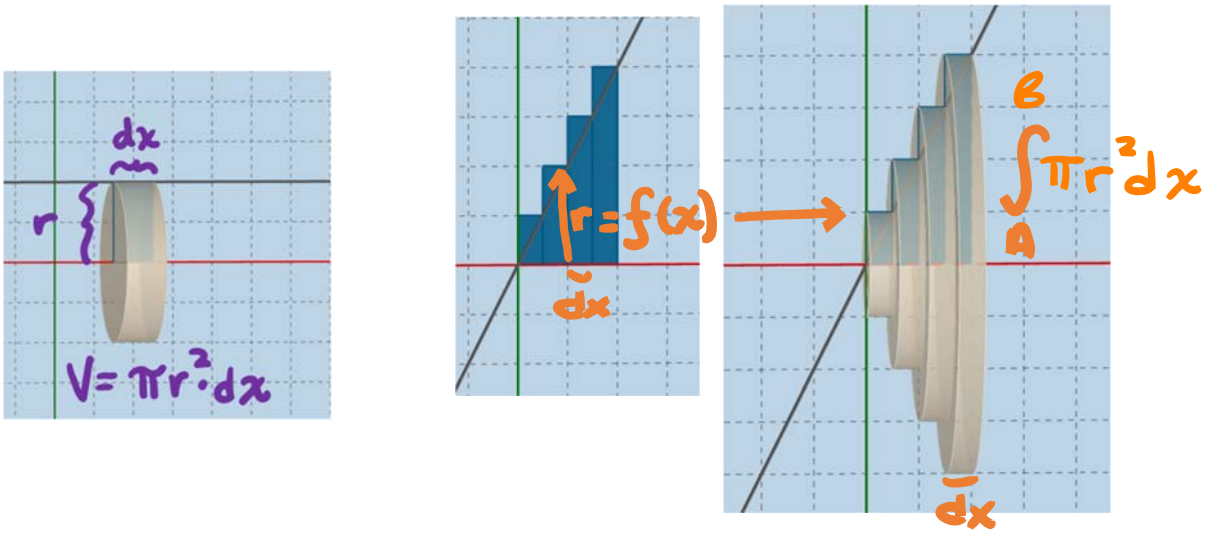
Accelerated AB Calculus **Name**_____

6.02 – Volumes of Revolution "Washers"

In the last episode of "Applications of Integrals - Disks"…

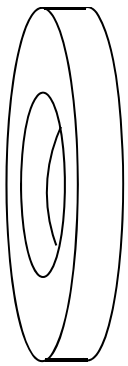


In each of the cases examined in the last class and on the homework, the **area region was flush up against the axis of revolution**, creating a bunch of "**disks**" (as shown above).

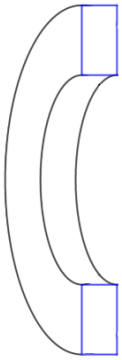
Let's start thinking about what would happen if we moved the axis of revolution so that the region was no longer against the axis of revolution….

Opener/Warm-Up

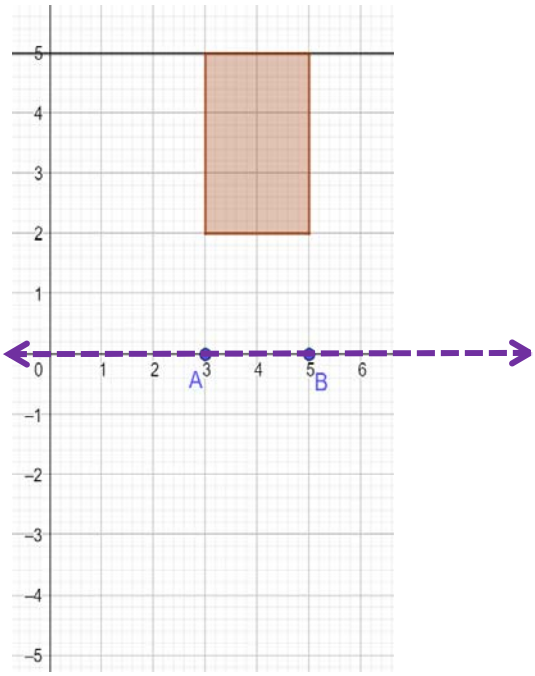
1a. Practice sketching the cylindrical "washer" shown below.



