Sección 1.5

27

$$(x + ye^{y})\frac{dy}{dx} = 1$$

• Considere y como variable independiente y x como dependiente

$$x + y \cdot e^{y} = \frac{dx}{dy}$$

$$\rho(y) = e^{\int p(y)\,dy}$$

$$\frac{dx}{dy} - 1 \cdot x = ye^{y}$$

$$\rho(y) = e^{\int -1\,dy}$$

$$\rho(y) = e^{-y}$$

$$e^{-y}\left(\frac{dx}{dy} - 1 \cdot x\right) = e^{-y}(ye^{y})$$

$$\frac{d}{dy}\left(e^{-y} \cdot x\right) = y$$

$$\int \frac{d}{dy}(e^{-y} \cdot x)\,dy = \int y\,dy$$

$$e^{-y}x = \frac{y^2}{2} + C$$

$$x(y) = e^{y}\left(\frac{1}{2}y^2 + C\right)$$

$$x(y) = \frac{1}{2}y^2e^{y} + Ce^{y}$$

29.

$$\frac{dy}{dx} = 1 + 2xy$$

$$\frac{dy}{dx} - 2x \cdot y = 1$$

$$\rho(x) = e^{\int p(x)dx}$$

$$\rho(x) = e^{\int -2x\,dx}$$

$$\rho(x) = e^{-x^2}$$

$$e^{-x^2}\left(\frac{dy}{dx} - 2x \cdot y\right) = e^{-x^2}$$

$$\frac{d}{dx}\left(e^{-x^2}y\right) = e^{-x^2}$$

$$d\left(e^{-x^2}y\right) = e^{-x^2}\,dx$$

$$\int d(e^{-x^2} y) = \int_0^x e^{-t^2} dt + c$$

$$e^{-x^2} y(x) = \int_0^x e^{-t^2} dt + c$$

$$y(x) = e^{x^2} \left(\int_0^x e^{-t^2} dt + c \right)$$

debemos reescribir $y(x)$ en terminos de

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

$$y(x) = e^{x^2} \left(\frac{\sqrt{\pi}}{2} \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt + c \right)$$

$$y(x) = e^{x^2} \left(\frac{\sqrt{\pi}}{2} erf(x) + c \right)$$

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$$2x \frac{dy}{dx} = y + 2x \cos x \quad ; \; y(1) = 0$$

$$2x \frac{dy}{dx} - y = 2x \cos x$$

$$\frac{dy}{dx} - \frac{1}{2x} y = \cos x$$

$$x^{-1/2} \left[\frac{dy}{dx} - \frac{1}{2x} y \right] = x^{-1/2} \cos x$$

$$\frac{d}{dx} \left(x^{-1/2} y \right) = x^{-1/2} \cos x$$

$$d(x^{-1/2} y) = x^{-1/2} \cos x \, dx$$

$$\rho(x) = e^{\int_1^x P(u) du}$$

$$\rho(x) = e^{\int_1^x -\frac{1}{2u} du}$$

$$\rho(x) = e^{\left[-\frac{1}{2} \ln u \right]_1^x}$$

$$\rho(x) = e^{-\frac{1}{2} \ln x + \cancel{\frac{1}{2} \ln 1}^{0}}$$

$$\rho(x) = e^{\ln x^{-1/2}} \qquad x^{-1/2}$$

$$\int d(x^{-1/2}y) = \int_1^x u^{-1/2}\cos u\,du + c$$

$$x^{-1/2}y = \int_1^x u^{-1/2}\cos u\,du + c$$

Como $y(1)=0$: $(1)^{-1/2}(0)=0=\int_1^1 u^{-1/2}\cos u\,du + c$

$$0 = 0 + c$$
$$0 = c$$

$$\rightarrow \quad y(x) = x^{1/2}\int_1^x u^{-1/2}\cos u\,du$$

36.

a. Debemos encontrar $x(t)$:

