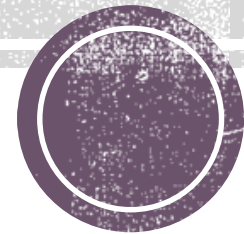


Discontinuidad

Cálculo I
Proyecto Primer Parcial

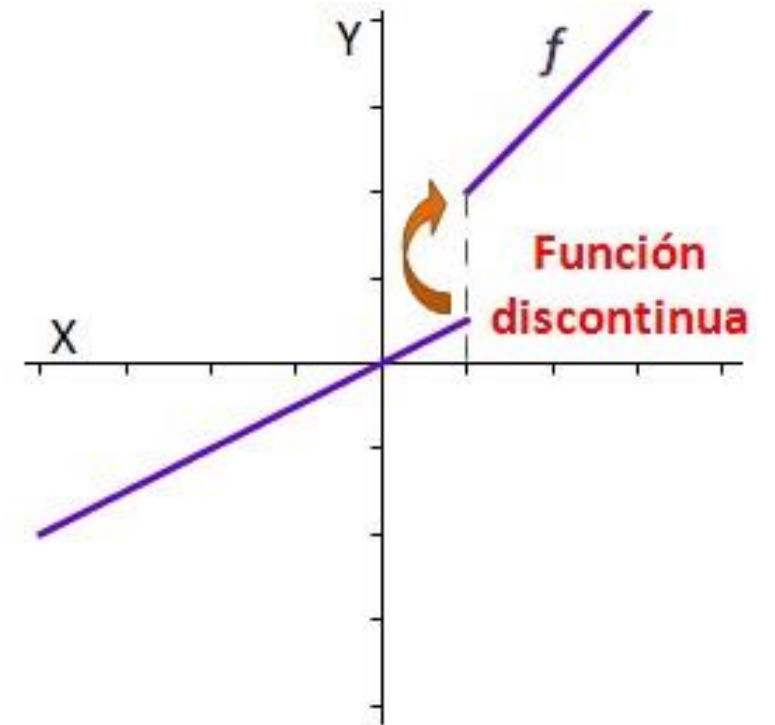
Isabella Ruiz- A01570125.

Arleth Robledo- A01570331.



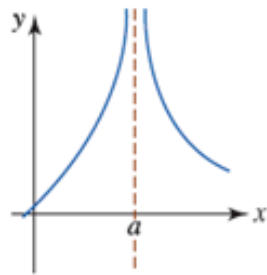
¿Qué es una discontinuidad?

- Es cuando se presenta un brusco cambio de valor en una gráfica, es decir que la línea de los valores es alterada al ser interrumpida o presentar un salto.

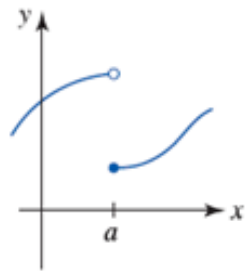


4 tipos discontinuidades

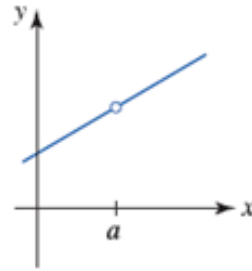
- Infinite discontinuity.
- Jump discontinuity.
- Removable discontinuity.
- Endpoint discontinuity.



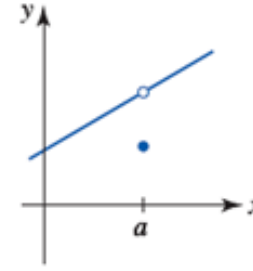
a) $\lim_{x \rightarrow a} f(x)$ no existe
y $f(a)$ no está
definida



