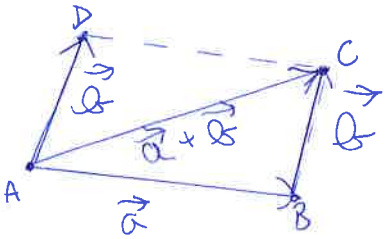
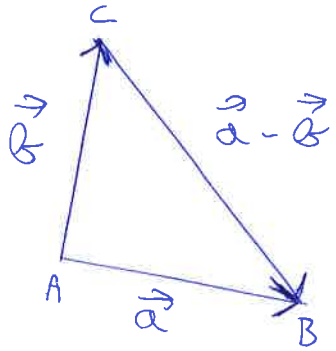
VEKTORI – MATURA

Roko Jurlina 3 E

VEKTORI - FORMULE

ZBROJ I RAZLIKA VEKTORA





KOMPONENTE VEKTORA

$T_1(x_1, y_1)$, $T_2(x_2, y_2)$

$$\vec{a} = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j}$$

$$= a_x\vec{i} + a_y\vec{j}$$

JEDNAKOST VEKTORA

$$\vec{a} = \vec{b} \longleftrightarrow a_x = b_x,\ a_y = b_y$$

DULJINA (NORMA VEKTORA)

$$|\vec{a}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$= \sqrt{a_x^2 + a_y^2}$$

SKALARNI UMNOŽAK

$$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi$$

$$= a_x b_x + a_y b_y$$

$$\varphi = \sphericalangle(\vec{a}, \vec{b})$$

$$\vec{a} \perp \vec{b} \longleftrightarrow a_x b_x + a_y b_y = 0$$

KOLINEARNOST

$$\vec{a} = k \cdot \vec{b}$$

JEDINIČNI VEKTOR

$$\vec{e} = \frac{\vec{a}}{|\vec{a}|}$$

1. KOLIKO IZNOSI KUT MEĐU VEKTORIMA $\vec{AB} = -3\vec{i} - 4\vec{j}$ I $\vec{CD} = 3\vec{i} - 4\vec{j}$?

$\varphi = ?$

$\vec{AB} \cdot \vec{CD} = |\vec{AB}| \cdot |\vec{CD}| \cdot \cos\varphi$

$= AB_x CD_x + AB_y CD_y$

$\vec{AB} \cdot \vec{CD} = -3 \cdot 3 + (-4) \cdot (-4)$

$\vec{AB} \cdot \vec{CD} = -9 + 16$

$= 7$

$7 = \sqrt{(-3)^2 + (-4)^2} \cdot \sqrt{3^2 + (-4)^2} \cdot \cos\varphi$

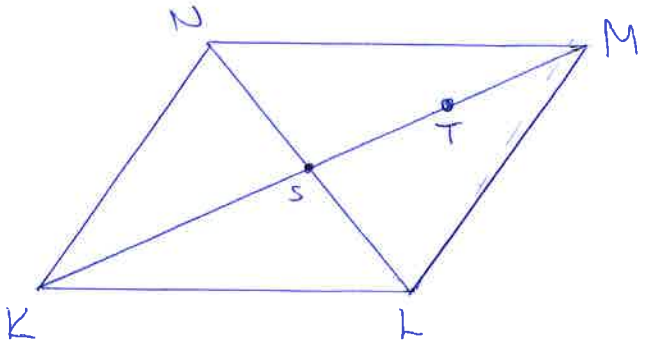
$7 = \sqrt{25} \cdot \sqrt{25} \cdot \cos\varphi$

$7 = 25 \cos\varphi$

$\cos\varphi = \frac{7}{25}$

$\varphi = 73^\circ 44' 23''$

2. ZADAN JE PARALELOGRAM KLMN I TOČKA T NA DIJAGONALI KM TAKVA DA VRIJEDI $|KT| = \frac{3}{4}|KM|$. IZRAZITE VEKTOR $\vec{NT}$ KAO LINEARNU KOMBINACIJU VEKTORA $\vec{KL}$ I $\vec{KM}$.



