Ejercicios de la unidad 15.2
Integrales de línea

En los ejercicios 7 a 10, evaluar la integral de línea a lo largo de la trayectoria dada.

7) $\int_C xy \, ds$

$C: r(t) = 4t\,i + 3t\,j$

$0 \leq t \leq 1$

$r'(t) = \langle 4, 3 \rangle$ $\qquad ds = \|r'(t)\| \, dt$

$ds = \sqrt{4^2 + 3^2} \, dt = \sqrt{16+9} \, dt = 5 \, dt$

$\int_0^1 (4t \cdot 3t) 5 \, dt = 60 \int_0^1 t^2 \, dt$

$60 \left[\frac{1}{3} t^3 \right]_0^1 = 60 \left[\frac{1}{3}(1)^3 - \frac{1}{3}(0)^3 \right]$

$= 60 \left[\frac{1}{3} - 0 \right] = 60 \left(\frac{1}{3} \right) = 60/3$

$= \boxed{20}$

9) $\int_C (x^2+y^2+z^2)\,ds$

$C: r(T) = \text{Sen}\,T\,i + \text{Cos}\,T\,j + 2k$

$0 \le T \le \frac{\pi}{2}$

$ds = \|r'(T)\|\,dT$

$r'(T) = \langle \cos T, -\text{sen}\,T, 0 \rangle$

$ds = \sqrt{(\cos T)^2 + (-\text{sen}\,T)^2 + 0^2}\,dT = \sqrt{\cos^2 T + \text{sen}^2 T}\,dT$

$= \sqrt{1}\,dT$

$= 1\,dT$

$\int_0^{\pi/2} (\text{sen}^2 T + \cos^2 T + 2^2)\,1\,dT$

$\int_0^{\pi/2} (\text{sen}^2 T + \cos^2 T + 4)\,dT$

$\int_0^{\pi/2} \text{sen}^2 T + \cos^2 T\,dT + \int_0^{\pi/2} 4\,dT$

$\int_0^{\pi/2} 1\,dT \quad + \quad 4T\Big|_0^{\pi/2}$

$T\Big|_0^{\pi/2} \quad + \quad 4[\pi/2 - 0]$

$(\pi/2 - 0) \quad + \quad 2\pi$

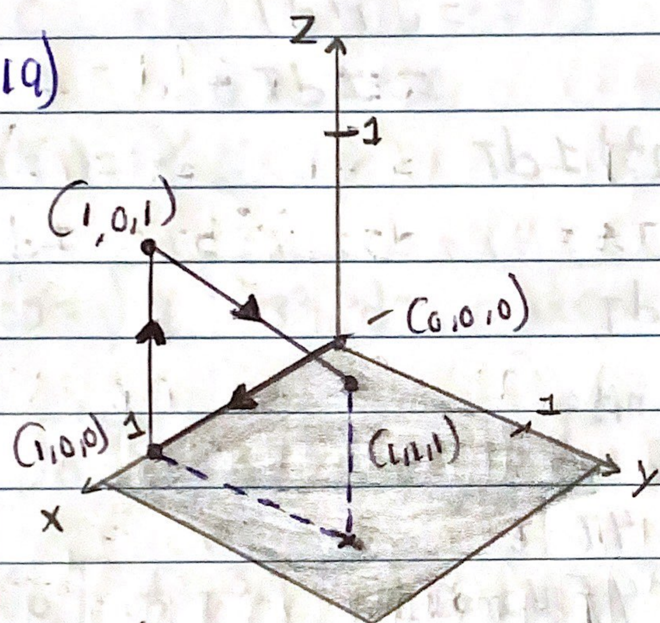
$\pi/2 + 2\pi$

$= \boxed{\frac{5\pi}{2}}$

En los ejercicios 19 y 20, a) encontrar una Parametrización Continua por secciones de la Trayectoria c que se muestra en la figura y b) evaluar

$\int_C (2x + y^2 - z)\, ds$, a lo largo de C.

