Test on Wednesday 2/14/24

- 75 minutes
- in-class
- bring your own writing implement
- basic calculators allowed (no graphing calculators)
  (you will almost certainly not need one)

Last thing from 7.8

- want to know if

$$\int_a^{\infty} f(x)\,dx$$

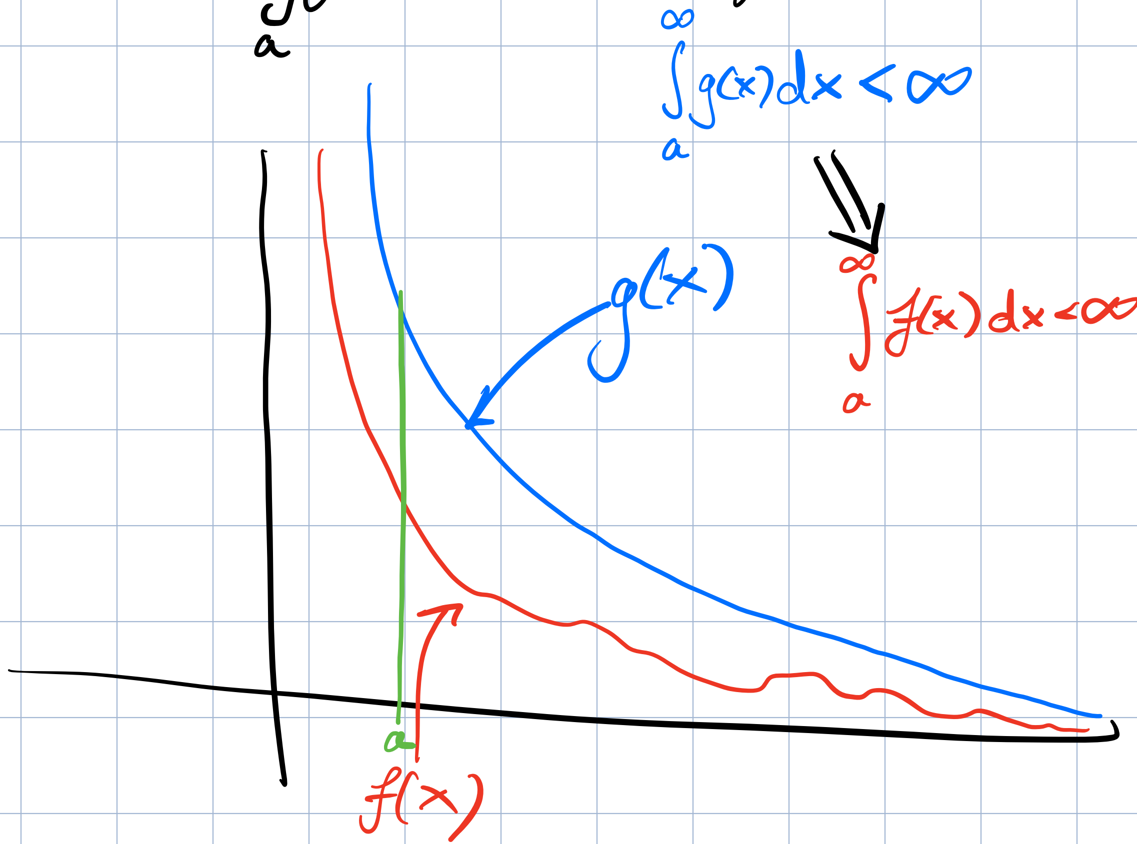
diverges or converges

(i) try to find some $g(x)$ with $g(x) \geq f(x)$

for all $x \geq a$

and show that

$\int_a^\infty g(x)dx$ converges



(ii) $g(x) \leq f(x)$ for $x \geq a$
and $\int_a^\infty g(x)dx$ diverges.

Then $\int_a^\infty f(x)dx$ also diverge

Test Review

§5.5

§6.1, 6.2, 6.3

§7.8

Basics:

~~★~~ - Sketching functions

- polynomials
- sin, cos, tan,
- e^x, $\ln(x)$, $\sqrt{x}$
- finding zeros, finding where functions intersect
- $\frac{1}{x}$, $\frac{1}{x^2}$, $\frac{1}{x^3}$

- standard equation for the circle with center at (a,b) w/ radius r

Standard Derivatives/Integrals

- derivs. and int.'s of e^x, $\ln(x)$, e^{ax}, polynoms., $\sin$, $\cos$, $\frac{1}{x^n}$, root functions
- product, quotient, chain rules

(don't need to know this stuff for tan, inverse trig functions, hyperbolic trig.)