$\vec{NT} = \vec{NM} + \vec{MT} = \vec{KL} - \frac{1}{4}\vec{KM}$

$\vec{NT} = \vec{KL} - \frac{1}{4}\vec{KM}$

3. MJERA KUTA IZMEĐU VEKTORA $\vec{p}$ I $\vec{r}$ IZNOSI $120°$. AKO JE $|\vec{p}|=8$, $|\vec{r}|=13$, KOLIKO JE $\vec{p}(\vec{p}-\vec{r})$?

$$\vec{p}(\vec{p}-\vec{r}) = \vec{p}^2 - \vec{p}\cdot\vec{r}$$

$$\vec{p}^2 = |\vec{p}|^2 = 8^2 = 64$$

$$\vec{p}\cdot\vec{r} = |\vec{p}|\cdot|\vec{r}|\cdot\cos\sphericalangle(\vec{p},\vec{r})$$

$$\vec{p}\cdot\vec{r} = 8\cdot 13\cdot\cos 120°$$

$$= -52$$

$$\vec{p}(\vec{p}-\vec{r}) = \vec{p}^2 - \vec{p}\cdot\vec{r} = 64 - (-52) = 116$$

4. ODREDITE REALNI BROJ k TAKO DA VEKTORI $\vec{a} = 6\vec{i} - 4\vec{j}$, $\vec{b} = 2\vec{i} + (2k+5)\vec{j}$ BUDU OKOMITI. $k \in \mathbb{R}$

$$\vec{a}\cdot\vec{b} = |\vec{a}|\cdot|\vec{b}|\cdot\cos\varphi$$

$\varphi = 90°$ $\cos 90° = 0$

$$\vec{a}\cdot\vec{b} = 0 \qquad \vec{a}\cdot\vec{b} = 6\cdot 2 + (-4)\cdot(2k+5) = 0$$

$$12 - 8k - 20 = 0$$

$$-8k = 20 - 12$$

$$8k = -8$$

$$k = -1$$

5. AKO SU VEKTORI $\vec{a}=3\vec{i}-7\vec{j}$ I $\vec{b}=x\vec{i}+4\vec{j}$ KOLINEARNI, KOLIKI JE x?

$$\vec{a}=k\cdot\vec{b}$$

$$3\vec{i}-7\vec{j}=k\cdot(x\vec{i}+4\vec{j})$$

$$3\vec{i}-7\vec{j}=kx\vec{i}+4k\vec{j}$$

$$3\vec{i}=kx\vec{i}$$

$$4k\vec{j}=-7\vec{j}$$

$$k=\frac{-7}{4}$$

$$\frac{-7}{4}x=3$$

$$x=\frac{3}{-\frac{7}{4}}=-\frac{12}{7}$$

6. VEKTOR $\vec{a}=-3\vec{i}-2\vec{j}$ KOLINEARAN JE VEKTORU $\vec{b}=\vec{AB}$ ODREĐENOM TOČKAMA $A(0,4)$ I $B(3,y)$. ODREDI y.

$$\vec{b}=\vec{AB}=(3-0)\vec{i}+(y-4)\vec{j}$$

$$=3\vec{i}+(y-4)\vec{j}$$

$$\vec{a}=k\vec{b}$$

$$-3\vec{i}-2\vec{j}=k\cdot(3\vec{i}+(y-4)\vec{j})$$

$$-3\vec{i}-2\vec{j}=3k\vec{i}+(y-4)k\vec{j}$$

$$-3\vec{i}=3k\vec{i}$$

$$3k=-3$$

$$k=-1$$

$$-2\vec{j}=(-y+4)\vec{j}$$

$$-y+4=-2$$

$$-y=-6$$

$$y=6$$

7. ZADANE SU TOČKE $M(2,3)$, $N(-1,4)$, $P(7,-3)$.

PRIKAŽI $\vec{MN} + \vec{MP}$ KAO LINEARNU KOMBINACIJU JEDINIČNIH VEKTORA $\vec{i}$ I $\vec{j}$.

