Sección 3,5.

(5)

$y'' + y' + y = sen^2 x$

Ec. Homogénea asociada

$y'' + y' + y = 0$

$r^2 + r + 1 = 0$

$r = \frac{-1 \pm \sqrt{1-4}}{2}$

$r = \frac{-1 \pm \sqrt{3} i}{2}$

$r = -\frac{1}{2} \pm \frac{\sqrt{3}}{2} i$

$y_c = e^{-\frac{1}{2}x} \left(A \cos\left(\frac{\sqrt{3}}{2} x\right) + B \, sen \left(\frac{\sqrt{3}}{2} x\right)\right)$

$r(x) = sen^2 x$

$r'(x) = 2 \, sen x \cos x = sen 2x$

$r''(x) = 2 \cos(2x)$

$y_p = A \, sen 2x + B \cos 2x + C$

$y_p' = 2A \cos 2x - 2B \, sen 2x$

$y_p'' = -4A \, sen 2x - 4B \cos 2x$

$-4A sen 2x - 4B \cos 2x + 2A \cos 2x - 2B sen x + A sen 2x +$
$B \cos 2x + C = sen^2 x$

$-3A sen 2x - 3B \cos 2x + 2A \cos 2x - 2B sen 2x + C =$
$sen^2 x$

$(2A - 3B) \cos 2x - (3A + 2B) sen 2x + C = sen^2 x$

$(2A - 3B) - 2(2A - 3B) sen^2 x - (3A + 2B) sen 2x +$
$C = sen^2 x$

- $2A - 3B + C = 0$
- $3A + 2B = 0$
- $4A - 6B = -1$

$B = -\frac{3}{2}A \qquad B = -\frac{3}{2}\left(-\frac{1}{13}\right) \qquad 2\left(-\frac{1}{13}\right) - 3\left(\frac{3}{26}\right) + C = 0$

$4A + 9A = -1$

$B = \frac{3}{26} \qquad -\frac{2}{13} - \frac{9}{26} + C = 0$

$13A = -1$

$A = -\frac{1}{13} \qquad C = \frac{2}{13} + \frac{9}{26} = \frac{1}{2}$

$y_p = \frac{3}{26}\cos 2x - \frac{1}{13}\,\text{sen}\,2x + \frac{1}{2}$

$y_p = \frac{1}{26}(3\cos 2x - 2\,\text{sen}\,2x + 13)$

(19)

$y'''' + 2y''' + 2y'' = 3x^2 - 1$

$r^5 + 2r^3 + 2r^2 = 0$

$(r^3 + 2r + 2) = 0$

$r_1 = 0,\ r_2 = 0,\ r_3 = -\frac{771}{1000}$

$y_c = (C_1 + C_2x)e^{0x} + C_3e^{-0{,}771x}$

$y_c = C_1 + C_2x + C_3e^{-0{,}771x}$

$y_p = (A + Bx + Cx^2)x^2$

$y_p = Ax^2 + Bx^3 + Cx^4$

$y_p' = 2Ax + 3Bx^2 + 4Cx^3$

$y_p'' = 2A + 6Bx + 12Cx^2$

$y_p''' = 6B + 24Cx$

$y_p'''' = 24C$

$y_p''''' = 0$

$0+2(6B+24Cx)+2(2A+6Bx+12Cx^2)=3x^2-1$

$24Cx^2+48Cx+12Bx+12B+4A=3x^2-1$

$24C=3$ $\qquad C=\frac{3}{24}=\frac{1}{8}$

$48C+12B=0$

$12B+4A=-1$

$48\left(\frac{1}{8}\right)+12B=0$

$6+12B=0$

$12B=-6$

$B=-\frac{1}{2}$

$12\left(-\frac{1}{2}\right)+4A=-1$

$-6+4A=-1$

$4A=5$

$A=\frac{5}{4}$

$y_p=\frac{5}{4}x^2-\frac{1}{2}x^3+\frac{1}{8}x^4$

$y_p=\frac{1}{8}(10x^2-4x^3+x^4)$

(25)

$y''+3y'+2y=x(e^{-x}-e^{-2x})$

$r^2+r+2=0$

$(r+2)(r+1)=0$

$r_1=-2\ ,\ r_2=-1$

$y_c=C_1e^{-2x}+Ce^{-x}$

$*x\ y_p=Ae^{-2x}+Bxe^{-2x}+Ce^{-x}+Dxe^{-x}$

$y_p=Axe^{-x}+Bx^2e^{-2x}+Cxe^{-x}+Dx^2e^{-x}$

(39) $y'''+y''=x+e^{-x}$; $y(0)=1$, $y'(0)=0$, $y''(0)=1$

$r^3+r^2=0$

$r^2(r+1)=0$

$r_1=0\ ,\ r_2=0,\ r_3=-1$

$$y_c = e^{0x}(c_1 + c_2 x) + c_3 e^{-x}$$
$$= c_1 + c_2 x + c_3 e^{-x}$$
$$y_c' = c_2 - c_3 e^{-x}$$
$$y_c'' = c_3 e^{-x}$$

$$y_p = A + Bx + Ce^{-x}$$
$$y_p = Ax^2 + Bx^3 + Cxe^{-x}$$
$$y_p' = 2Ax + 3Bx^2 + Ce^{-x} - Cxe^{-x}$$
$$y_p'' = 2A + 6Bx - Ce^{-x} - Ce^{-x} + Cxe^{-x}$$
$$= 2A + 6Bx - 2Ce^{-x} + Cxe^{-x}$$
$$y_p''' = 6B + 2Ce^{-x} + Ce^{-x} - Cxe^{-x}$$
$$= 6B + 3Ce^{-x} - Cxe^{-x}$$

$$6B + 3Ce^{-x} - Cxe^{-x} + 2A + 6Bx - 2Ce^{-x} + Cxe^{-x} = x + e^{-x}$$
$$6B + 2A + 6Bx + Ce^{-x} = x + e^{-x}$$

$C = 1$ $\quad 6B + 2A = 0$

$B = \frac{1}{6}$ $\quad 1 + 2A = 0$

$A = -\frac{1}{2}$

$$y_p = -\frac{1}{2}x^2 + \frac{1}{6}x^3 + xe^{-x}$$

$$y = c_1 + c_2 x + c_3 e^{-x} - \frac{1}{2}x^2 + \frac{1}{6}x^3 + xe^{-x}$$
$$1 = c_1 + c_3$$

$$y' = c_2 - c_3 e^{-x} - x + \frac{1}{2}x^2 + e^{-x} - xe^{-x}$$
$$0 = c_2 - c_3 + 1$$
$$-1 = c_2 - c_3$$

$$y'' = c_3 e^{-x} - 1 + x - e^{-x} - e^{-x} + xe^{-x}$$
$$1 = c_3 - 1 - 1 - 1$$
$$1 = c_3 - 3$$
$$4 = c_3$$

$$y = -3 + 3x - \frac{1}{2}x^2 + \frac{1}{6}x^3 + 4e^{-x} + xe^{-x}$$

$-1 = c_2 - c_3$ $\quad 1 = c_1 + c_3$

$-1 = c_2 - 4$ $\quad 1 = c_1 + 4$

$3 = c_2$ $\quad -3 = c_1$