Sección 3.3

19.

$$y^{(3)} + y'' - y' - y = 0$$

$$r^3 + r^2 - r - 1 = 0$$

$$(r+1)(r+1)(r-1) = 0$$

$r_1 = -1$
$r_2 = -1$
$r_3 = 1$

$$y_c = C_1 e^{-x} + C_2 x e^{-x} + C_3 e^{x}$$

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$$y(x) = (A + Bx + Cx^2)e^{2x}$$

$$y_1 = e^{rx} \qquad e^{rx} = e^{2x}$$

$$r = 2 \qquad r - 2 = 0$$

$$(r-2)^3 = 0 \qquad r^3 - 6r^2 + 12r - 8 = 0$$

$$y''' - 6y'' + 12y' - 8y = 0$$

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$$y(x) = (A + Bx + Cx^2)\cos 2x + (D + Ex + Fx^2)\operatorname{sen} 2x$$

$$y(x) = A\cos 2x + Bx\cos 2x + Cx^2\cos 2x + D\operatorname{sen} 2x + Ex\operatorname{sen} 2x + Fx^2\operatorname{sen} 2x$$

$$y(x) = (A\cos 2x + D\operatorname{sen} 2x) + x(B\cos 2x + E\cos 2x) + x^2(C\cos 2x + F\operatorname{sen} 2x)$$

$y(x) = e^{0\cdot x}[A\cos 2x + D\operatorname{sen} 2x] + e^{0x}x[B\cos 2x + E\operatorname{sen} 2x]$

$+ e^{0x}x^2[C\cos 2x + F\operatorname{sen} 2x]$

Por teorema, las raíces de la ecuación característica son $r = 0 \pm 2i$ con multiplicidad $k = 3$ $\longrightarrow$ Entonces:

$$[(r-(0+2i))(r-(0-2i))]^3 = 0$$

$$[(r-0)-2i \quad (r-0)+2i]^3 = 0$$

$$[(r-0)^2-(2i)^2]^3 = 0$$

$$[r^2 - 4i^2]^3 = 0$$

$$[r^2 + 4]^3 = 0$$

$$(r^2)^3 + 3(r^2)^2\cdot 4 + 3(r^2)4^2 + 4^3 = 0$$

$$r^6 + 12r^4 + 48r^2 + 64 = 0$$

Entonces:

Edo orden superior lineal homogénea

$$y^{(6)} + 12y^{(4)} + 48y'' + 64y = 0$$

55 :

$$x^3y''' - x^2y'' + xy' = 0$$

$$v = \ln|x|$$

$$\frac{dv}{dx} = \frac{1}{x}$$

$$y' = \frac{dy}{dx} = \frac{dy}{dv}\cdot\frac{dv}{dx} = \frac{dy}{dv}\cdot\frac{1}{x}$$

$$y'' = \frac{d^2y}{dx} = \frac{d^2y}{dv^2}\cdot\frac{dv}{dx}\cdot\frac{1}{x} - \frac{dy}{dv}\cdot\frac{1}{x^2} = \frac{d^2y}{dv^2}\cdot\frac{1}{x^2} - \frac{dy}{dv}\cdot\frac{1}{x^2}$$

$$y''' = \frac{d^3y}{dx^3} = \frac{d^3y}{dv^3}\frac{dv}{dx}\frac{1}{x^2} - \frac{d^2y}{dv^2}\frac{2}{x^3} - \frac{d^2y}{dv^2}\cdot\frac{dv}{dx}\frac{1}{x^2} + \frac{dy}{dv}\cdot\frac{2}{x^3}$$

$$= \frac{d^3y}{dv^3}\cdot\frac{1}{x^3} - \frac{d^2y}{dv^2}\frac{3}{x^3} + \frac{dy}{dv}\cdot\frac{2}{x^3}$$

$$\cancel{x^3}\left(\frac{d^3y}{dv^3}\frac{1}{\cancel{x^3}} - \frac{d^2y}{dv^2}\frac{3}{\cancel{x^3}} + \frac{dy}{dv}\cdot\frac{2}{\cancel{x^3}}\right) -$$