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§5.5: U-Sub.

Idea: "chain rule in reverse"

Look for integrals look like

$$\int f'(g(x))\, \underbrace{g'(x)\,dx}_{\frac{du}{dx}}$$

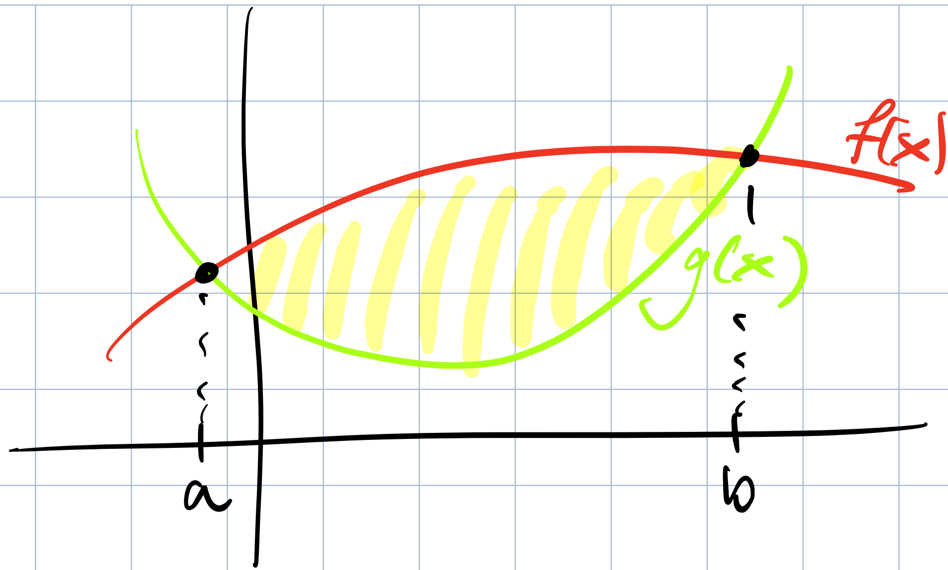
$g(x)=u$

- pick u to be the "ugly" part

- for indef. ints, need to sub back in for u at the end

- for def. ints, need to change bounds, but no substitution is req'd after that

§6.1 Area between curves



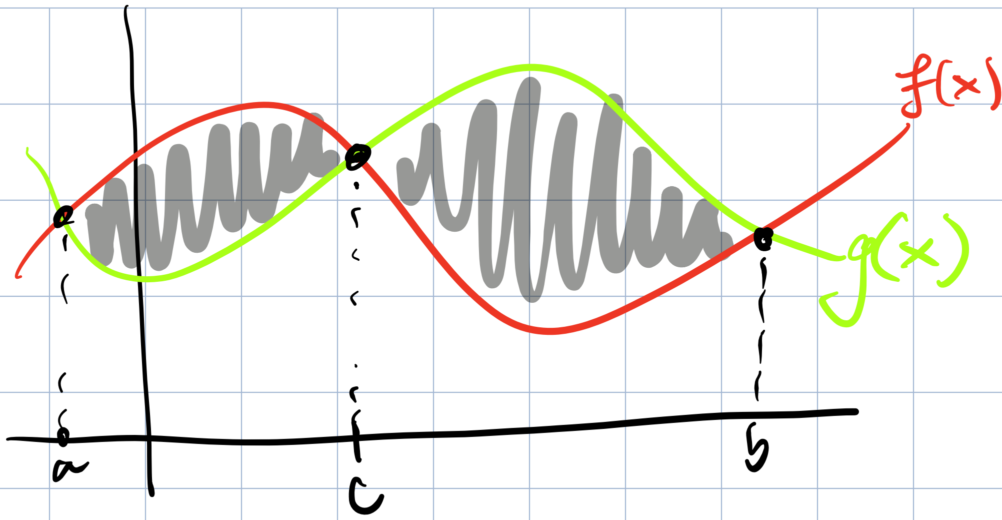
- compute the area between f(x) and g(x) (sometimes other bounds)

i.e. $$\int_a^b \big(f(x)-g(x)\big)\,dx$$

- need to find where g and f intersect (i.e. $f=g$)