b) $\lim_{x \rightarrow a} f(x)$ no existe
pero $f(a)$ está
definida



c) $\lim_{x \rightarrow a} f(x)$ existe
pero $f(a)$ no está
definida

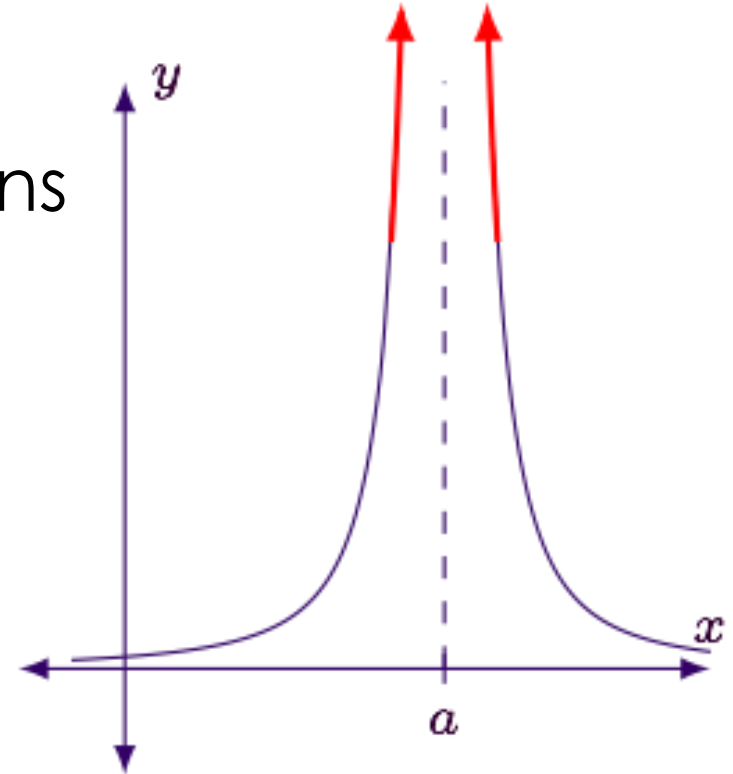


d) $\lim_{x \rightarrow a} f(x)$ existe,
 $f(a)$ está definida,
pero $\lim_{x \rightarrow a} f(x) \neq f(a)$



Infinite discontinuity.

- The function at the singular point goes to infinity in different directions on the two sides.

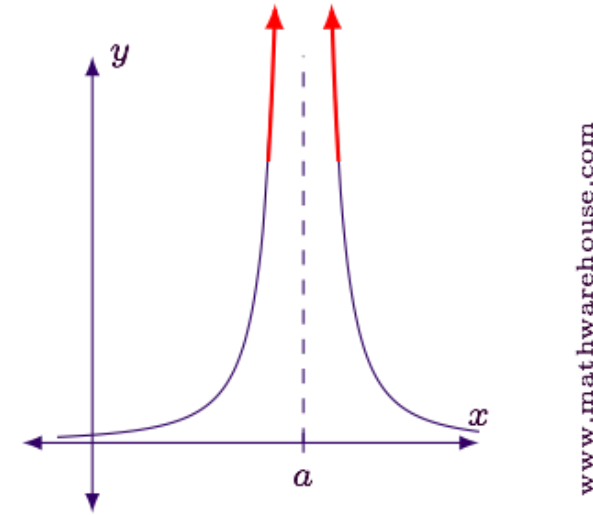


Example of an Infinite Discontinuity



Ejemplo 1

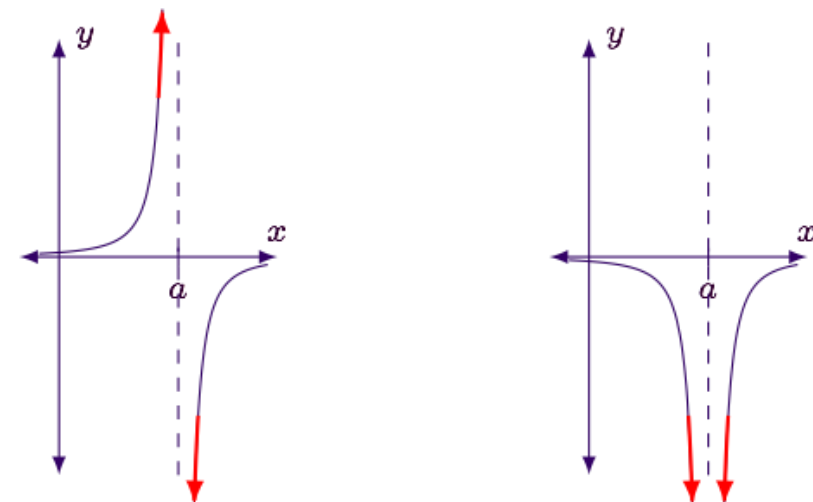
- La grafica muestra que la función es discontinua en $x = a$
- Las flechas en la función indican que crecerá infinitamente positivo como x se va aproximando a a . Como la función no se aproxima a un valor en particular definido el limite no existe. Esto es una discontinuidad infinita.



Example of an Infinite Discontinuity

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- *Los gráficos siguientes también son ejemplos de discontinuidades positivas y negativas.



