Cálculo vectorial

Ejercicios Para entregar
13.10 → Multiplicadores de Lagrange

En los ejercicios 5 a 10, utilizar multiplicadores de Lagrange para hallar el extremo indicado, suponer que x y y son positivos

5) Minimizar: $f(x,y) = x^2 + y^2$

Restricción: $x + 2y - 5 = 0$

$\nabla f = \nabla g \lambda$

$\langle f_x, f_y \rangle = \lambda \langle g_x, g_y \rangle$

$\langle 2x, 2y \rangle = \lambda \langle 1, 2 \rangle$

$$\left.\begin{aligned} 2x &= \lambda \\ 2y &= 2\lambda \\ x+2y-5 &= 0 \end{aligned}\right\} \Rightarrow \begin{aligned} x &= \lambda/2 \\ y &= \lambda \end{aligned}$$

Justituir x y y en ec 3

$\frac{\lambda}{2} + 2(\lambda) - 5 = 0$

$\lambda/2 + 2\lambda = 5$

$5\lambda/2 = 5$

$5\lambda = 10$

$\boxed{\lambda = 2} \rightarrow \boxed{y = 2}, \; x = \lambda/2 = 2/2$

$\boxed{x = 1}$

P.C $(1,2)$

$f(1,2) = (1)^2 + (2)^2$

$= 1 + 4$

$= 5$

$\therefore$ Minimo $= f(1,2) = 5$

7) Maximizar $f(x,y) = 2x + 2xy + y$

Restricción $= 2x + y = 100$

$\nabla f = \lambda \nabla g$

$\langle f_x, f_y \rangle = \lambda \langle g_x, g_y \rangle$

$\langle 2+2y, 2x+1 \rangle = \lambda \langle 2, 1 \rangle$

$$\left.\begin{aligned} 2+2y &= 2\lambda \\ 2x+1 &= \lambda \\ 2x+y &= 100 \end{aligned}\right\} \begin{aligned} &\Rightarrow y = \frac{2\lambda - 2}{2} = \lambda - 1 \\ &\Rightarrow x = \frac{\lambda - 1}{2} \end{aligned}$$

Sustituir x y y en ec 3

$2\left(\frac{\lambda-1}{2}\right) + \lambda - 1 = 100$

$\lambda - 1 + \lambda - 1 = 100$

$2\lambda - 2 = 100$

$\lambda = 102/2 = 51$

$x = \frac{\lambda - 1}{2} = \frac{51-1}{2} = 50/2 = 25$; $y = \lambda - 1 \Rightarrow y = 50$

$f(25,50) = 2(25) + 2(25)(50) + (50)$

$= 50 + 2{,}500 + 50$

$= \boxed{2{,}600}$

$\therefore$ Maximo $= f(25,50) = 2{,}600$

11) Minimizar $f(x,y,z) = x^2+y^2+z^2$

Restricción $= x+y+z-9=0$

$\nabla f = \lambda \nabla g$

$\langle f_x, f_y, f_z \rangle = \lambda \langle g_x, g_y, g_z \rangle$

$\langle 2x, 2y, 2z \rangle = \lambda \langle 1,1,1 \rangle$

$$\left.\begin{array}{l} 2x = \lambda \\ 2y = \lambda \\ 2z = \lambda \\ x+y+z-9=0 \end{array}\right\} \quad \begin{array}{l} x = \lambda/2 \quad ; z = \lambda/2 \\ y = \lambda/2 \\ \\ \\ \end{array}$$

Sustituir x, y, z en ec 3

$(\lambda/2) + (\lambda/2) + (\lambda/2) - 9 = 0$

$3\lambda/2 = 9$

$3\lambda = 18$

$\boxed{\lambda = 6} \Rightarrow \boxed{x=3, \ y=3, \ z=3}$

$f(3,3,3) = 3^2+3^2+3^2$

$= 9+9+9$

$= 27$

$\therefore$ Minimo $= f(3,3,3) = 27$

17) Maximizar = $f(x,y,z) = xyz$

Restriccion = $x + y + z = 32$

$x - y + z = 0$

$\nabla f = \lambda \nabla g + \mu \nabla h$

$\langle yz, xz, xy \rangle = \lambda \langle 1,1,1 \rangle + \mu \langle 1,-1,1 \rangle$

$$\left.\begin{array}{ll} 1) & yz = \lambda + \mu \\ 2) & xz = \lambda - \mu \\ 3) & xy = \lambda + \mu \\ 4) & x + y + z - 32 = 0 \\ 5) & x - y + z = 0 \end{array}\right\}$$

Restar 5 de 4

$$\begin{array}{rl} & x - y + z = 0 \\ - & x + y + z = 32 \\ \hline & 0 - 2y + 0 = -32 \end{array}$$

$-2y = -32$

$\boxed{y = 16}$

Ahora igualar 1 y 2

$yz = xy$

$(16)z = x(16)$

$16z = 16x$

$z = x$

Sustituir en 4 $z = x$ y $y = 16$

$x + y + z = 32$

$x + 16 + x = 32$

$2x = 32 - 16$

$2x = 16$

$\boxed{x = 8}$

Sustituir $x = 8$ y $y = 16$ en 5

$x - y + z = 0$

$8 - 16 + z = 0$

$-8 + z = 0$

$\boxed{z = 8}$

Ahora =

$f(8,16,8) = (8)(16)(8) = 1{,}024$

$\therefore$ El máximo es = 1,024

$f(8,16,8) = 1{,}024$