19)



Parametrizar =

$r(T) = (1-T)\vec{r}_1 + T\vec{r}_2 \quad , \quad 0 \leq T \leq 1$

$= (1-T)\langle 0,0,0\rangle + T\langle 1,0,0\rangle$

$= \langle 0,0,0\rangle + \langle T,0,0\rangle$

$= \langle T,0,0\rangle \quad , \quad x = T,\ y = 0,\ z = 0$

$r(T) = (1-T)\langle 1,0,0\rangle + T\langle 1,0,1\rangle \quad , \quad 0 \leq T \leq 1$

$= \langle 1-T, 0, 0\rangle + \langle T, 0, T\rangle$

$= \langle 1, 0, T\rangle \quad , \quad x = 1,\ y = 0,\ z = T$

$r(T) = (1-T)\langle 1,0,1\rangle + T\langle 1,1,1\rangle \quad , 0 \le T \le 1$

$= \langle 1-T, 0, 1-T\rangle + \langle T, T, T\rangle$

$= \langle 1, T, 1\rangle = x=1,\ y=T,\ z=1$

$$r(T) = \begin{cases} Ti & 0 \le T \le 1 \\ i + TK & 0 \le T \le 1 \\ i + TJ + K & 0 \le T \le 1 \end{cases}$$

$$r'(T) = \begin{cases} 1, & 0 \le T \le 1 & ds = \sqrt{1^2}\,dT = 1\,dT \\ 0+1, & 0 \le T \le 1 & ds = \sqrt{1^2}\,dT = 1\,dT \\ 0+1+0, & 0 \le T \le 1 & ds = \sqrt{1^2}\,dT = 1\,dT \end{cases}$$

$$\int_0^1 (2(T) + 0^2 - 0)\,1\,dT + \int_0^1 (2(1) + 0^2 - T)\,1\,dT + \int_0^1 (2(1) + T^2 - 1)\,1\,dT$$

$$\int_0^1 2T\,dT + \int_0^1 2 - T\,dT + \int_0^1 1 + T^2\,dT$$

$$\frac{2T^2}{2}\Big|_0^1 + 2T - \frac{T^2}{2}\Big|_0^1 + T + \frac{T^3}{3}\Big|_0^1$$

$$1 + 2(1) - \frac{1}{2} + 1 + \frac{1}{3}$$

$$2 + 2 - \frac{1}{2} + \frac{1}{3} = 4 - \frac{1}{2} + \frac{1}{3}$$

$$= \frac{7}{2} + \frac{1}{3}$$

$$= \frac{21+2}{6} = \boxed{\frac{23}{6}}$$

En los ejercicios 27 a 32, evaluar

$\int_C \vec{F} \cdot d\vec{r}$, donde C está representa por $r(t)$.