Other Examples of Infinite Discontinuities

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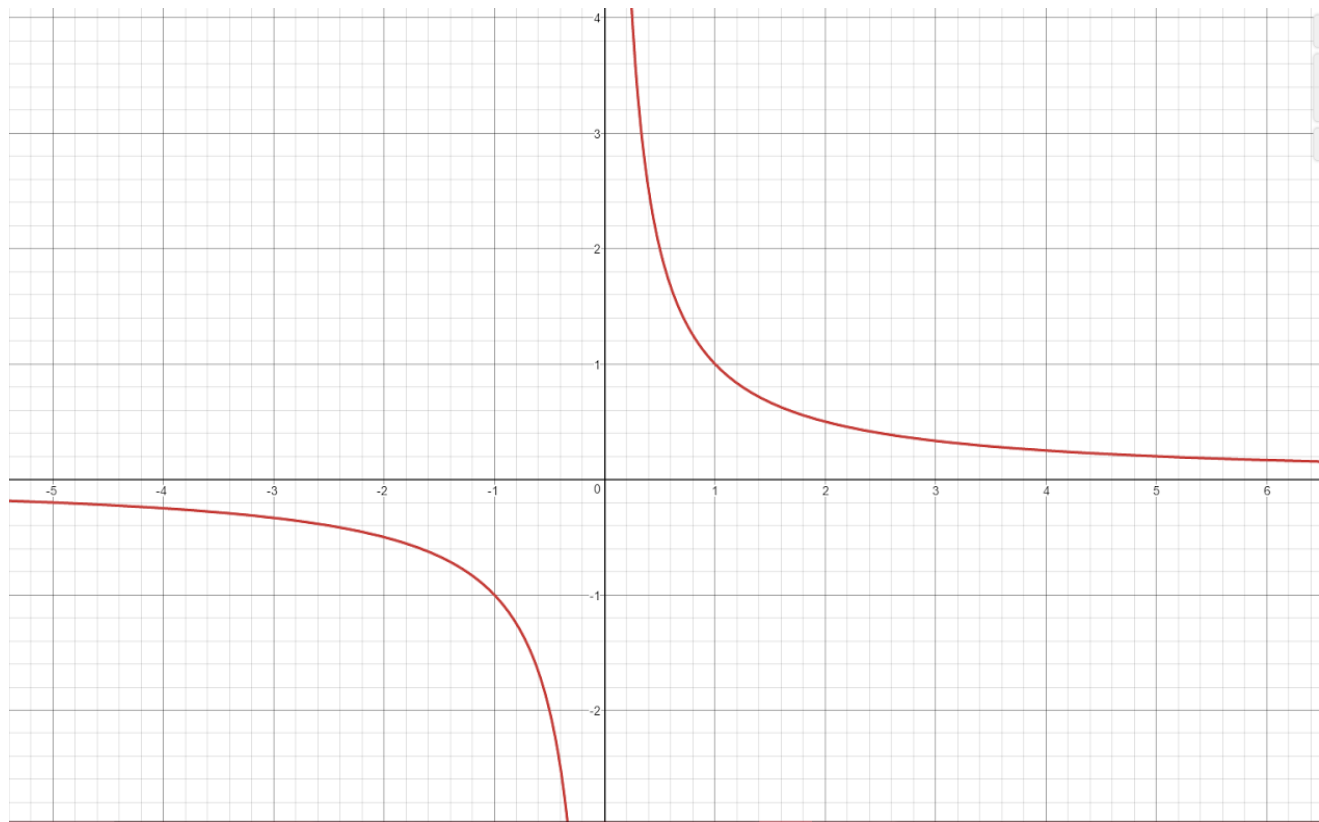