$$\vec{MN} = (x_N - x_M)\vec{i} + (y_N - y_M)\vec{j}$$

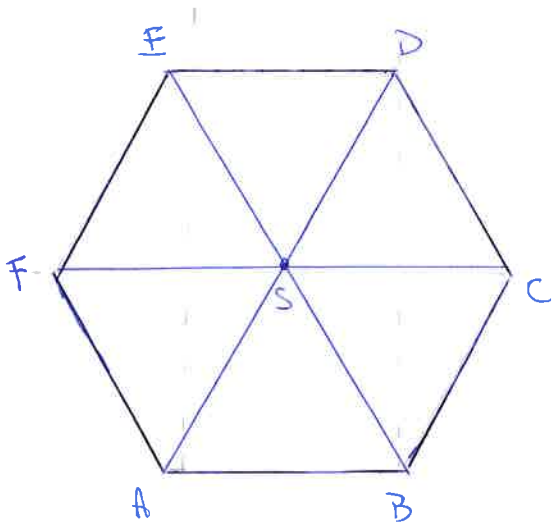
$$= -3\vec{i} + \vec{j}$$

$$\vec{MP} = 5\vec{i} - 6\vec{j}$$

$$\vec{MP} + \vec{MN} = -3\vec{i} + \vec{j} + 5\vec{i} - 6\vec{j}$$

$$= 2\vec{i} - 5\vec{j}$$

8. AKO SU A, B, C, D, E, F VRHOVI PRAVILNOG ŠESTEROKUTA A S SJECIŠTE DIJAGONALA ŠESTEROKUTA, KOLIKO JE $\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF}$?



$$\vec{AB} + \vec{AC} + \vec{AD} + \vec{AE} + \vec{AF} =$$

$$= \vec{AB} + \vec{AC} + \vec{AD} + \vec{BD} + \vec{CD}$$

$$= \vec{AB} + \vec{BD} + \vec{AC} + \vec{CD} + \vec{AD}$$

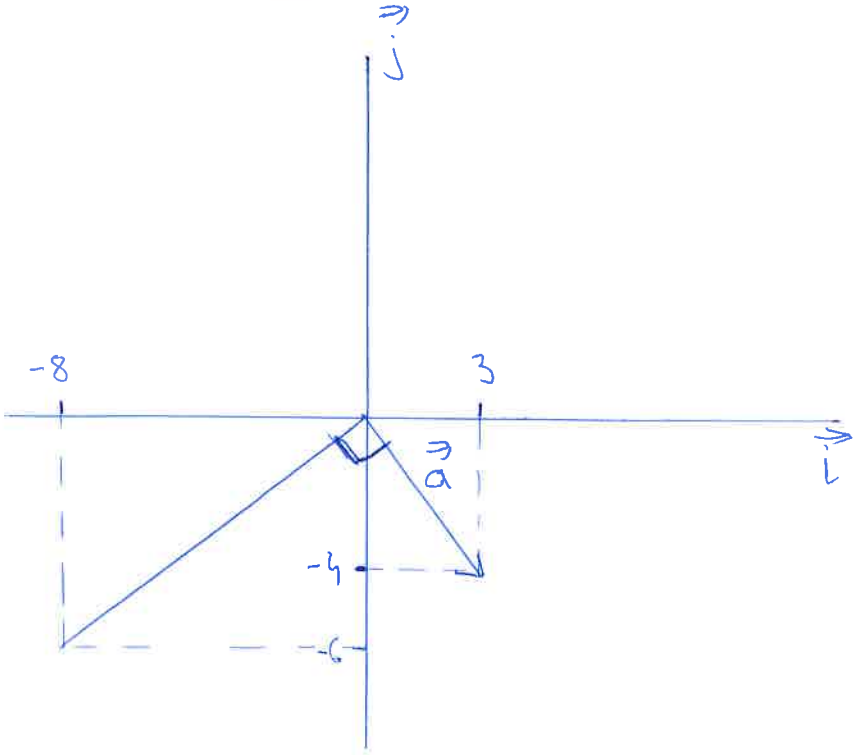
$$= \vec{AD} + \vec{AD} + \vec{AD}$$

$$= 3\vec{AD}$$

9. ODREDITE VEKTOR $\vec{b}$ OKOMIT NA VEKTOR $\vec{a} = 3\vec{i} - 4\vec{j}$ TAKAV DA S OSI y ZATVARA TUPI KUT I IMA DULJINU 10.

