y = -0.2x + 4.2$
L_5	$y = -x + 5$
L_1	$y = -3x + 15$
L_4	` $y = 4$
L_6	$x = 5$

5. (a)



(b) $y = -x + 2, \; y = x - 2, \; y = -x - 2, \; y = x + 2$

(c) Area = 8

B. Paper 2 questions (*LONG*)

6. (a) $Z(\frac{7}{2},1)$

(b) $m_{CD}=\frac{3-1}{2-6}=\frac{2}{-4}=-\frac{1}{2}, \quad m_{\perp}=2$

Midpoint of [CD]: M(4,2)

Perpendicular bisector: $y-2=2(x-4)\Rightarrow 2x-y=6$

(c) $Y(\frac{37}{6},\frac{19}{3})$

(d) $d_{ZY}=\sqrt{\left(\frac{7}{2}-\frac{37}{6}\right)^2+\left(1-\frac{19}{3}\right)^2}=\frac{8\sqrt{5}}{3}$

(e) $m_{ZY}=2$

$m_{AD}=2\Rightarrow m_{XZ}=-\frac{1}{2}$

$m_{ZY}\times m_{XZ}=-1$ hence perpendicular

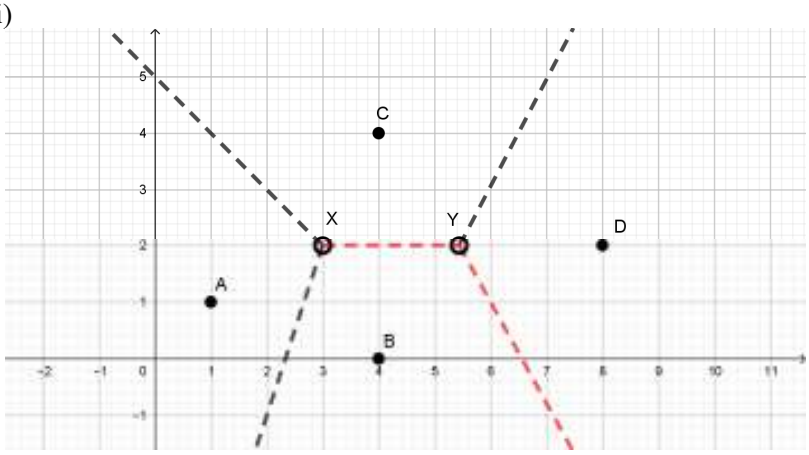
(f) (i) $\cos\theta=\frac{ZY}{XZ}=\frac{\frac{8\sqrt{5}}{3}}{\frac{10\sqrt{5}}{3}}=\frac{8}{10}=\frac{4}{5}$

(ii) A right-angled triangle with sides 3,4,5 shows that $\sin\theta=\frac{3}{5}$

(**OR** for HL, using the Pythagorean identity $\cos^2\theta+\sin^2\theta=1$ we obtain $\sin\theta=\frac{3}{5}$)

(g) $Area=\frac{1}{2}\times XY\times YZ\times\sin\theta=\frac{1}{2}\frac{10\sqrt{5}}{3}\frac{8\sqrt{5}}{3}\frac{3}{5}=\frac{40}{3}$

7. (a) (i)



(ii) $y=2$, $y=-2x+13$

(b) X(3, 2), Y(5.5, 2)

(c) $d_{AD}=\sqrt{(8-1)^2+(2-1)^2}=\sqrt{50}=5\sqrt{2}$

(d) $A_{ABCD}=A_{ABC}+A_{DBC}=\frac{1}{2}\times 4\times 3+\frac{1}{2}\times 4\times 4=14$.