Ejemplo 2

$$f(x) = \frac{1}{x}$$

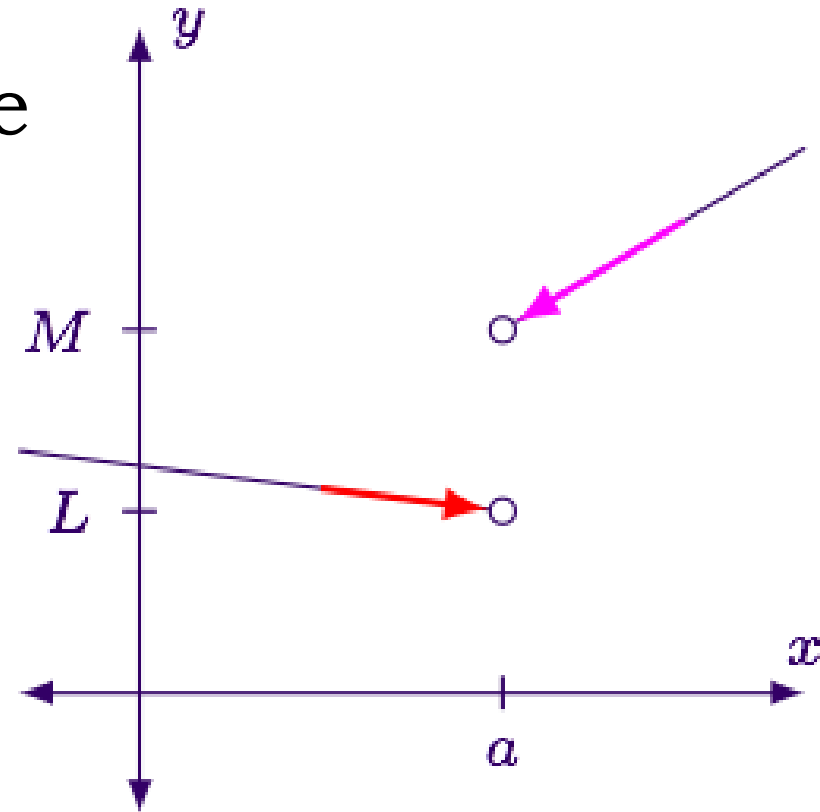
$$\lim_{x \rightarrow 0^-} \frac{1}{x} = -\infty$$

$$\lim_{x \rightarrow 0^+} \frac{1}{x} = +\infty$$



Jump discontinuity

- Una función que brinca de un punto a otro.



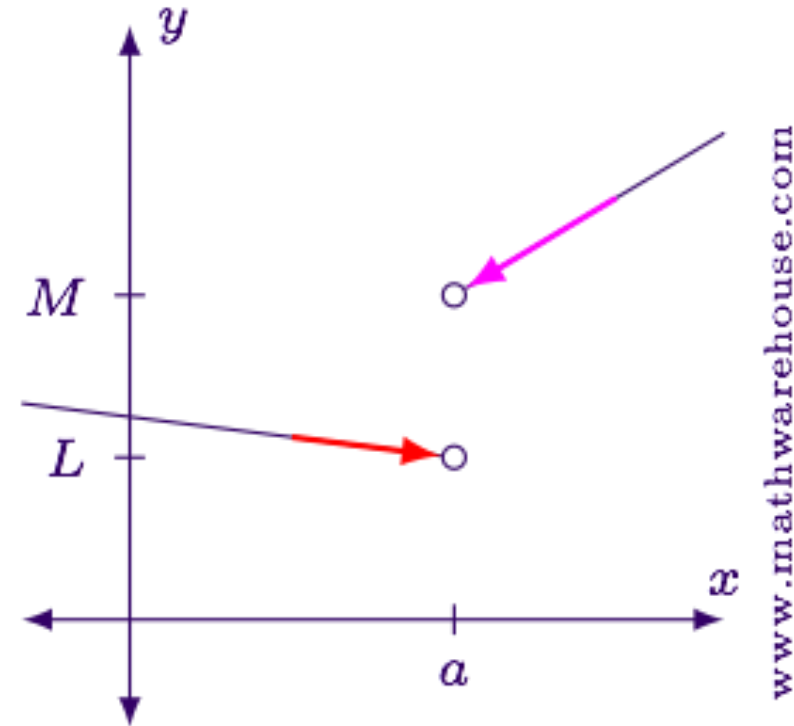
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Example of a Jump Discontinuity



Ejemplo 1

- En esta gráfica, se puede ver fácilmente que el límite $\lim_{x \rightarrow a^-} f(x) = L$ y $\lim_{x \rightarrow a^+} f(x) = M$.
- La función se aproxima a puntos diferentes dependiendo de donde venga la dirección de x . Cuando esto sucede, decimos que la función tiene un brinco de discontinuidad en $x = a$.

