ROTATIONAL KINEMATICS NOTES

This is one-dimensional motion, where the "dimension" is the angle $\theta(t)$. This angle is *always measured in radians*, and is measured positive in the counterclockwise direction from the positive x-axis. The rate of change of this angle as an object moves in a circular path is the angular velocity $\omega(t)$ in radians/second:

$$\omega(t) \equiv \frac{d\theta}{dt}$$

The rate of change of the angular velocity is the angular acceleration α in radians/second^2

$$\alpha(t) \equiv \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$$

For most problems the angular acceleration will be constant (but not necessarily zero). Just as with translational kinematics, if the acceleration α is not constant then we must use calculus to analyze the motion.

It was shown in an earlier document that the tangential (or "linear") velocity v_T is

$$v_T = \omega R \tag{1}$$

where R is the radius of the circle. Then, taking the time derivative of this, we have the tangential or "linear" acceleration a_T (taking the angular acceleration to be constant):

$$a_T = \frac{dv_T}{dt} = \frac{d}{dt}(\omega R) = \frac{d\omega}{dt}R = \alpha R \tag{2}$$

We also established earlier that the radial acceleration is

$$a_R = \frac{v^2}{R} = \omega^2 R \tag{3}$$

so that the magnitude of the total linear acceleration can be written

$$a = R\sqrt{\alpha^2 + \omega^4} \tag{4}$$

With these definitions we can proceed to develop the rotational kinematics formulas, in exactly the same manner as we did for linear kinematics. Here are the results*, all of which assume a constant acceleration*.

TRANSLATIONAL	ROTATIONAL	
$x_t = x_0 + \overline{v}_t\, t$	$\theta_t = \theta_0 + \overline{\omega}_t\, t$	(5)
$x_t = x_0 + v_0 t + \frac{1}{2}a t^2$	$\theta_t = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$	(6)
$x_t = x_0 + v_t t - \frac{1}{2}a t^2$	$\theta_t = \theta_0 + \omega_t t - \frac{1}{2}\alpha t^2$	(7)
$v_t = v_0 + a t$	$\omega_t = \omega_0 + \alpha t$	(8)
$\overline{v}_t = \frac{v_0 + v_t}{2} = v_0 + \frac{1}{2}at$	$\overline{\omega}_t = \frac{\omega_0 + \omega_t}{2} = \omega_0 + \frac{1}{2}\alpha t$	(9)
$v_t^2 = v_0^2 + 2a\left(x_t - x_0\right)$	$\omega_t^2 = \omega_0^2 + 2\alpha\left(\theta_t - \theta_0\right)$	(10)