$$\cancel{x^2}\left(\frac{d^2y}{dv^2}\cdot\frac{1}{\cancel{x^2}} - \frac{dy}{dv}\cdot\frac{1}{\cancel{x^2}}\right) + \cancel{x}\left(\frac{dy}{dv}\cdot\frac{1}{\cancel{x}}\right) = 0$$

$$\frac{d^3y}{dv^3} - \cancel{\frac{3d^2y}{dv^2}} + \cancel{\frac{2dy}{dv}} - \cancel{\frac{d^2y}{dv^2}} + \cancel{\frac{dy}{dv}} + \cancel{\frac{dy}{dv}} = 0$$

$$\frac{d^3y}{dv^3} - 4\frac{d^2y}{dv^2} + 4\frac{dy}{dv} = 0$$

$r^3 - 4r^2 + 4r = 0$ $\quad r_1 = 0$

$r(r^2 - 4r + 4) = 0$ $\quad r_2 = 2$

$r(r-2)(r-2) = 0$ $\quad r_2 = 2$

$$y(v) = C_1 e^{(0)v} + C_2 e^{(2)v} + C_3 e^{(2)v}$$

$$y(x) = C_1 + C_2 e^{2\ln x} + C_3 \ln x\, e^{2\ln x}$$

$$= C_1 + (C_2 + C_3 \ln x)\, x^2$$

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$$x^3 y''' + 6x^2 y'' + 7xy' + y = 0$$

$$v = \ln|x|$$

$$y' = \frac{dy}{dx} = \frac{dy}{dv} \cdot \frac{dv}{dx} = \frac{dy}{dv} \cdot \frac{1}{x}$$

$$\frac{dv}{dx} = \frac{1}{x}$$

$$y'' = \frac{d^2y}{dx} = \frac{d^2y}{dv^2} \cdot \frac{dv}{dx} \frac{1}{x} - \frac{dy}{dv} \cdot \frac{1}{x^2}$$

$$= \frac{d^2y}{dv^2} \cdot \frac{1}{x^2} - \frac{dy}{dv} \cdot \frac{1}{x^2}$$

$$y''' = \frac{d^3y}{dx^3} = \frac{d^3y}{dv^3} \cdot \frac{dv}{dx} \frac{1}{x^2} - \frac{d^2y}{dv^2} \frac{2}{x^3} - \frac{d^2y}{dv} \frac{dv}{dx} \frac{1}{x^2} +$$

$$\frac{dy}{dv} \cdot \frac{2}{x^3} = \frac{d^3y}{dv^3} \cdot \frac{1}{v^3} - \frac{d^2y}{dv^2} \frac{3}{x^3} + \frac{dy}{dv} \cdot \frac{2}{x^3}$$

$$\cancel{x^3}\left(\frac{d^3y}{dv^3} \frac{1}{\cancel{x^3}} - \frac{d^2y}{dv^2} \frac{3}{\cancel{x^3}} + \frac{dy}{dv} \frac{2}{\cancel{x^3}}\right) + 6\cancel{x^2}\left(\frac{d^2y}{dv^2} \frac{1}{\cancel{x^2}} - \frac{dy}{dv} \frac{1}{\cancel{x^2}}\right) +$$

$$7\cancel{x}\left(\frac{dy}{dv} \frac{1}{\cancel{x}}\right) + y = 0$$

$$\frac{d^3y}{dv^3} - \cancel{3\frac{d^2y}{dv^2}} + 2\frac{dy}{dv} + \cancel{6\frac{d^2y}{dv^2}} - 6\frac{dy}{dv} + 7\frac{dy}{dv} + y = 0$$

$$\frac{d^3y}{dv^3} + 3\frac{d^2y}{dv^2} + 3\frac{dy}{dv} + y = 0$$

$$r^3 + 3r^2 + 3r + 1 = 0$$

$$(r+1)^3 = 0 \qquad r_1 = r_2 = r_3 = -1$$

$$y(v) = (c_1 + c_2 v + c_3 v^2)\,e^{-v}$$

$$y(x) = (c_1 + c_2 \ln x + c_3 (\ln x)^2)\,e^{-\ln x} =$$

$$(c_1 + c_2 \ln x + c_3^{(\ln x)^2})\,x^{-1}$$