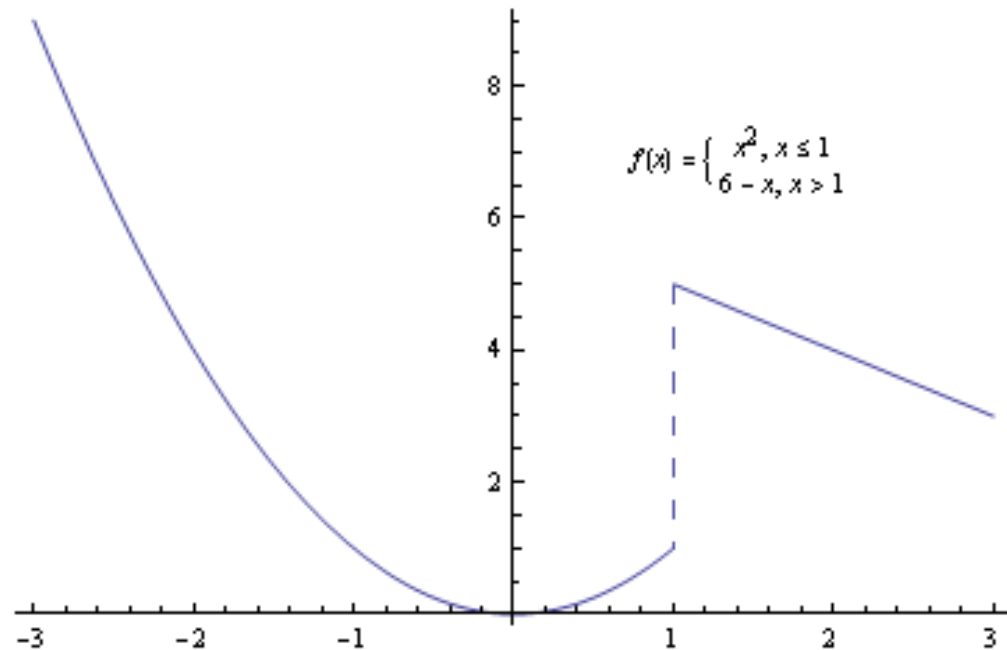


Example of a Jump Discontinuity



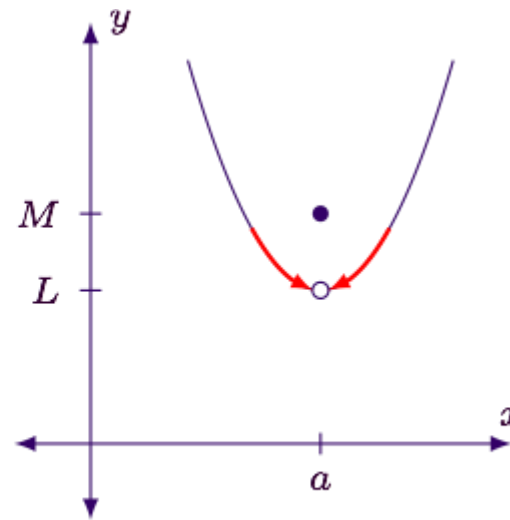
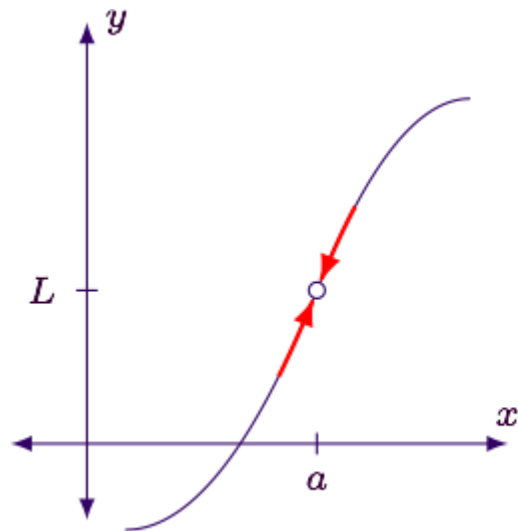
Ejemplo 2

- Las dos funciones tienen un valor diferente en $x = 1$ y podemos ver que en la gráfica f brinca de una rama a otra. Notase que el brinco hace a la función



Removable discontinuity

- Es cuando un punto en la gráfica es indefinido, es decir que hay un hueco.



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Examples of Removable Discontinuities

Notice that for both graphs, even though there are holes at $x=a$, the limit value at $x=a$ exists.