$|\vec{b}| = 10$

$\varphi = 90° \rightarrow \vec{a} \perp \vec{b} \rightarrow \vec{a} \cdot \vec{b} = 0$



$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$

$0 = 3b_x - 4b_y$

$10 = \sqrt{b_x^2 + b_y^2} \quad /^2$

$0 = 3b_x - 4b_y$

$100 = b_x^2 + b_y^2$

$3b_x = 4b_y$

$b_x = \frac{4}{3} b_y$

$100 = \frac{16}{9} b_y^2 + b_y^2$

$\frac{25}{9} b_y^2 = 100$

$b_y^2 = 36 \quad /\sqrt{}$

$b_y = \pm 6$

$b_x = \frac{4}{3} \cdot \frac{6}{1}$

$= \pm 8$

$\vec{b} = 8\vec{i} + 6\vec{j}$

$= \boxed{-8\vec{i} - 6\vec{j}}$

10. ODREDI VEKTOR $\vec{b}$ KOLINEARAN S VEKTOROM $\vec{a} = 2\vec{i} - 3\vec{j}$ I KOJI ZADOVOLJAVA UVJET $\vec{a} \cdot \vec{b} = -26$.

$\vec{a} = k \cdot \vec{b}$

$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$

$-26 = 2b_x - 3b_y$

$2\vec{i} - 3\vec{j} = k \cdot b_x\vec{i} + kb_y\vec{j}$

$kb_x = 2$

$k = \frac{2}{b_x}$

$kb_y = -3$

$k = \frac{-3}{b_y}$

$\frac{2}{b_x} = \frac{-3}{b_y}$

$2b_y = -3b_x$

$b_y = \frac{-3}{2} b_x$

$-26 = 2b_x + \frac{9}{2} b_x$

$-26 = 6.5\, b_x$

$b_x = -4$

$b_y = \frac{-3}{2} \cdot \frac{-4}{1} = 6$

$\vec{b} = -4\vec{i} + 6\vec{j}$

11. ODREDI DULJINU VEKTORA $\vec{a}+\vec{b}$, AKO JE $|\vec{a}|=11$, $|\vec{b}|=23$, $|\vec{a}-\vec{b}|=30$.

$|\vec{a}|=11 \rightarrow \vec{a}^2 = \vec{a}\cdot\vec{a} = |\vec{a}|\cdot|\vec{a}|\cdot\cos\varphi^{\cos 0^\circ} = 121$

$|\vec{b}|=23 \rightarrow \vec{b}^2 = \vec{b}\cdot\vec{b} = |\vec{b}|\cdot|\vec{b}|\cdot\cos\varphi = 529$

$|\vec{a}-\vec{b}|=30$

$30 = \sqrt{(\vec{a}-\vec{b})^2} = \sqrt{\vec{a}^2 - 2\vec{a}\vec{b} + \vec{b}^2}$

$30 = \sqrt{121 - 2\vec{a}\vec{b} + 529} \quad /^2$

$900 = 121 - 2\vec{a}\vec{b} + 529$

$650 - 2\vec{a}\vec{b} = 900$

$-2\vec{a}\vec{b} = 250$

$2\vec{a}\vec{b} = -250$

$|\vec{a}+\vec{b}| = \sqrt{(\vec{a}+\vec{b})^2}$

$= \sqrt{(\vec{a}^2 + 2\vec{a}\vec{b} + \vec{b}^2)}$

$= \sqrt{121 - 250 + 529}$

$= 20$

12. AKO JE $|\vec{a}| = |\vec{b}| = |\vec{c}| = 1$, TE $\vec{a} + \vec{b} + \vec{c} = 0$, KOLIKO JE $\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}$?

$\vec{a}^2 = \vec{a}\cdot\vec{a} = |\vec{a}|\cdot|\vec{a}|\cdot\cos\varphi$
$= 1\cdot 1\cdot 1$
$= 1$

$\vec{a} + \vec{b} + \vec{c} = 0 \quad /^2$

$\vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a}\vec{b} + 2\vec{a}\vec{c} + 2\vec{b}\vec{c} = 0$

$1 + 1 + 1 + 2\vec{a}\vec{b} + 2\vec{a}\vec{c} + 2\vec{b}\vec{c} = 0$

$2(\vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c} + \vec{b}\cdot\vec{c}) = -3$

