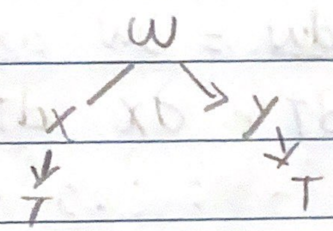
Cálculo Vectorial

Ejercicios Unidad 13.5 → Regla de la Cadena para funciones de varias variables

En los ejercicios 1 a 4, hallar dw/dT utilizando la reglas de la cadena apropiadas.

3) $w = x \operatorname{sen} y$

$x = e^T, \quad y = \pi - T$



$$\frac{dw}{dT} = \frac{\partial w}{\partial x}\frac{dx}{dT} + \frac{\partial w}{\partial y}\frac{dy}{dT}$$

$$= \left[\left(x \cdot \cos y(0) + \operatorname{sen} y\,(1)\right) \cdot e^T \cdot (1)\right] + \left[\left(x \cos y\,(1) + \operatorname{sen} y \cdot 0\right)(-1)\right]$$

$$= \operatorname{sen} y\, e^T + (x \cos y)(-1)$$

$$= \operatorname{sen} y\, e^T - x \cos y$$

$$= \operatorname{sen}(\pi - T)\, e^T - e^T \cos(\pi - T)$$

En los ejercicios 5-10, hallar dw/dT

a) utilizando la regla de la cadena apropiada

b) Convirtiendo w en función de T antes de derivar.

7) $w = x^2 + y^2 + z^2, \quad x = \cos T, \quad y = \operatorname{sen} T, \quad z = e^T$

$$\frac{dw}{dT} = \frac{\partial w}{\partial x}\frac{dx}{dT} + \frac{\partial w}{\partial y}\frac{dy}{dT} + \frac{\partial w}{\partial z}\frac{dz}{dT}$$

$$= 2x \cdot -\operatorname{sen} T + 2y \cdot \cos T + 2z \cdot e^T(1)$$

$$= -2x \operatorname{sen} T + 2y \cos T + 2z e^T$$

$$w = (\cos T)^2 + (\operatorname{sen} T)^2 + (e^T)^2$$

$$w = \cos^2 T + \operatorname{sen}^2 T + e^{2T}$$

$$\frac{dw}{dT} = 2\cos T(-\operatorname{sen}(T)) + 2\operatorname{sen} T(\cos(T)) + e^{2T} \cdot 2$$

$$= -2\cos T \operatorname{sen} T + 2\operatorname{sen} T \cos T + 2e^{2T}$$

$$= 2e^{2T}$$

En los ejercicios 13 y 14, hallar d^2w/dT^2 utilizando la regla de la Cadena apropiada. Evaluar d^2w/dT^2 en el valor de T dado.

14) $w = \dfrac{x^2}{y}$, $x = T^2$, $y = T+1$, $T = 1$

$$\frac{dw}{dT} = \frac{\partial w}{\partial x}\frac{dx}{dy} + \frac{\partial w}{\partial y}\frac{dy}{dT}$$

$$= \frac{2x}{y} \cdot 2T - \frac{x^2}{y^2}(1)$$

$$= \frac{2x \cdot 2T}{y} - \frac{x^2}{y^2} = \frac{4xT}{y} - \frac{x^2}{y^2} = \frac{4xTy - x^2}{y^2}$$

$$\frac{d^2w}{dT^2} = \frac{\partial w}{\partial x}\frac{dx}{dT} + \frac{\partial w}{\partial y}\frac{dy}{dT} + \frac{\partial w}{\partial T}$$

$$= \frac{(4Ty - 2x)(2T)}{y^2} + \frac{2x^2 - 4xTy}{y^3}(1) + \frac{4x}{y}$$

$$= \frac{(4T(T+1) - 2(T^2))2T}{(T+1)^2} + \frac{2(T^2)^2 - 4(T^2)T(T+1)}{y^3} + \frac{4(T^2)}{(T+1)}$$

$$\left.\frac{d^2w}{dT^2}\right|_{T=1} = \frac{(4(1)(2) - 2(1))2(1)}{(4)} + \frac{2(1) - 4(1)(1)(2)}{8} + \frac{4(1)}{(2)}$$