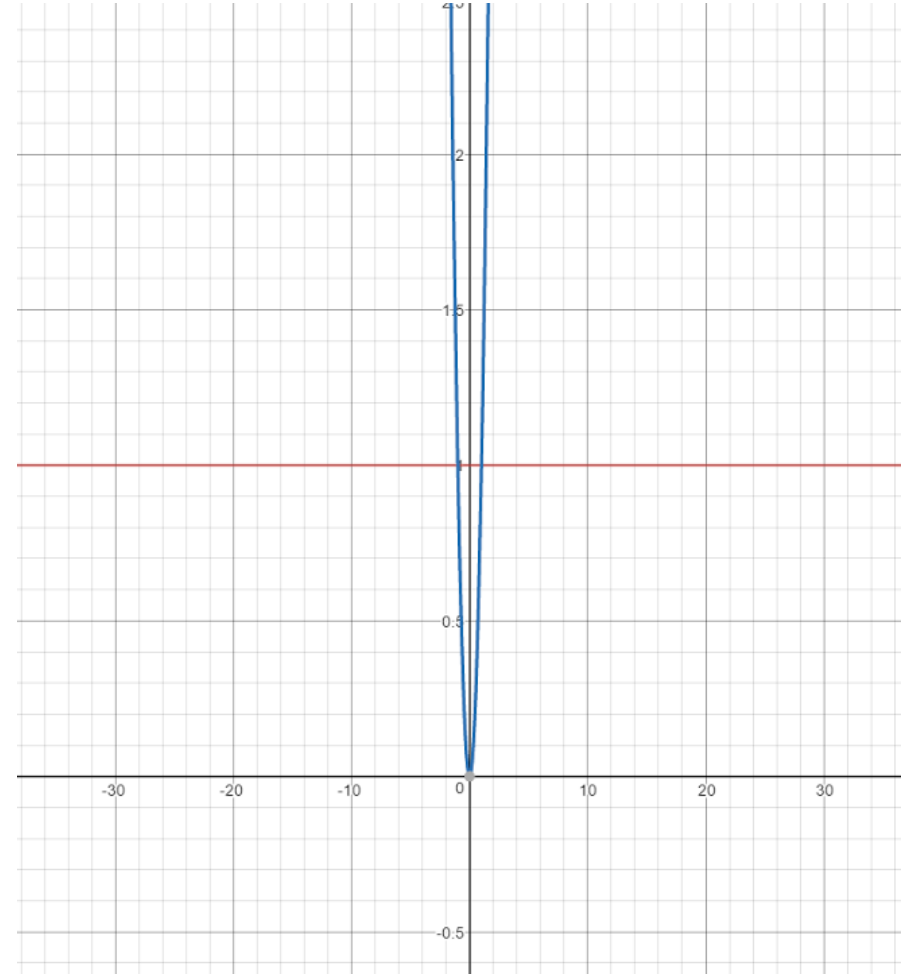


Ejemplo 1

Considerando la función

$$f(x) = \begin{cases} 1, & x = 3 \\ x^2, & \text{all other real } x - \text{values} \end{cases}$$

We defined the value of the function to be 1 at the point $x = 3$, yet, the rest of the function is dictated by $f(x) = x^2$. We can see in the graph that the function is continuous except for the tiny hole in the curve at $x = 1$.