$\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{3}{2}$

13. AKO SU TOČKE $A(-4,3)$, $B(-1,-1)$ i $C(4,0)$ VRHOVI TROKUTA, ODREDITE SVE KUTOVE TROKUTA.

$\vec{CA} = -8\vec{i} + 3\vec{j}$

$|\vec{CA}| = \sqrt{64+9} = \sqrt{73}$

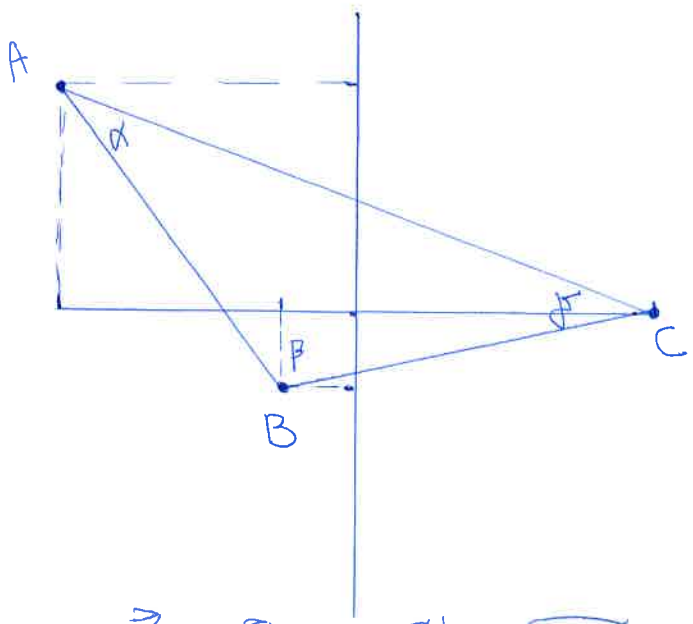
$\vec{CB} = -5\vec{i} - \vec{j} = \sqrt{26} = |\vec{CB}|$

$\vec{CA}\cdot\vec{CB} = 40 - 3 = 37$

$37 = \sqrt{73}\cdot\sqrt{26}\cdot\cos\gamma$

$\cos\gamma = \frac{37}{\sqrt{73}\cdot\sqrt{26}}$

$\gamma = 31^\circ\ 51'\ 58''$



$\vec{AB} = +3\vec{i} - 4\vec{j}$ $\quad |\vec{AB}| = \sqrt{9+16} = 5$

$\vec{CB} = -5\vec{i} - \vec{j}$ $\quad |\vec{CB}| = \sqrt{25+1} = \sqrt{26}$

$\vec{AB}\cdot\vec{CB} = -15 + 4 = -11$

$\alpha + \beta + \gamma = 180^\circ$

$\alpha = 180^\circ - 115^\circ 33' 36'' - 31^\circ 51'$

$\alpha = 32^\circ\ 34'\ 26''$

$-11 = 5\cdot\sqrt{26}\cdot\cos\beta$

$\cos\beta = \frac{-11}{5\cdot\sqrt{26}}$

$\beta = 115^\circ\ 33'\ 36''$

14. KOLIKI KUT ZATVARAJU VEKTORI $\vec{a}+\vec{b}$ i $\vec{a}-\vec{b}$, AKO JE $\vec{a}=\vec{i}+4\vec{j}$, $\vec{b}=-3\vec{i}+2\vec{j}$?

$\vec{a}+\vec{b}=-2\vec{i}+6\vec{j} \longrightarrow |\vec{a}+\vec{b}|=\sqrt{(-2)^2+6^2}=\sqrt{40}$

$\vec{a}-\vec{b}=4\vec{i}+2\vec{j} \longrightarrow |\vec{a}-\vec{b}|=\sqrt{4^2+2^2}=\sqrt{20}$

$(\vec{a}+\vec{b})(\vec{a}-\vec{b})=-8+12=4$

$4=\sqrt{40}\cdot\sqrt{20}\cdot\cos\varphi$

$\cos\varphi=\dfrac{4}{\sqrt{40}\cdot\sqrt{20}}=\dfrac{\sqrt{2}}{10}$

$\varphi=\arccos\dfrac{\sqrt{2}}{10}=81^\circ 52' 12''$