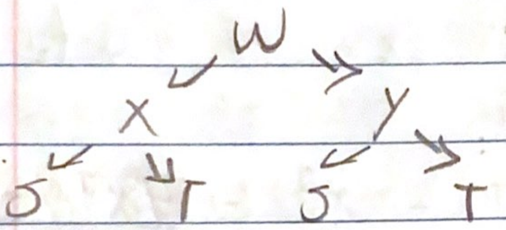
$$= \frac{12}{(4)} + \frac{-6}{8} + 2$$

$$= 3 + 2 - \frac{3}{4} = 5 - \frac{3}{4} = \boxed{\frac{17}{4}}$$

En los ejercicios 15 a 18, hallar $\partial w/\partial s$ y $\partial w/\partial t$ utilizando la regla de la cadena apropiada y evaluar cada derivada parcial en los valores de s y t dados.

17) $w = \operatorname{sen}(2x + 3y)$ $\quad s = 0, \; t = \pi/2$

$x = s + t, \quad y = s - t$



$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s}$$

$$= \cos(2x+3y)\cdot(2)\cdot 1 + \cos(2x+3y)\cdot(3)\cdot 1$$

$$= 2\cos(2x+3y) + 3\cos(2x+3y)$$

$$= 5\cos(2x+3y)$$

$$= 5\cos(2(s+t) + 3(s-t))$$

$$\left.\frac{\partial w}{\partial s}\right|_{s=0,\,t=\pi/2} = 5\cos\left(2\left(0+\pi/2\right) + 3\left(0 - \pi/2\right)\right)$$

$$= 5\cos\left(\pi - \frac{3\pi}{2}\right) = 5\cos(-\pi/2)$$

$$= 5 \cdot 0$$

$$= \boxed{0}$$

$$\frac{\partial w}{\partial T} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial T} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial T}$$

$$= \cos(2x+3y)(2)\cdot(1) + \cos(2x+3y)(3)(-1)$$

$$= 2\cos(2x+3y) - 3\cos(2x+3y)$$

$$= -\cos(2x+3y)$$

$$= -\cos(2(s+T)+3(s-T))$$

$$\left.\frac{\partial w}{\partial T}\right|_{T=\pi/2,\, s=0} = -\cos(2(0+\pi/2)+3(0-\pi/2))$$

$$= -\cos(\pi - 3\pi/2)$$

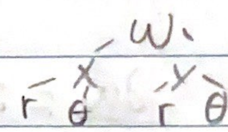
$$= -\cos(-\pi/2)$$

$$= -\cdot 0 = \boxed{0}$$

En los ejercicios 19 a 22 hallar $\partial w/\partial r$ y $\partial w/\partial \theta$

a) utilizando la regla de la cadena apropiada

b) Convirtiendo w en una función de r y θ antes de derivar



20) $w = x^2 - 2xy + y^2$; $x = r+\theta$; $y = r-\theta$

a) $$\frac{\partial w}{\partial r} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial r} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial r}$$

$-2(r^2-\theta^2) = 2r^2 - 2\theta^2$

$$= 2x - 2y\cdot(1) + (-2x+2y)(1)$$

$$= 2x - 2y - 2x + 2y = 0$$

a) $\frac{\partial w}{\partial \theta} = \frac{\partial w}{\partial x}\frac{dx}{d\theta} + \frac{\partial w}{\partial y}\frac{dy}{d\theta}$