- need to figure out which function is larger

Beware of "negative area"



naively compute

$$\int_a^b (f(x) - g(x))\,dx$$

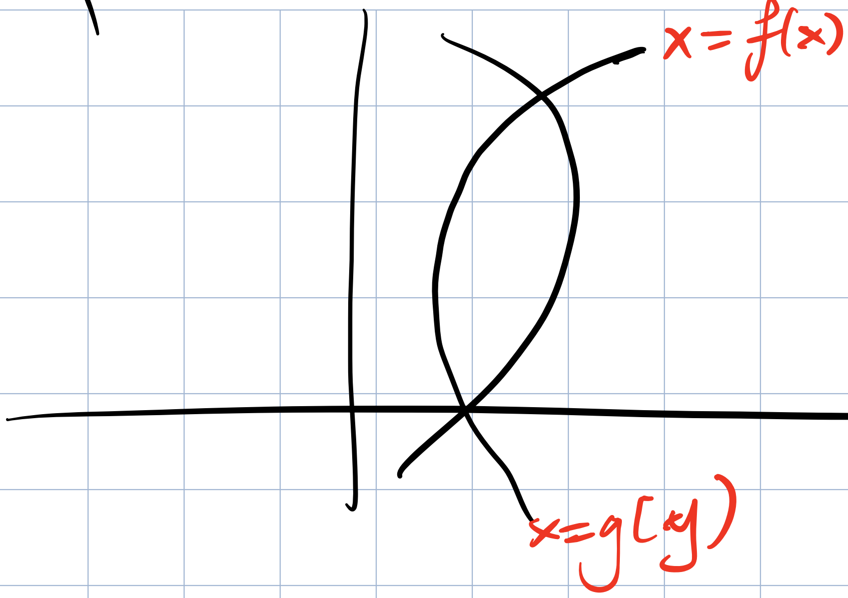
- instead find c and compute

$$\int_a^c f(x) - g(x)\,dx + \int_c^b g(x) - f(x)\,dx$$

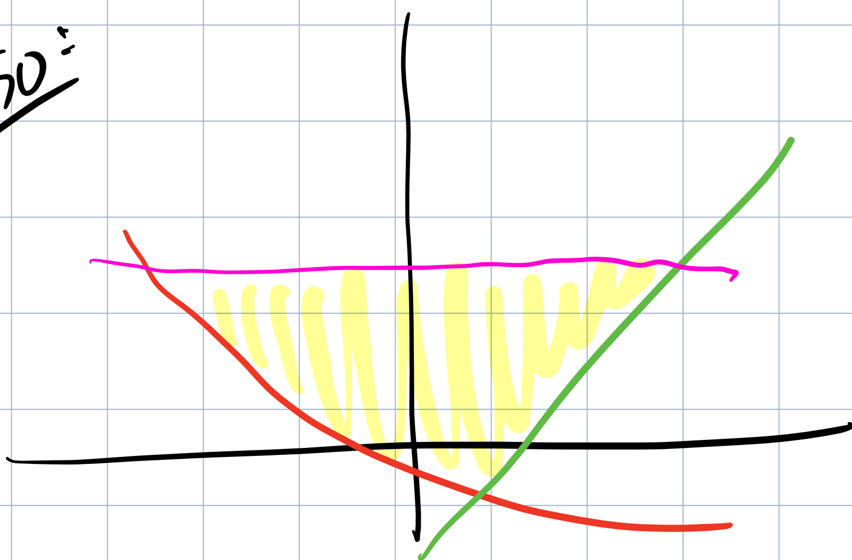
Might need to switch variables!

sibility:

One possi

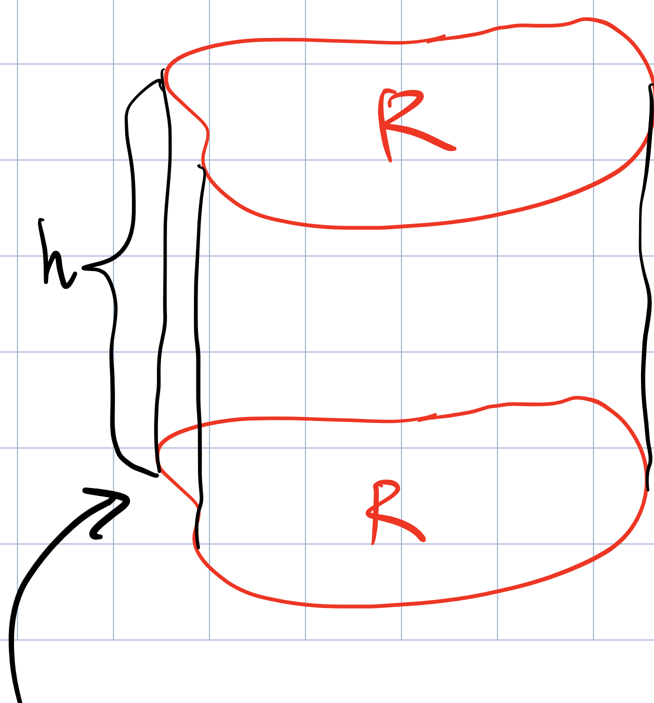


Also:



Volumes

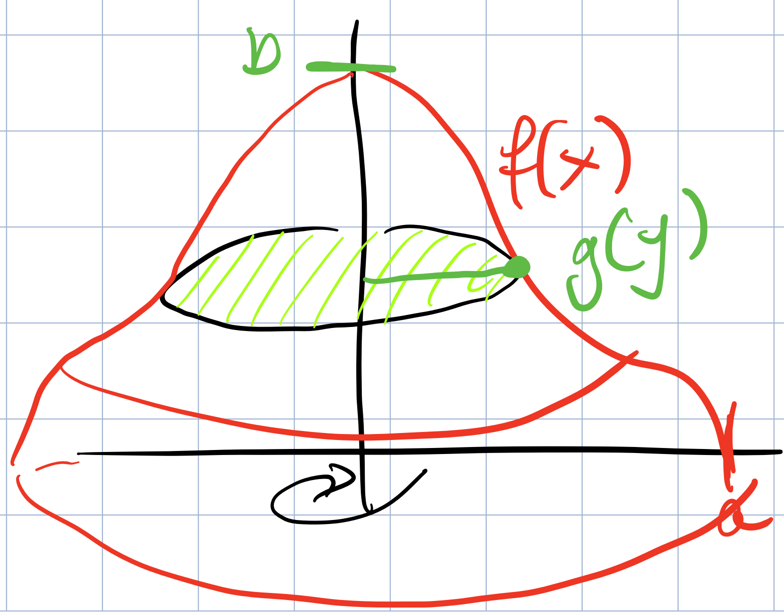
- Volume of cylinders



volume of this is
h·area(R)
- use this to approximate and ultimately define volume for more complex solids

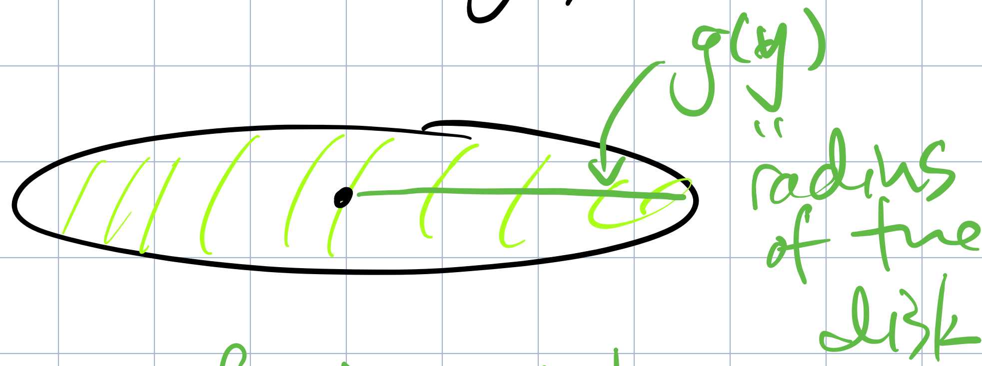
Motto: Think of area as "infinitesimal volume" and then add up all the areas (by an integral)

Disks/Washers



rotating about y-axis

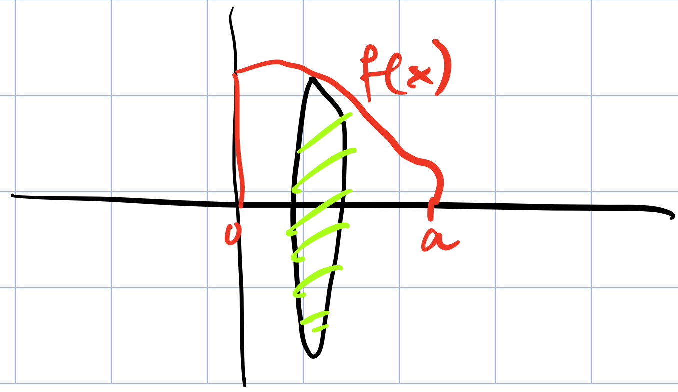
(i) solve for x, i.e. write the curve as $x = g(y)$ for some g