$v(0) = 60$ gal

$x(0) = 0.16$

$C_e = \frac{1b}{gal}$

$r_e = 2\,\frac{gal}{min}$

$r_s = 3\,\frac{gal}{min}$

$V(60)=0$

$$\frac{dx}{dt} = C_e r_e - r_s\frac{x}{V_0 + (r_e - r_s)t}$$

$$\frac{dx}{dt} = (1)(2) - \frac{(3)x}{60 - 1t}$$

$$\frac{dx}{dt} = 2 - \frac{3}{60-t}x$$

$$\frac{dx}{dt} + \frac{3}{60-t}x = 2$$

$$\int \frac{3}{60-t}\,dt$$

$$(60-t)^{-3}\left[\frac{dx}{dt} + \frac{3}{60-t}x\right] = (60-t)^{-3}(2)$$

$\rho(t) = e$

$\rho(t) = e^{-3\ln|60-t|}$ $\quad t<60$

$$\frac{d}{dt}\left[(60-t)^{-3}x\right] = 2(60-t)^{-3}$$

$\rho(t) = e^{-3\ln(60-t)}$

$$\int \frac{d}{dt}\left[(60-t)^{-3}x\right]dt = \int 2(60-t)^{-3}\,dt$$

$\rho(t) = e^{\ln(60-t)^{-3}} = (60-t)^{-3}$

$$(60-t)^{-3} x = \frac{1}{(60-t)^2} + C$$

$$x(t) = \frac{(60-t)^3}{(60-t)^2} + C(60-t)^3$$

$$x(t) = 60 - t + C(60-t)^3$$

como $x(0)=0$

$$(60-0) + C(60-0)^3 = 0$$

$$60 - C\,60^3 = 0$$

$$C = \frac{1}{3600}$$

$$x(t) = (60-t) - \frac{1}{3600}(60-t)^3$$

Cantidad de sal present en el deposito en todo instante $t \in [0,60]$ min

b. Hallamos el maximo de $x(t)$ en $t \in [0,60]$

$$x'(t) = -1 - \frac{3}{3600}(60-t)^2(-1)$$

$$x'(t) = -1 + \frac{1}{1200}(60-t)^2 = 0$$

$$\frac{1}{1200}(60-t)^2 = 1$$

$$(60-t)^2 = 1200$$

$$60 - t = \pm 20\sqrt{3}$$

$60 - t = 20\sqrt{3}$ ∨ $60 - t = -20\sqrt{3}$

$60 - 20\sqrt{3} = t$ ∨ $60 + 20\sqrt{3} = t$

$t \approx 25.36$ min ∨ $t \approx \cancel{94.64}$ min

$t \notin [0,60]$ min

$\rightarrow x(0) = 0$ lb

$x(25.36) \simeq 23.09$ lb $= x_{max}$

$x(60) = 0$ lb

38.

$V_1(0) = 100$ gal

$V_2(0) = 200$ gal

$X(0) = 50$ lb

$y(0) = 50$ lb

$r_{e_1} = r_{s_1} = 5 \frac{gal}{min}$

$r_{s_2} = 5 \frac{gal}{min}$ $\qquad c_{e_1} = 0$

$\rightarrow r_{e_2} = r_{s_1}$

$\rightarrow c_{e_2} = c_{s_1}$

a

Tanque 1:

$$\frac{dx}{dt} = c_{e_1} \cdot r_{e_1} - r_{s_1} \cdot c_{s_1}$$

$$\frac{dx}{dt} = (0)\cdot(5) - (5)\frac{x(t)}{V_1(t)}$$

$$\frac{dx}{dt} = -5 \frac{x}{V_1(0) + \underbrace{(r_{e_1} - r_{s_1})}_{\text{cero: } r_{e_1} = r_{s_1}} t}$$

$$\frac{dx}{dt} = -\frac{5x}{100}$$

$$\frac{dx}{x} = -\frac{5}{100} dt$$

$$\int \frac{dx}{x} = \int -\frac{1}{20} dt$$

$$\ln x = -\frac{1}{20} t + c_1$$

$X(0) = 50$

$\ln 50 = -\frac{1}{20}(0) + c_1$

$\ln 50 = c_1$

$$\ln x = \frac{-1}{20} t + \ln 50$$

$$e^{\ln x} = e^{-\frac{1}{20}t + \ln 50} = e^{\frac{-1}{20}t} \; e^{\ln 50}$$

$$x(t) = 50 \, e^{\frac{-1}{20}t}$$

b.

Tanque 2

$$\frac{dy}{dt} = C_{e_2} \, r_{e_2} - C_{s_2} \, r_{s_2}$$

$$\frac{dy}{dt} = \underbrace{\frac{x(t)}{V_1(t)}}_{C_{s_1}} \underbrace{(5)}_{r_{s_1}} - \frac{y(t)}{V_2(t)} (5)$$

$$\frac{dy}{dt} = \frac{x}{V_1(0) + (\underbrace{r_{e_1} - r_{s_1}}_{0})t} \cdot 5 - \frac{y}{V_2(0) + (\underbrace{r_{e_2} - r_{s_2}}_{0})t} \cdot 5$$

$$\frac{dy}{dt} = \frac{5x}{100} - \frac{5y}{200}$$

$$\rightarrow \frac{dy}{dt} = \frac{5}{100} \left(50 e^{-\frac{1}{20}t}\right) - \frac{5}{200} y$$

