13.5 Ejercicios

- Hallar dw/dt utilizando la regla de la cadena apropiada.

(3) $w = x \operatorname{sen} y$

$x = e^t, \quad y = \pi - t.$

$$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{\partial t} + \frac{\partial w}{\partial y}\frac{dy}{dt}$$

$$= (e^t)\sin y + x\cos y(-1)$$

$$= \sin(\pi - t)\, e^t - e^t\cos(\pi - t)$$
$$= e^t \sin t + e^t \cos t.$$

- Hallar $\frac{dw}{dt}$ a) utilizando la regla de la cadena apropiada y b) convirtiendo w en función de t antes de derivar.

(7) $w = x^2 + y^2 + z^2, \quad x = \cos t, \quad y = \operatorname{sen} t, \quad z = e^t$

a) $$\frac{dw}{dt} = \frac{\partial w}{\partial x}\frac{dx}{dt} + \frac{\partial w}{\partial y}\frac{dy}{dt} + \frac{\partial w}{\partial z}\frac{dz}{dt}$$

$$= 2x(-\sin t) + 2y(\cos t) + 2z(e^t)$$
$$= -2\cos t \sin t + 2\sin t\cos t + 2e^{2t} = \boxed{2e^{2t}}$$

b) $w = \cos^2 t + \sin^2 t + e^{2t} = 1 + e^{2t}$

$$\frac{dw}{dt} = \boxed{2e^{2t}}$$

• Hallar $\frac{d^2w}{dt^2}$ utilizando la regla de la cadena apropiada. Evaluar $\frac{d^2w}{dt^2}$ en el valor de t dado.

(14) $w = \frac{x^2}{y}$, $x = t^2$, $y = t+1$, $t = 1$.

$x = t^2$

$y = t+1$

$t = 1$

$$\frac{dw}{dt} = \frac{\partial w}{\partial y}\frac{dy}{dt} = \frac{2x}{y}(2t) + \frac{(-x^2)}{y^2}(1)$$

$$= \frac{2t^2(2t)}{t+1} - \frac{t^4}{(t+1)^2} = \frac{(t+1)(4t^3) - t^4}{(t+1)^2}$$

$$= \frac{3t^4 + 4t^3}{(t+1)^2}$$

$$\frac{d^2w}{dt^2} = \frac{(t+1)^2(12t^3 + 12t^2) - (3t^4 + 4t^3)\,2(t+1)}{(t+1)^4}$$

$$t = 1;\ \frac{d^2w}{dt^2} = \frac{4(24) - (7)(4)}{16}$$

$$= \frac{68}{18} = \boxed{4.25}$$

- Hallar $\frac{\partial w}{\partial s}$ ∧ $\frac{\partial w}{\partial t}$ utilizando la regla de la cadena apropiada y evaluar cada derivada en los valores de s ∧ t dados.

(17) $w = \text{sen}(2x + 3y)$.

$x = s + t$, $y = s - t$

$$\frac{\partial w}{\partial s} = 2\cos(2x + 3y) + 3\cos(2x + 3y)$$

$$= 5\cos(2x + 3y) = \boxed{5\cos(5s - t)}$$

$$\frac{\partial w}{\partial t} = 2\cos(2x + 3y) - 3\cos(2x + 3y)$$

$$= -\cos(2x + 3y) = -\cos(5s - t)$$

Cuando $s = 0$ ∧ $t = \frac{\pi}{2}$, $\frac{\partial w}{\partial s} = 0$ ∧ $\frac{\partial w}{\partial t} = 0$.

- Hallar $\frac{\partial w}{\partial r}$ ∧ $\frac{\partial w}{\partial \theta}$ a) utilizando la regla de la cadena apropiada
b) Convirtiendo w en una función de r y θ antes de derivar

(20) $w = x^2 - 2xy + y^2$; $x = r + \theta$; $y = r - \theta$

a) $$\frac{\partial w}{\partial r} = (2x - 2y)(1) + (-2x + 2y)(1) = 0$$

$$\frac{\partial w}{\partial \theta} = (2x - 2y)(1) + (-2x + 2y)(-1)$$

$$= 4x - 4y$$

$$= 4(x - y) = 4[(r + \theta) - (r - \theta)] = 4[\cancel{x} + \theta - \cancel{r} + \theta]$$

$$= 4\theta + 4\theta = \boxed{8\theta}$$

b) $w = (r+\theta^2) - 2(r+\theta)(r-\theta) + (r-\theta)^2$

$= (r^2 + 2r\theta + \theta^2) - 2(r^2 - \theta^2) + (r^2 - 2r\theta + \theta^2)$

$= r^2 \cancel{+2r\theta} + \theta^2 - 2r^2 + 2\theta^2 + r^2 \cancel{-2r\theta} + \theta^2$

$= \cancel{2r^2} - \cancel{2r^2} + 2\theta^2 + 2\theta^2 = \boxed{4\theta^2}$

- Hallar $\frac{\partial w}{\partial s}$ ∧ $\frac{\partial w}{\partial t}$ utilizando la regla de la cadena apropiado

(25) $w = ze^{xy}$, $x = s$, $y = s+t$, $z = st$.

$$\frac{\partial w}{\partial s} = yze^{xy}(1) + xze^{xy}(1) + e^{xy}(t)$$

$$= e^{(s-t)(s+t)}[(s+t)st + (s-t)st + t]$$

$$= e^{(s-t)(s+t)}[2s^2t + t]$$

$$= te^{s^2-t^2}(2s^2+1)$$

$$\frac{\partial w}{\partial t} = yze^{xy}(-1) + xze^{xy}(1) + e^{xy}(s)$$

$$= e^{(s-t)(s+t)}[-(s+t)(st) + (s-t)st + s]$$

$$= e^{(st)(s+t)}[-2st^2 + s]$$

$$= se^{s^2-t^2}(1-2t^2)$$

- Hallar dy/dx por derivación implícita

(27) $x^2 - xy + y^2 - x + y = 0$

$$\frac{dy}{dx} = \frac{F_x(x,y)}{F_y(x,y)} = \frac{2x-y-1}{-x+2y+1}$$

$$= \boxed{\frac{y-2x+1}{2y-x+1}}$$

- Hallar las primeras derivadas parciales de z por derivación implícita.

(33) $x^2 + 2yz + z^2 = 1$.

$$\frac{\partial z}{\partial x} = \frac{F_x(x,y,z)}{F_z(x,y,z)} = \frac{-2x}{2y+2z} = \frac{-x}{y+z}$$

$$\frac{\partial z}{\partial y} = \frac{F_y(x,y,z)}{F_z(x,y,z)} = \frac{-2z}{2y+2z} = \frac{-z}{y+z}$$

- Hallar las primeras derivadas parciales de w por derivación implícita.

(39) $xy + yz - wz + wx = 5$

$F(x,y,z,w) = xy + yz - wz + wx - 5$.

$F_x = y + w$ | $F_z = y - w$

$F_y = x + z$ | $F_w = -z + w$

$$\frac{\partial w}{\partial x} = -\frac{F_x}{F_w} = -\frac{y+w}{-z+x} = \frac{y+w}{z-x}$$

$$\frac{\partial w}{\partial y} = \frac{F_y}{F_w} = \frac{x+z}{-z+x} = \frac{x+z}{z-x}$$

$$\frac{\partial w}{\partial z} = \frac{F_z}{F_w} = -\frac{y-w}{-z+x} = \frac{y-w}{z-x}$$