Sara Kolar, 3.e

PRAVCI - ZADATCI

6.1.

14. Odredi jednadžbu pravca koji je simetričan pravcu $3x - 4y + 8 = 0$ s obzirom na osi apscisa i ordinata

a) apscisa

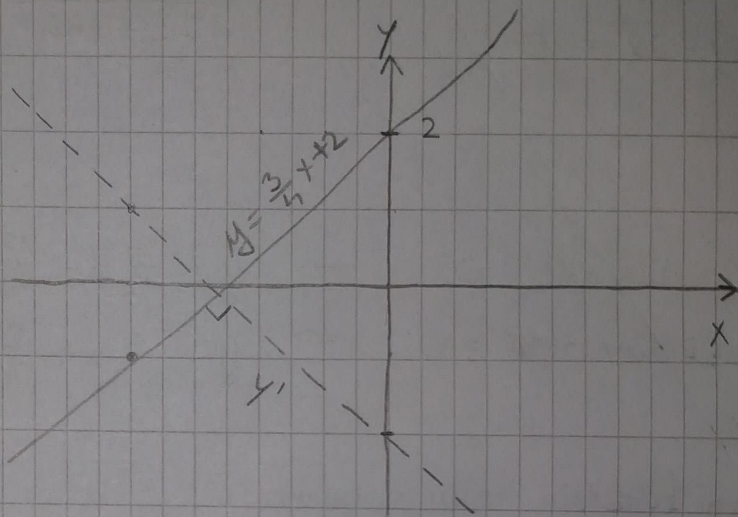
$3x - 4y + 8 = 0$

$3x + 8 = 4y$

$\frac{3}{4}x + 2 = y$

$y \perp y'$

$y' = -\frac{4}{3}x - 2$



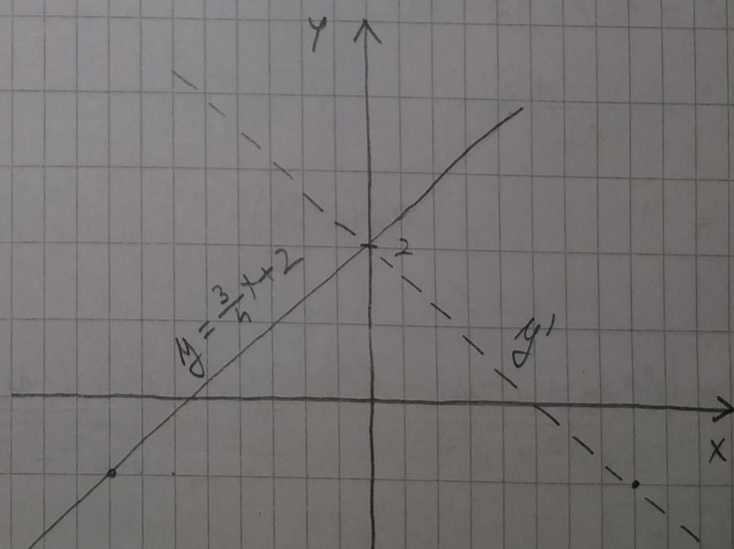
b) ordinata

$3x - 4y + 8 = 0$

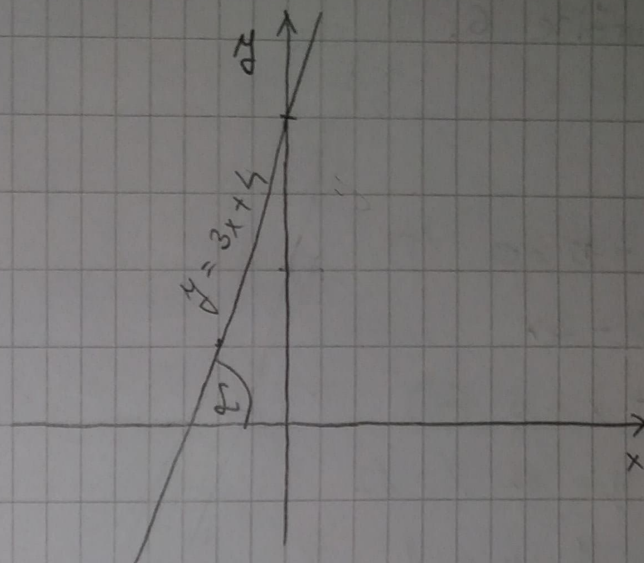
$y = \frac{3}{4}x + 2$

$y \perp y'$

$y' = -\frac{4}{3}x + 2$



17. Napiši jednadžbu pravca koji prolazi točkom $T(2,2)$, a s pozitivnim dijelom osi apscisa zatvara dvostruko veći kut od pravca $y = 3x + 4$