1b. Practice drawing the letter "C" which is half a "washer." [Sometimes it is easier to visualize just half of the washer as the diagram looks less cluttered.]



1c. Draw the **axis of revolution** as a dotted line in your sketches above. Also label the **inner radius**, R_i, and an outer radius, R_o on your sketches.

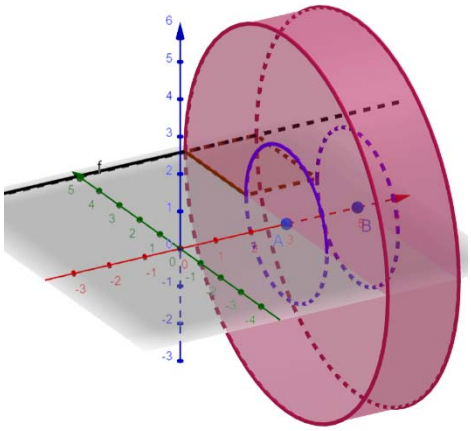


2. A solid is generated when revolving the region bound by the x-axis, the function $f(x) = 5$, $y = 2$, $x = 3$, and $x = 5$ around the x-axis.

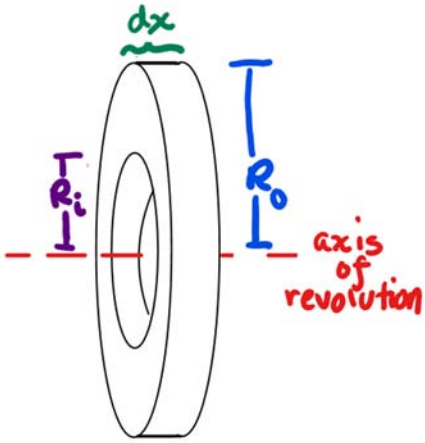
a. On the grid on the left, sketch a perspective of drawing of this solid (or half of it using the above practice in #1 as your guide.

Axis of Revolution = x-axis

b. Using geometry, find the volume of the washer you have drawn.

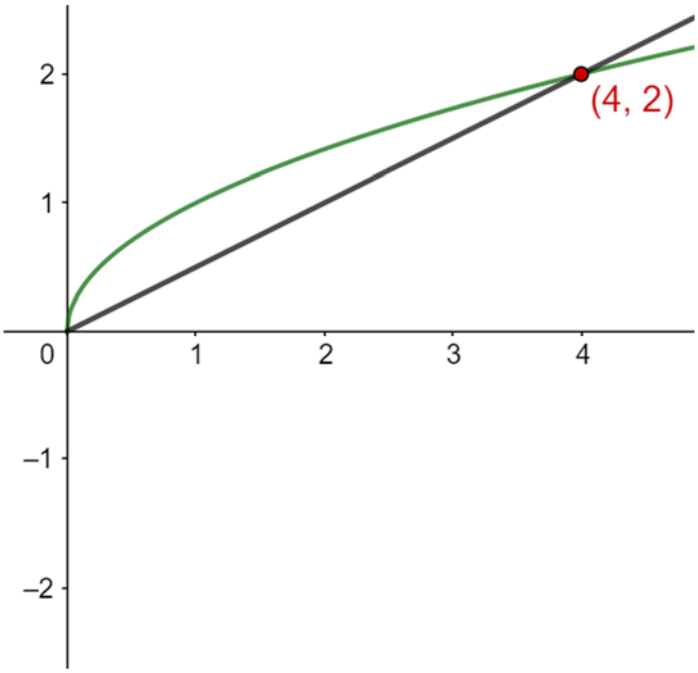


This is what it would look like if the z-axis was pointed upward instead of out of the page.



c. Write a generalized equation for the volume of a generic "skinny washer" using the diagram to the left.