- area of the disk is $\pi g(y)^2$

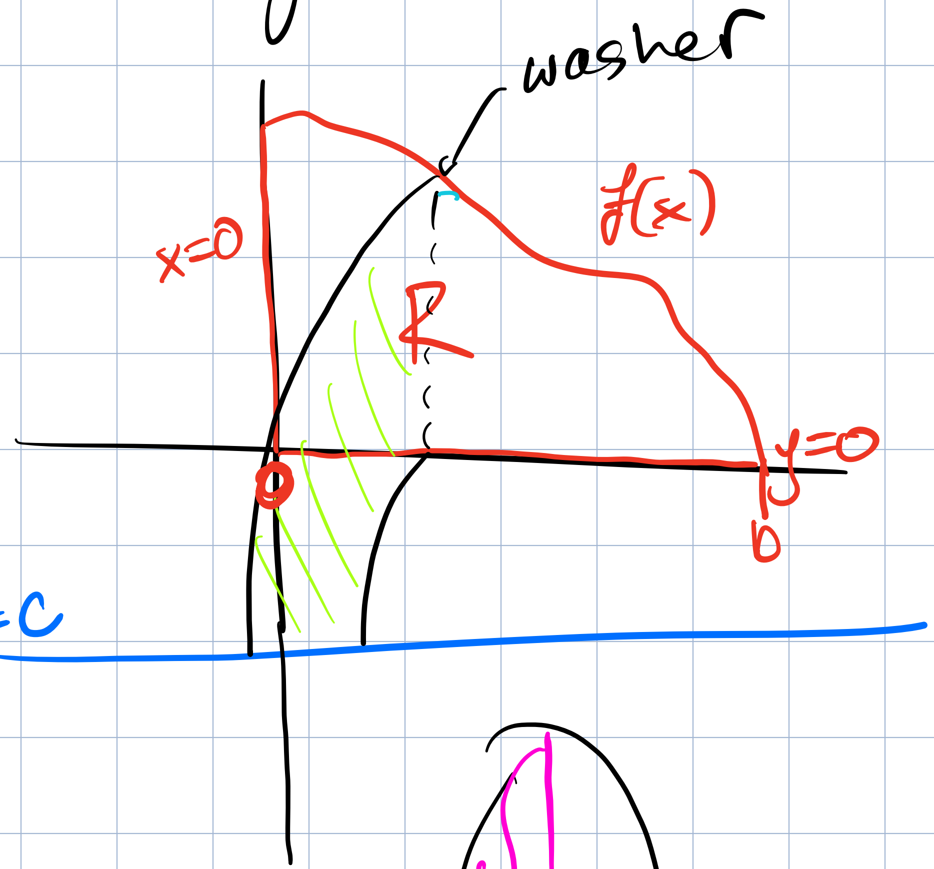
- add these up by $\displaystyle\int_0^b \pi g(y)^2\, dy$