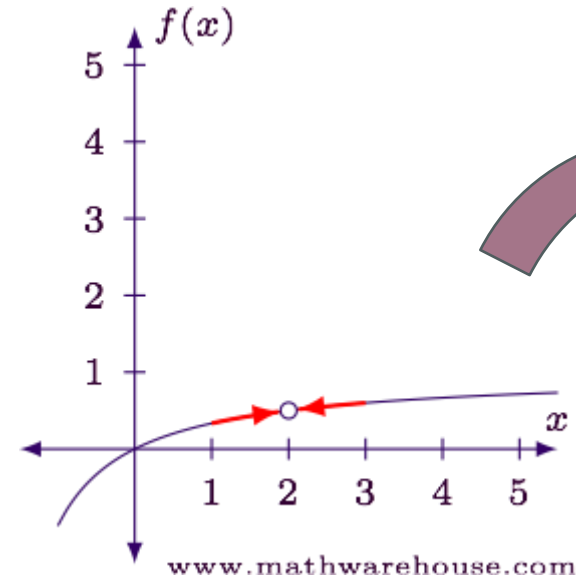


Ejemplo 2

$$f(x) = \frac{x^2 - 2x}{x^2 - 4}$$



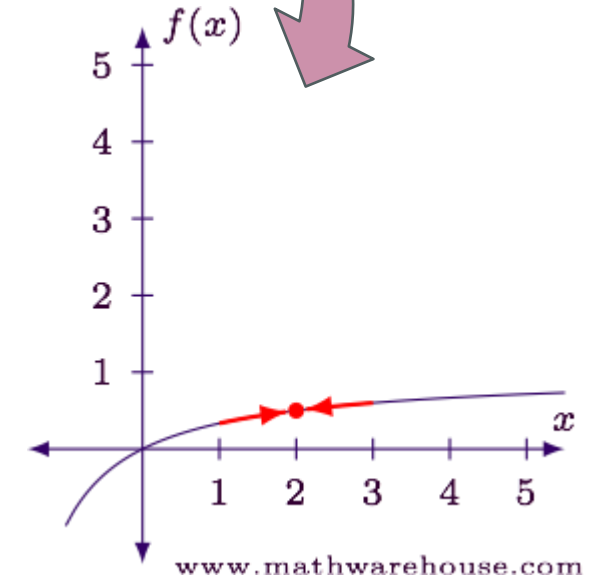
Redefine the equation so it becomes continuous at $x=2$



$$\lim_{x \rightarrow 2} \frac{x^2 - 2x}{x^2 - 4} = \frac{(2)^2 - 2(2)}{(2)^2 - 4} = \frac{0}{0}$$



We need to know Y value to find the hole. The division in tell us there's a discontinuity



$$\lim_{x \rightarrow 2} f(x) = \frac{1}{2}$$



So we redefine it into a piecewise function

$$f(x) = \begin{cases} \frac{x^2 - 2x}{x^2 - 4}, & \text{for all } x \neq 2 \\ \frac{1}{2}, & \text{for } x = 2 \end{cases}$$

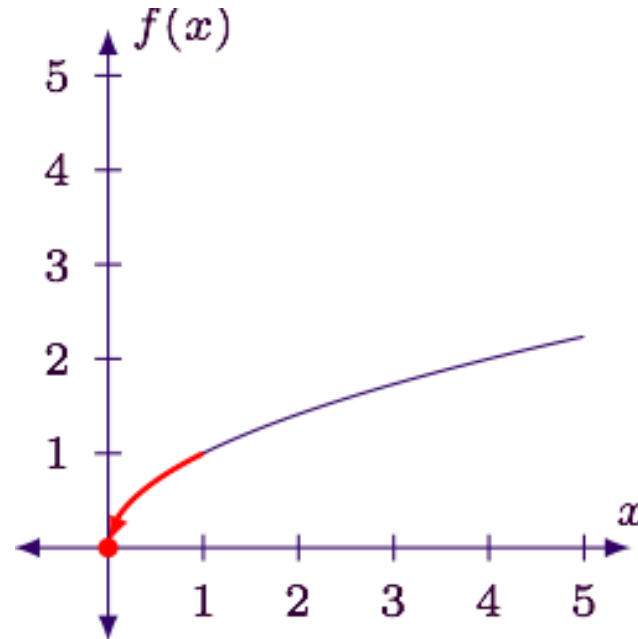


The first piece preserves the overall behavior of the function, while the second piece plugs the hole.



Endpoint discontinuity

- When a function is defined on an interval with a closed endpoint, the limit cannot exist at that endpoint. This is because the limit has to examine the function values as x approaches from both sides.
- For example, consider finding $\lim_{x \rightarrow 0} \sqrt{x}$ (see the graph below).