6. Let **R** be the region bounded by $f(x)=\sqrt{x}$ and $g(x)=\frac{1}{2}x$.

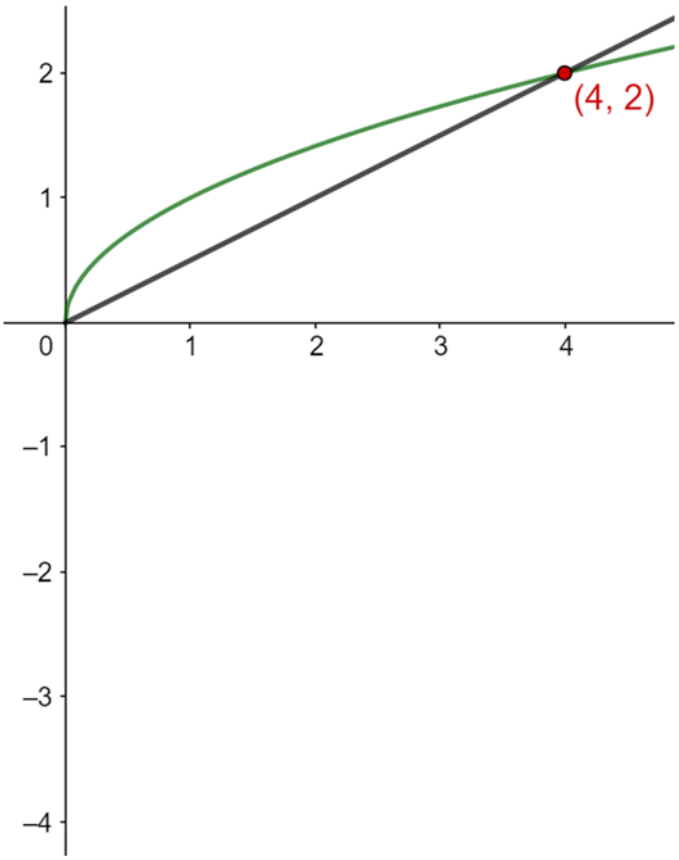


A solid is formed by rotating region R about:

i. An axis of revolution of the **x-axis**.

a. Sketch a single arbitrary rectangle in the region and its corresponding washer.

b. Write the integral that represents the volume of the solid formed by rotating R about the **x-axis**

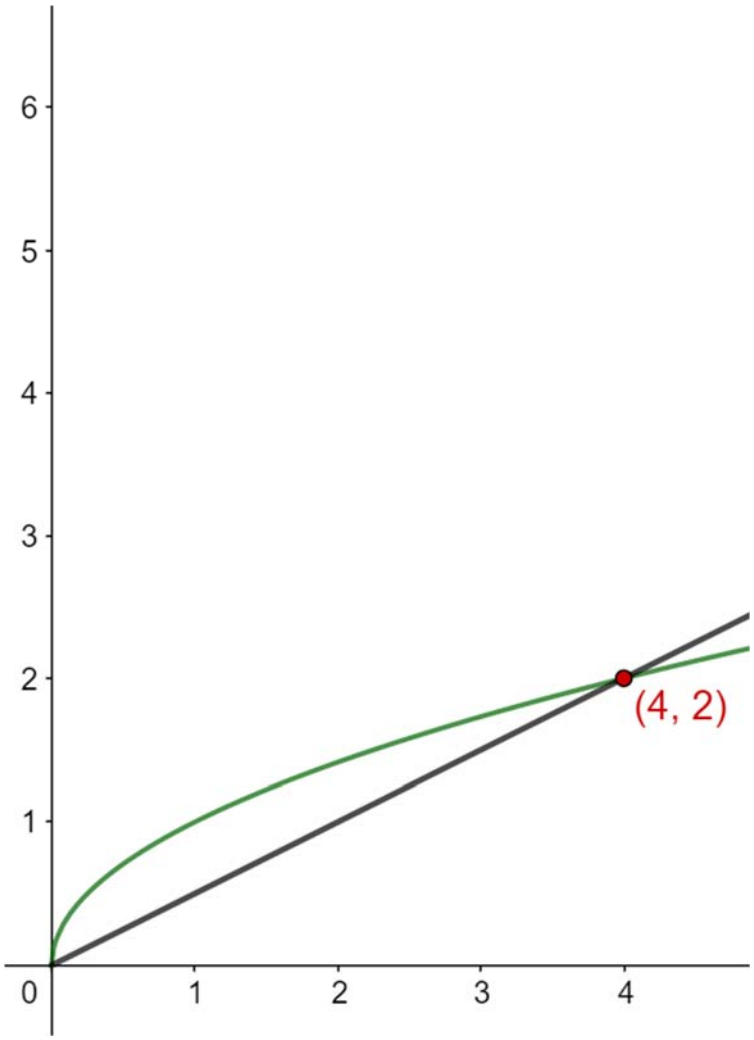


ii. An axis of revolution of $y=-1$

a. Sketch a single arbitrary rectangle in the region and its corresponding washer.

b. Write the integral that represents the volume of the solid formed by rotating R about $y=-1$

Using the same region R bounded by $f(x)=\sqrt{x}$ and $g(x)=\frac{1}{2}x$.