31) $\vec{F}(x,y,z) = xy\,i + xz\,j + yz\,k$

$C: \vec{r}(t) = t\,i + t^2\,j + 2t\,k, \quad 0 \leq t \leq 1$

$x = t, \quad y = t^2, \quad z = 2t \qquad 0 \leq t \leq 1$

$\vec{r}'(t) = \langle 1, 2t, 2 \rangle dt$

$\vec{F} = (t^3 + 2t^2 + 2t^3)$

$W = \int_C \vec{F} \cdot d\vec{r}$

$\int_0^1 (t^3 + 2t^2 + 2t^3) \cdot (1 + 2t + 2)\, dt$

$\int_0^1 (t^3 + 4t^3 + 4t^3)\, dt$

$\int_0^1 9t^3\, dt = \frac{9}{4} t^4 \Big|_0^1$

$= \frac{9}{4}[1^4 - 0^4]$

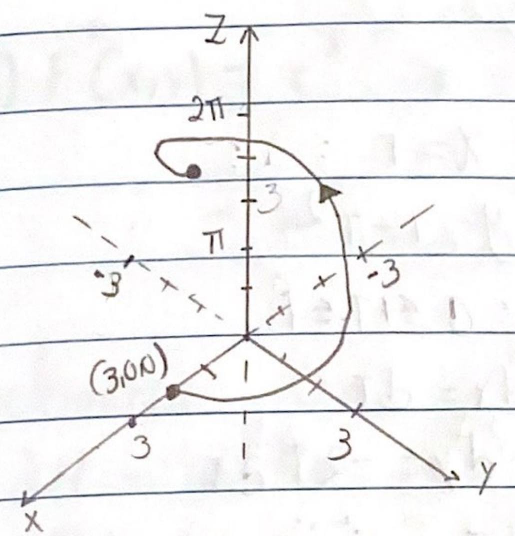
$= \frac{9}{4}(1)$

$= \boxed{\frac{9}{4}}$

Trabajo. En los ejercicios 35 a 40, hallar el Trabajo realizado Por el Campo de fuerzas F sobre una Particula que se mueve a lo largo de la trayectoria dada.

39) $\vec{F}(x,y,z) = x\,i + y\,j - 5z\,k$

$C: \vec{r}(T) = 2\cos T\,i + 2\,\text{sen}\,T\,j + T\,k, \quad 0 \le T \le 2\pi$



$x = 2\cos T$

$y = 2\,\text{sen}\,T$

$z = T$

$\vec{r}\,'(T) = \langle -2\,\text{sen}\,T, 2\cos T, 1 \rangle\, dT$

$\vec{F} = (2\cos T\; i + 2\,\text{sen}\,T - 5T)\, dT$

$$W = \int_C \vec{F} \cdot d\vec{r}$$

$$= \int_0^{2\pi} (2\cos T + 2\,\text{sen}\,T - 5T) \cdot (-2\,\text{sen}\,T + 2\cos T + 1)\, dT$$

$$\int_0^{2\pi} (-4\,\text{sen}\,T\cos T + 4\,\text{sen}\,T\cos T - 5T)\, dT$$

$$\int_0^{2\pi} -5T\, dT = -\left[5\frac{T^2}{2}\right]_0^{2\pi} = -\left(\frac{5(2\pi)^2}{2} - \frac{5(0)^2}{2}\right)$$

$$= -\left[\frac{5 \cdot 4\pi^2}{2} - 0\right)$$

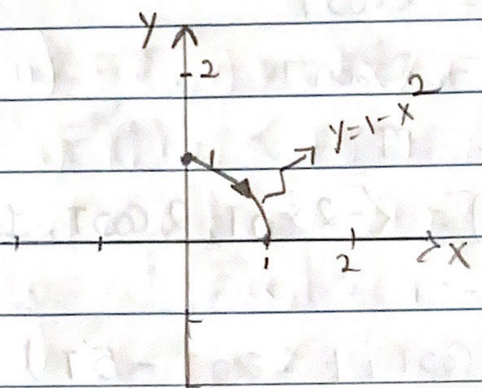
$$= \boxed{-10\pi^2 \text{ Joule}}$$

En los ejercicios 55 a 62, evaluar la integral

$$\int_C (2x-y)\,dx + (x+3y)\,dy$$

a lo largo de la trayectoria C.

59) C: arco sobre $\underbrace{y=1-x^2}$ desde $(0,1)$ hasta $(1,0)$.



$x = T$

$y = 1-T^2$

$0 \le T \le 1$

$dx = dT$

$dy = -2T\,dT$

$$\int_0^1 (2(T)-(1-T^2))(dT) + (T+3(1-T^2))(-2T)dT$$

$$\int_0^1 (2T-1+T^2)dT + (T+3-3T^2)(-2T\,dT)$$

$$\int_0^1 (2T-1+T^2)dT + (-2T^2-6T+6T^3)dT$$

$$\int_0^1 (2T-1+T^2-2T^2-6T+6T^3)dT$$

$$\int_0^1 (-4T-T^2-1+6T^3)dT = -\frac{4T^2}{2} - \frac{T^3}{3} - T + \frac{6}{4}T^4 \Big|_0^1$$

$$= -2(1) - \frac{(1)}{3} - (1) + \frac{3}{2} = -2 - \frac{1}{3} - 1 + \frac{3}{2} = \boxed{\frac{-11}{6}}$$