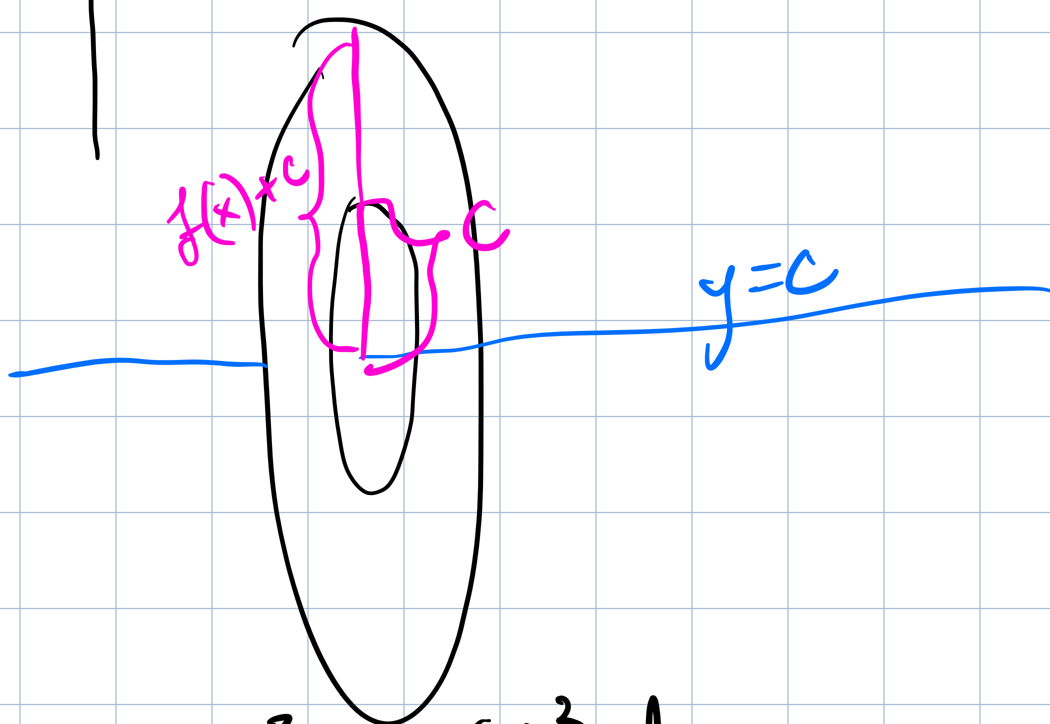
Alternatively Rotate about x-axis

- 

$$\int_0^a \pi f(x)^2\, dx$$

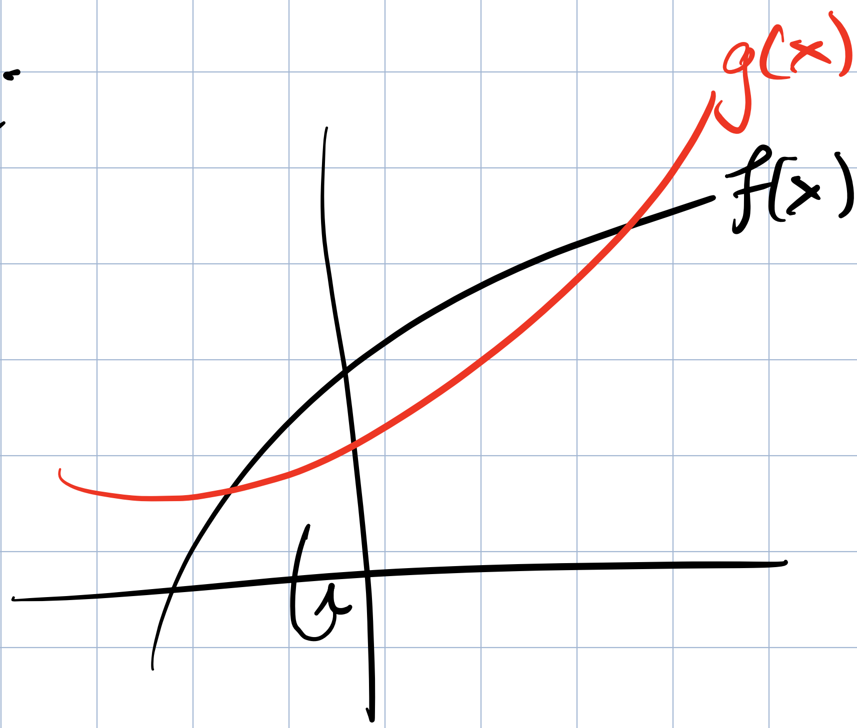
- rotate about some other line altogether

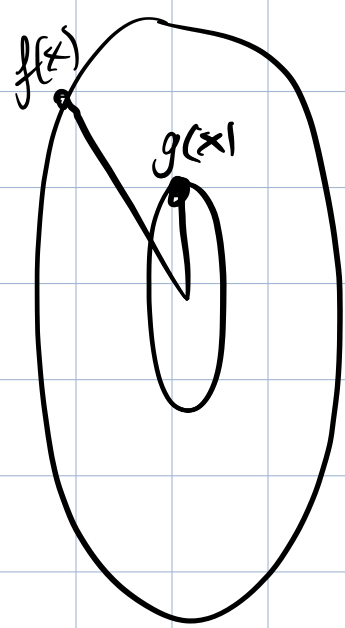




$$\int_0^b \underbrace{\pi\left(f(x)+c\right)^2-\pi(c)^2}_{\pi\left(f(x)^2+2cf(x)+\cancel{c^2}\right)-\cancel{\pi c^2}}\,dx$$