$$\rho(t) = e^{\int \frac{1}{40} dt}$$

$$\frac{dy}{dt} + \frac{1}{40} y = \frac{5}{2} e^{-\frac{1}{20}t}$$

$$\rho(t) = e^{\frac{1}{40}t}$$

$$e^{\frac{1}{40}t} \left[\frac{dy}{dt} + \frac{1}{40} y \right] = e^{\frac{1}{40}t} \, \frac{5}{2} \, e^{-\frac{1}{20}t}$$

$$\frac{d}{dt} \left(e^{\frac{1}{40}t} y \right) = \frac{5}{2} e^{-\frac{1}{40}t}$$

$$\int \frac{d}{dt} \left(e^{\frac{1}{40}t} y \right) dt = \int \frac{5}{2} e^{\frac{-1}{40}t} dt$$

$$e^{\frac{1}{40}t} y = -100 \, e^{\frac{-1}{40}t} + C_2$$

$$y(t) = e^{-\frac{1}{40}t} \left(-100\, e^{-\frac{1}{40}t} + C_2\right)$$

$$y(t) = -100\, e^{-\frac{1}{20}t} + C_2\, e^{-\frac{1}{40}t}$$

y como $y(0) = 50$

$$-100\, e^{0} + C_2\, e^{0} = 5$$

$$C_2 = 150$$

$$y(t) = 100\, e^{-\frac{1}{20}t} + 150\, e^{-\frac{1}{40}t}$$

c)

$$y'(t) = -100\, e^{-\frac{1}{20}t}\left(-\frac{1}{20}\right) + 150\, e^{-\frac{1}{40}t}\left(-\frac{1}{40}\right)$$

$$y'(t) = 5\, e^{-\frac{1}{20}t} - \frac{15}{4}\, e^{-\frac{1}{40}t} = 0$$

$$5\, e^{-\frac{1}{20}t} = \frac{15}{4}\, e^{-\frac{1}{40}t}$$

$$\frac{20}{15} = e^{-\frac{1}{40}t}\, e^{\frac{1}{20}t}$$

$$\frac{20}{15} = e^{\frac{1}{40}t}$$

$$\ln(4/3) = \frac{1}{40}t$$

$$40 \ln(4/3) = t$$

$$11.5 \text{ min} = t$$

44.

a. $y' = x + y$

$\frac{dy}{dx} - y = x$

$e^{-x}\left(\frac{dy}{dx} - y\right) = x e^{-x}$

$\frac{d}{dx}(e^{-x} y) = x e^{-x}$

$d(e^{-x} y) = x e^{-x} dx$

$\int d(e^{-x} y) = \int x e^{-x} dx$

$e^{-x} y = -x e^{-x} - e^{-x} + c$

$y(x) = -x - 1 + c e^{x}$

$\rho(x) = e^{\int p(x) dx}$

$\rho(x) = e^{\int -1 dx}$

$\rho(x) = e^{-x}$

$\int x e^{-x} dx \quad u = x \quad dv = e^{-x} dx$

$du = dx \quad v = -e^{-x}$

$= -x e^{-x} - \int -e^{-x} dx$

$= -x e^{-x} + \int e^{-x} dx$

$= -x e^{-x} - e^{-x} + c.$

Cuando $x \to \infty$ $y(x) \to -x-1$ asimptoticamente pues $ce^{x} \to 0$ conforme $x \to -\infty$

b. $y(x) = -x - 1 + Ce^{x}$

* $y(5) = -10$

$-5 - 1 + Ce^{5} = -10$

$Ce^{5} = -4$

$C = -\frac{4}{e^{5}}$

$y(-5) = -(-5) - 1 - \frac{4}{e^{5}} e^{-5}$

$= 4 - 4e^{-10}$

$= 3.99982$

$y(5) = -5$

$-5 - 1 + Ce^{5} = -5$

$Ce^{5} = 1$

$C = e^{-5}$

$y(-5) = -(-5) - 1 + e^{-5} \cdot e^{-5}$

$= 4 + e^{-10}$

$= 4,00005$

$y(5) = 0$
$-5 - 1 + ce^{5} = 0$
$ce^{5} = 6$
$c = \frac{6}{e^{5}}$

$y(5) = 5$
$-5 - 1 + ce^{5} = 5$
$ce^{5} = 11$
$c = \frac{11}{e^{5}}$

$y(-5) = -(5) - 1 + \frac{6}{e^{5}} e^{-5}$
$= 4 + 6e^{-10}$
$= 4{,}00027$

$y(-5) = -(-5) - 1 + \frac{11}{e^{5}} e^{-5}$
$= 4 + 11e^{-10}$
$= 4{,}00050$

$y(5) = 10$
$-5 - 1 + ce^{5} = 10$
$ce^{5} = 16$
$c = \frac{16}{e^{5}}$

$y(-5) = -(-5) - 1 + \frac{16}{e^{5}} e^{-5}$
$= 4 + 16e^{-10}$
$= 4{,}00073$