$= (2x-2y)(1) + (-2x+2y)(-1)$

$= 2x-2y + (2x-2y)$

$= 4x-4y$

b) $w = x^2 - 2xy + y^2 = (r+\theta)^2 - 2(r+\theta)(r-\theta) + (r-\theta)^2$

$w = (r+\theta)^2 - 2(r^2-\theta^2) + (r-\theta)^2$

$\frac{\partial w}{\partial r} = 2(r+\theta)\cdot(1) - 2(2r) + 2(r-\theta)(1)$

$= 2(r+\theta) - 4r + 2(r-\theta) = \boxed{0}$

$\frac{\partial w}{\partial \theta} = 2(r+\theta)(1) - 2(-2\theta) + 2(r-\theta)(-1)$

$= 2(r+\theta) + 4\theta - 2(r-\theta)$

$= 2r + 2\theta + 4\theta - 2r + 2\theta$

$\partial w = 4\theta + 4\theta = \boxed{8\theta}$

En las ejercicios 23 a 26, hallar $\partial w/\partial s$ y $\partial w/\partial T$ utilizando la regla de la cadena apropiada.

25) $w = Ze^{xy}$, $x = s-T$, $y = s+T$, $Z = sT$

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial s} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial s} + \frac{\partial w}{\partial Z}\frac{\partial Z}{\partial s}$$

$$= Ze^{xy}(y)(1) + Ze^{xy}(x)(1) + e^{xy}(T)$$

$$= Ze^{xy}y + Ze^{xy}x + e^{xy}(T)$$

$$= e^{xy}(Zy + ZX + (T))$$

$$= e^{(s-T)(s+T)}((sT)(s+T) + (sT)(s-T) + (T))$$

$$= e^{s^2T^2}(s^2T + sT^2 + s^2T - sT^2 + T)$$

$$= e^{s^2-T^2}(2s^2T + T)$$

$$\frac{\partial w}{\partial T} = \frac{\partial w}{\partial x}\frac{\partial x}{\partial T} + \frac{\partial w}{\partial y}\frac{\partial y}{\partial T} + \frac{\partial w}{\partial Z}\frac{\partial Z}{\partial T}$$

$$= Ze^{xy}(y)(-1) + Ze^{xy}(x)(1) + e^{xy}(s)$$

$$= -Ze^{xy}y + Ze^{xy}x + e^{xy}s$$

$$= e^{xy}(-Zy + ZX + s)$$

$$= e^{s^2-T^2}(-(sT)(s+T) + (sT)(s-T) + s)$$

$$= e^{s^2-T^2}(-s^2T - sT^2 + s^2T - sT^2 + s)$$

$$= e^{s^2-T^2}(-2sT^2 + s)$$

En los ejercicios 27 a 30, hallar dy/dx Por derivación implícita

27) $x^2 - xy + y^2 - x + y = 0$

$$\frac{dy}{dx} = \frac{-\partial f/\partial x}{\partial f/\partial y} = -\frac{f_x(x,y)}{f_y(x,y)}$$

$$= \frac{-(2x - y - 1)}{-x + 2y + 1} = \frac{-2x + y + 1}{-x + 2y + 1}$$

En los ejercicios 31 a 38, hallar las Primeras derivadas Parciales de z Por derivación implícita.

33) $x^2 + 2yz + z^2 = 1$

$$\frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = \frac{-2x}{2y + 2z} = \frac{-x}{y + z}$$

$$\frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = \frac{-2z}{2y + 2z} = \frac{-z}{y + z}$$

En los ejercicios 39 a 42, hallar las primeras derivadas parciales de w por derivación implícita.

39) $xy + yz - wz + wx = 5$

$$\frac{\partial w}{\partial x} = \frac{-F_x}{F_w} = -\frac{y+w}{-z+x} = -\frac{y+w}{x-z}$$

$$\frac{\partial w}{\partial y} = \frac{-F_y}{F_w} = -\frac{x+z}{x-z} = -\frac{x+z}{x-z}$$

$$\frac{\partial w}{\partial z} = \frac{-F_z}{F_w} = -\frac{y-w}{x-z} = -\frac{y-w}{x-z}$$