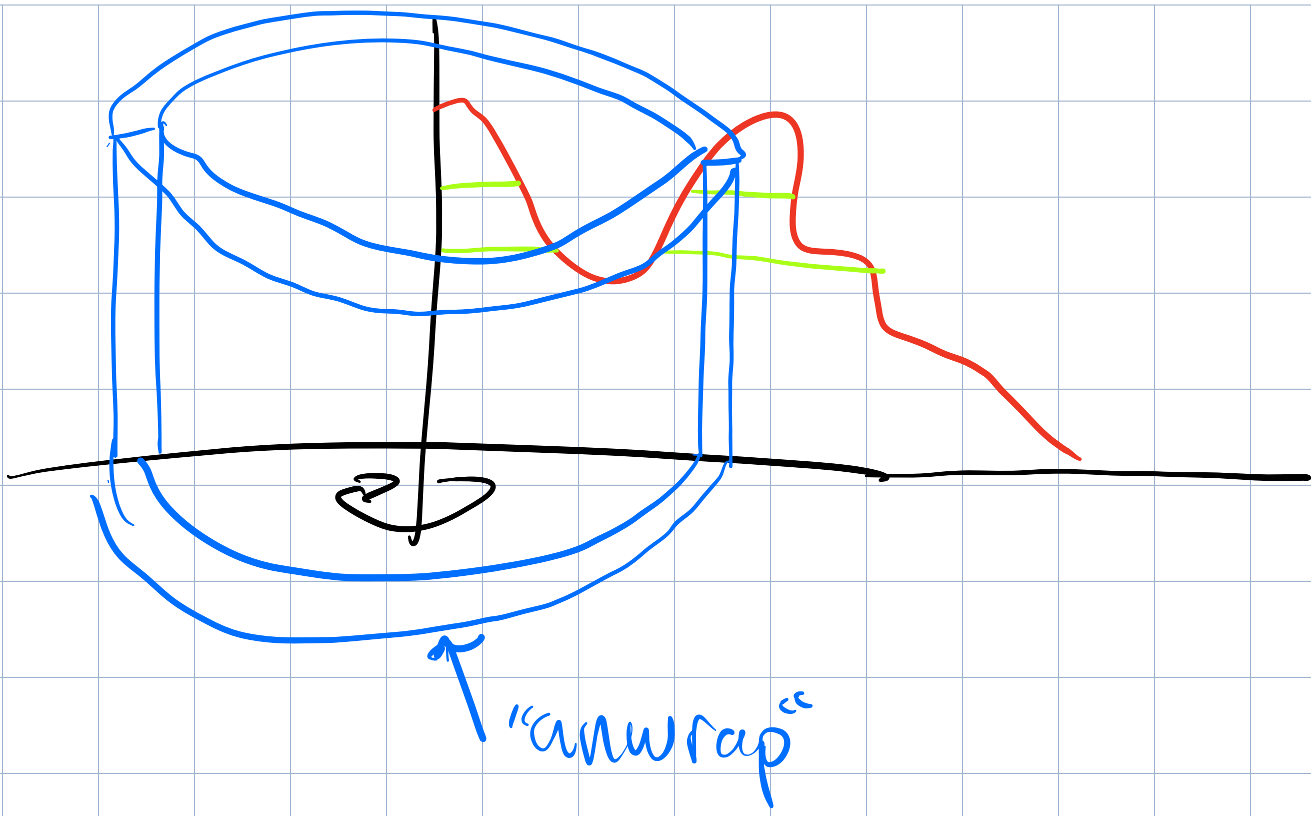
Ex:

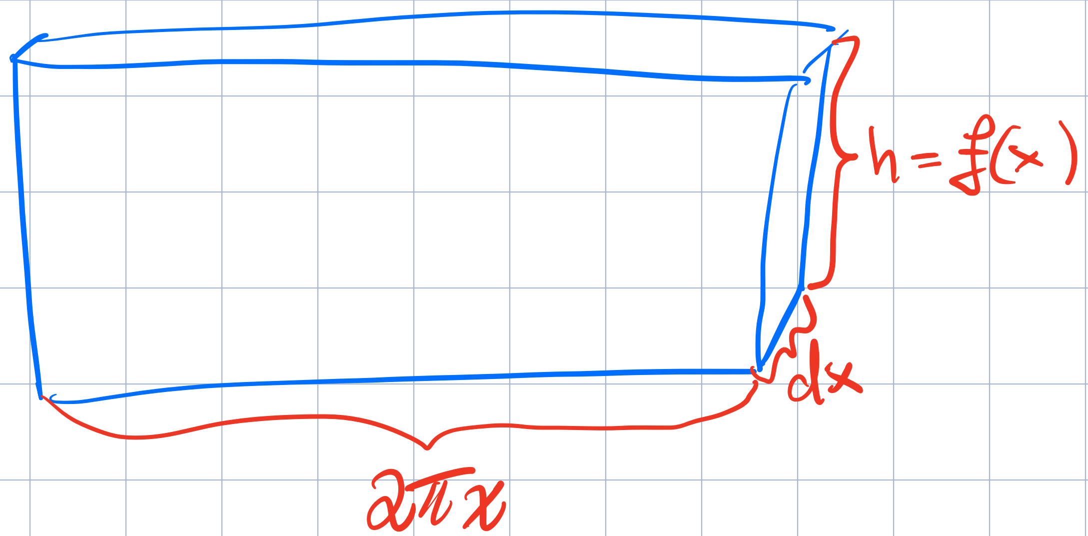




$$\int \pi f(x)^2 - \pi g(x)^2 = \pi \int \left(f(x)^2 - g(x)^2\right) dx$$

Cylindrical shells





$$\int 2\pi x f(x)\,dx$$

§7.8: Improper Integrals

Type I:

- letting x go to ∞, $-\infty$ or **both**

(i) $\displaystyle\int_a^\infty f(x)\,dx = \lim_{t\to\infty}\int_a^t f(x)\,dx$

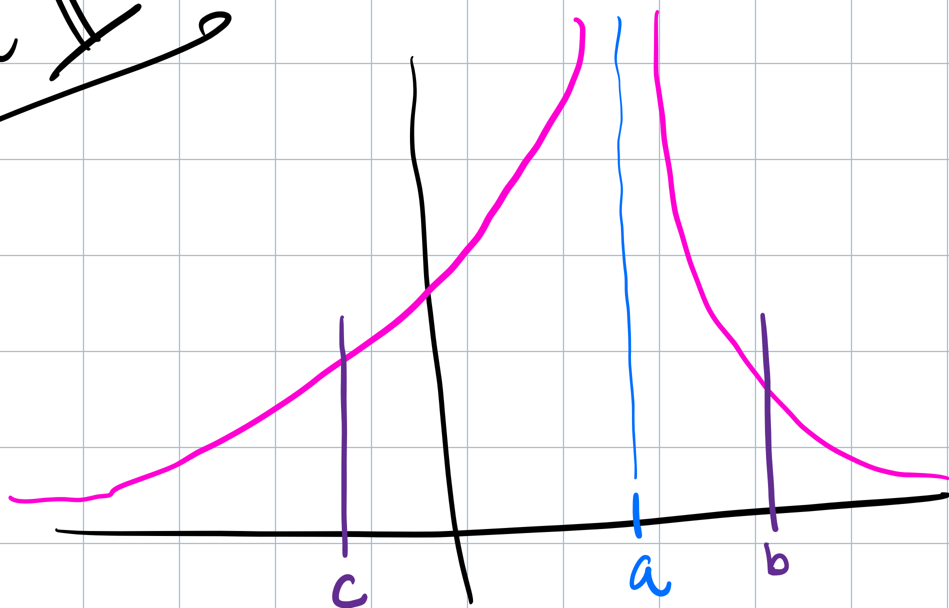
(ii) $\displaystyle\int_{-\infty}^a f(x)\,dx = \lim_{t\to\infty}\int_{-t}^a f(x)\,dx$

(iii) $\displaystyle\int_{-\infty}^\infty f(x)\,dx = \underbrace{\int_{-\infty}^{c} f(x)\,dx + \int_c^\infty f(x)\,dx}$

pick a c

(both need to exist for this integral to conv.

Type II



(i) $\displaystyle\int_a^b f(x)\,dx = \lim_{t\to a^+} \int_t^b f(x)\,dx$

(ii) $\displaystyle\int_c^a f(x)\,dx = \lim_{t\to a^-} \int_c^t f(x)\,dx$

(iii) $$\int_c^b f(x)\,dx = \lim_{t\to a^-}\int_c^t f(x)\,dx + \lim_{t\to a^+}\int_t^b f(x)\,dx$$

Warnings:

- limits need not exist
- you may not be told that there is a discontinuity