

NEB-GRADE XII
2079(2022)
Mathematics [0081'K']
New Course

For regular students (excluding technical stream's students)

Candidates are required to give their answer in their own words as far as practicable. The figures in the margin indicate full marks.

Time: 3 hrs

Full Marks: 75

Attempt **All** The Questions.

Group 'A'

[11 × 1 = 11]

Rewrite the correct options of each question in your answer sheet.

1. The property of cube roots of unity where ω is cube root of unity is
A. $1+\omega\omega^2=0$ B. $\omega^2+1=0$ C. $\omega=-\omega^2$ D. $\omega+\omega^2=1$
2. Which of the following is not true ?
 - A. The set of integers under addition is an abelian group.
 - B. The set of two order matrices is an abelian group under multiplication.
 - C. The set of rational numbers under multiplication is an abelian group.
 - D. The set of natural numbers is an abelian group under addition.
3. $\sin^{-1} x$ is equal to
A. $\cos^{-1} \sqrt{x^2-1}$ B. $\tan^{-1} \sqrt{1-x^2}$ C. $\operatorname{cosec}^{-1} \sqrt{1-x^2}$
D. $\cos^{-1} \sqrt{1-x^2}$

4. If $\tan x + \frac{1}{\sqrt{3}} = 0$, is the trigonometric equation then the general value for x is
A. $n\pi \pm \frac{\pi}{6}$
B. $2n\pi + \frac{\pi}{6}$
C. $2n\pi + \frac{5\pi}{6}$
D. $n\pi + \frac{5\pi}{6}$
5. If $\vec{a}$ and $\vec{b}$ are two vectors then the value of $(\vec{a} - \vec{b})(\vec{a} + \vec{b})$ is equal to
A. $2(\vec{a} \times \vec{b})$ B. $\vec{a}^2 - \vec{b}^2$ C. $-2(\vec{a} \times \vec{b})$ D. $2(\vec{a}^2 - \vec{b}^2)$
6. The eccentricity of the ellipse $\frac{x^2}{16} - \frac{y^2}{64} = 1$ is
A. $\frac{3}{5}$ B. $\frac{5}{3}$ C. $\frac{3}{4}$ D. $\frac{4}{3}$
7. The range of the probability of any event A is
A. $0 \leq P(A) \leq 1$
B. $0 \leq P(A) < 1$
C. $-1 \geq P(A) \leq 1$
D. $0 < P(A) < 1$
8. Tangent to the curve $y = x^2$ at $x = 2$ and $x = -2$ are
A. parallel B. perpendicular C. intersectiong D. Does not exist
9. The integrating factor of differential equation $\sin x \frac{dy}{dx} + y \cos x$ is
A. $\sin x$ B. $\cos x$ C. $\tan x$ D. $\cot x$
10. During the process of solving system of linear equation in x, y, z if we get $0 \times z = 0$, then the system is
A. inconsistent and has no solution

- B. consistent and has unique solution
- C. inconsistent and but has solution
- D. consistent and has infinitely solution

11. A force acts on a body of mass m for a time t and changes its velocity from u to v . The change in momentum of the body in time t is

A. $\frac{mu - mv}{t}$ B. $\frac{mv - mu}{t}$ C. $\frac{tm - tv}{uv}$ D. $\frac{u - v}{mt}$

OR

In an input-output model if A = input coefficient of matrix, I be unit matrix then each diagonal elements of the $(I - A)$ is positive and $[I - A]$ is positive then the system is called

- A. viable
- B. unviable
- C. unit elastic
- D. elastic

Group 'B'

[8 × 5 = 40]

12. (a) Define binomial expansion. (1)
 (b) Find the middle term of $(2 + 3x)^{30}$. (2)
 (c) Find the sum of n binomial coefficients in expansion of $(1 + x)^n$ when $x = 1$. (1)
 (d) What do you mean by argument of complex number? (1)
13. (a) Prove that $1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$ by mathematical induction. (4)
 (b) Find the sum of square of first 20 natural numbers. (1)
14. (a) Find the equation of plane through $(-3, 2, -1)$ and parallel to the plane $2x + 5y - 2 = 0$ (2)
 (b) Using vector method, prove that $\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$ in any ΔABC . (3)

15. (a) Find the derivative of $(\tan x)^{\log x}$. (2)

(b) Solve: $(1 + x)dy = dx(1 + xy - x)$ (3)

16. Find the regression equations related to age (x) and blood pressure (y) are given as below:

$$0.22x - y + 81.1175 = 0 \text{ and } x - 1.623y + 105.695 = 0$$

(a) Find the regression coefficients. (2)

(b) Find the correlation coefficient. (1)

(c) Find the mean of x and mean of y . (2)

(d) What does ' C ' represent in the expression $\int \frac{dx}{3 \sin x + 4 \cos x} = \frac{1}{5} \ln \left| \tan \left(\frac{x}{2} + \frac{1}{2} \tan^{-1} \frac{4}{3} \right) \right| + C$?

17. Define proper rational function with an example. Integrate $\int \frac{2x^2 + 3}{x^3 + 3x^2 + 2x} dx$ using concept of partial function. (1 + 4)

18. Use simplex method and maximize:
 $Z(x, y) = 7x + y$ subject to constraints $2x + 3y \leq 12, y - x \leq 14, xy \geq 0$. (5)

19. A particle is thrown with an initial velocity of $120ms^{-1}$ at an angle of 30° above the horizon. If $g = 10ms^{-2}$ then find

(a) the time to attain the highest point (1)

(b) the time of flight (1)

(c) the horizontal range (1)

(d) the greatest height reached (1)

(e) the maximum horizontal range. (1)

OR

Given the transition matrix (5)

Producing sector	Purchasing sector		Final Demand
	Agriculture	Industry	
Agriculture	300	600	100
Industry	400	1200	400

- (a) Write the input coefficient matrix.
 (b) Find the total output to meet the final demand of 100 units of agriculture and 400 units of industry.

Group 'C' [3 × 8 = 24]

20. A mixture is to be prepared of three foods A , B and C which contain nutrients P , Q and R as shown in table below. The total quantity of 30 units of P , 36 units of Q and 30 units of R respectively be required.

Foods	P	Q	R
A	2	2	4
B	3	5	0
C	4	3	5

- (a) Express the information in equation form. (1)
 (b) Solve the equations using matrix. (5)
 (c) If the cost per kg of the foods A , B , C are Rs. 40, Rs. 60 and Rs. 80 respectively, find the total cost of the mixture by matrix method. (2)
21. (a) Solve $\sqrt{3}\sin\theta + \cos\theta = 2$. (3)
 (b) Find the co-ordinates of the point which divides the line segment joining $(1, -2, 3)$ and $(1, 2, 3)$ internally in the ratio $1 : 2$. (2)
 (c) Find the point where the line through the points $(1, 2, 3)$ and $(4, -4, 9)$ meets the ZX -plane. (3)

22. A water tank is filled in a such a way that the rate at which the depth of water increases is proportional to the square roots of the depth. Initially the depth is ' n ' meters.

- (a) Write down a differential equation for h . (2).
 (b) Show that $\sqrt{h} = \frac{kt}{2} + \sqrt{n}$ where k is a constant. (4)
 (c) When $h = 16$ and $t = 6$ hours, prove that $n = (4 - 3k)^2$ (2)

Ambik