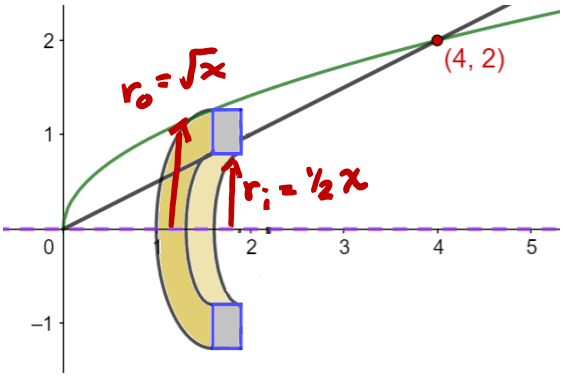


A solid is formed by rotating region R about:

iii. An axis of revolution of $y=3$

a. Sketch a single arbitrary rectangle in the region and its corresponding washer.

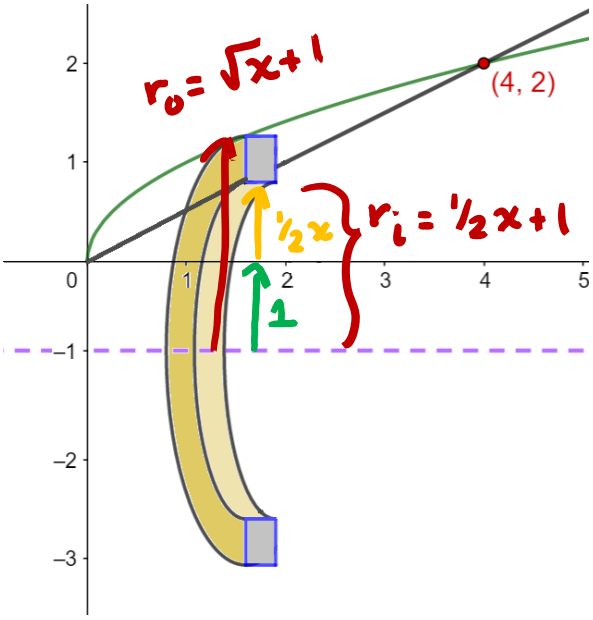
b. Write the integral that represents the volume of the solid formed by rotating R about $y=3$



$$\int_{x=0}^{x=4} \pi r_o^2 - \pi r_i^2 \, dx =$$

$$\pi \int_0^4 (\sqrt{x})^2 - (\tfrac{1}{2}x)^2 \, dx$$

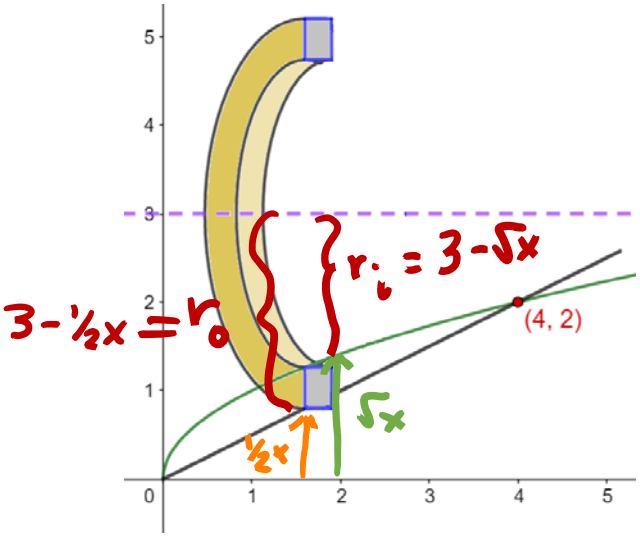
$$= \frac{8}{3}\pi$$



$$\int_0 \pi r_o^2 - \pi r_i^2 \, dx$$

$$= \pi \int_0^4 (\sqrt{x}+1)^2 - (\tfrac{1}{2}x+1)^2 \, dx$$

$$\approx 16.755$$



$$\int_0^4 \pi r_o^2 - \pi r_i^2 \, dx$$

$$\pi \int_0^4 (3-\tfrac{1}{2}x)^2 - (3-\sqrt{x})^2 \, dx$$

$$= 16.755$$