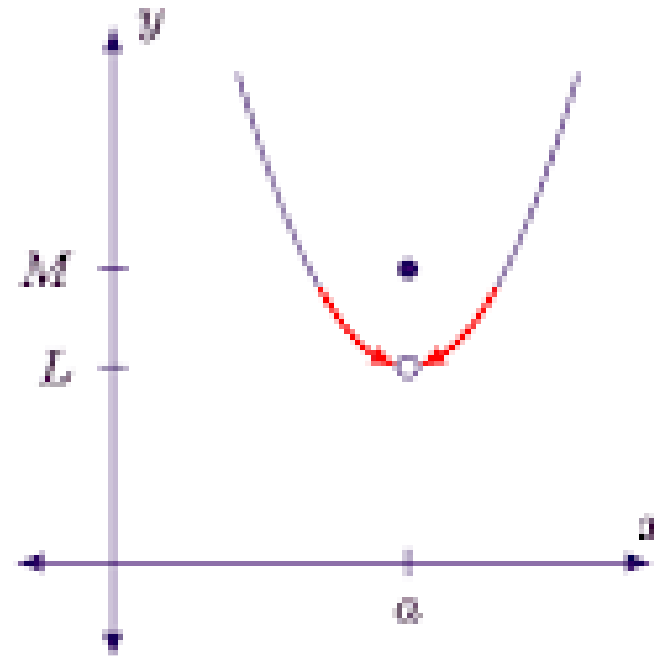
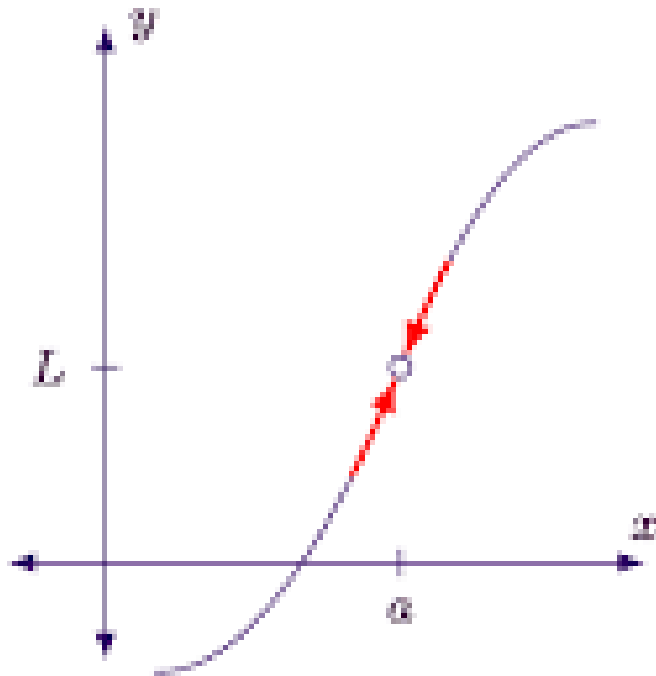


Note that $x=0$ is the left-endpoint of the function's domain: $[0, \infty)$, and the function is technically not continuous there because the limit doesn't exist (because x can't approach from both sides).

We should note that the function is right-hand continuous at $x=0$ which is why we don't see

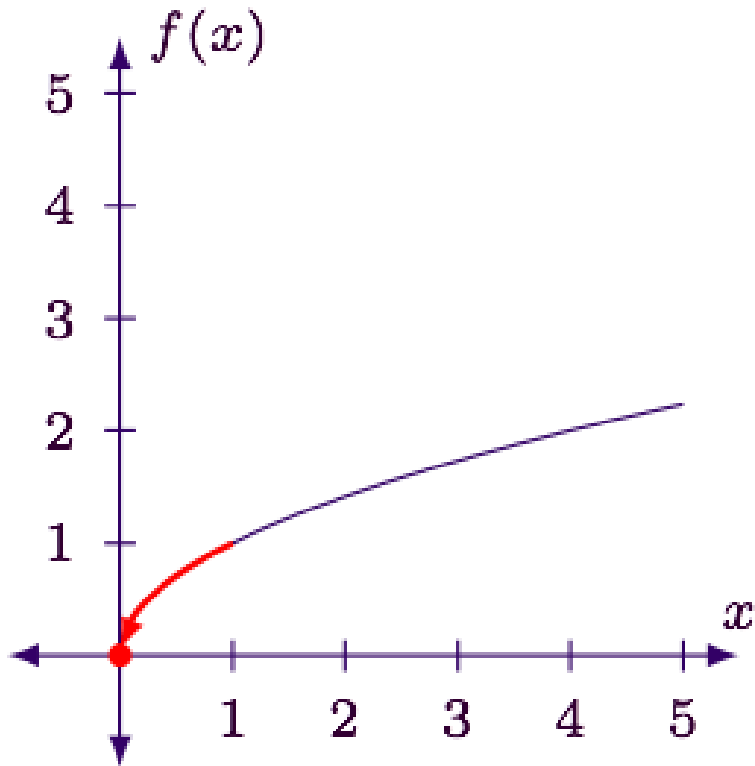
Ejemplo 1

- We have a limit since the $(a, \infty]$ from both sides of (a) , but never touching it.



Ejemplo 2

- consider finding $\lim_{x \rightarrow 0^+} x \sqrt{\lim_{x \rightarrow 0^+} x}$ (see the graph below)



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Note that $x=0$ is the left-endpoint of the function's domain: $[0, \infty)$, and the function is technically not continuous there because the limit doesn't exist (because x can't approach from both sides).

We should note that the function is right-hand continuous at $x=0$ which is why we don't see any jumps, or holes at the endpoint.



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