$T(2,2)$

$y_1 = 3x + 4$

$y_2 = ?$

$k_1 = tg\,\alpha_1$

$3 = tg\,\alpha$

$\alpha_1 = 71°33'54''$

$\alpha_2 = 2 \cdot \alpha_1$

$\alpha_2 = 143°\,7'\,48''$ $\Rightarrow$ $k_2 = tg\,\alpha_2 = -\frac{3}{4}$

$y = kx + l$

$y = -\frac{3}{4}x + l$

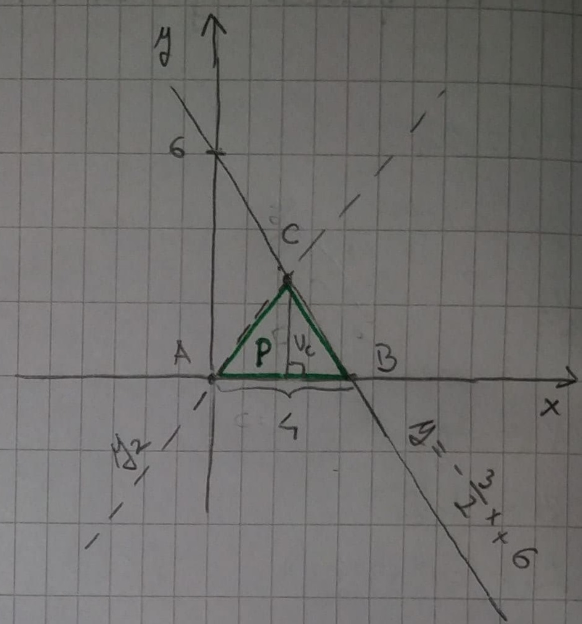
$2 = -\frac{6}{4} + l$

$l = 2 + \frac{6}{4}$

$l = 3.5$

$y = -\frac{3}{4}x + \frac{7}{2}$

24. Ishodištem koordinatnog sustava položi pravac koji će s pravcem $3x+2y-12=0$ i osi apscisa tvoriti trokut površine 6.



$P=6$

$3x+2y-12=0$ (I)

$2y=12-3x$

$y=-\frac{3}{2}x+6$ (E)

$3x+2y=12 \quad /:12$

$\frac{1}{4}x+\frac{1}{6}y=1$

$\frac{x}{4}+\frac{y}{6}=1$ (S)

$\frac{c \cdot v_c}{2}=P$

$\boxed{c=4}$

$v_c=\frac{2P}{c} \Rightarrow v_c=3$

$T=0,0$

$y_1 \ldots 3x+2y-12=0$

$y_2 \ldots y=kx$

$C \ldots y_1 \cap y_2 \Rightarrow C(x_c, \pm 3)$

$y_c=3$

$3x_c+2\cdot 3-12=0$

$x_c=2$

$T(0,0)$

$C_1(2,3)$

$k=\frac{3}{2}$

$y_c=-3$

$3x_c-6-12=0$

$x_c=6$

$C(6 \ -3)$

$k=-\frac{1}{2}$

$y_2 \ldots y=\frac{3}{2}x$

$y_2' \ldots y=-\frac{1}{2}x$

6.2

12. Odredi jednadžbu pravca koji prolazi točkom $T(5,1)$ i na koordinatnim osima odsijeca odsječke jednakih duljina

$T(5,1)$

$m = \pm n$

$\frac{x}{m} + \frac{y}{m} = 1$

$\frac{5}{m} + \frac{1}{m} = 1$

$\frac{6}{m} = 1 \quad / \cdot \frac{m}{6}$

$1 = \frac{m}{6} \quad / \cdot 6$

$m = 6$

$n = 6$

$\frac{x}{6} + \frac{y}{6} = 1$ (S)

$y = -x + 6$ (E)

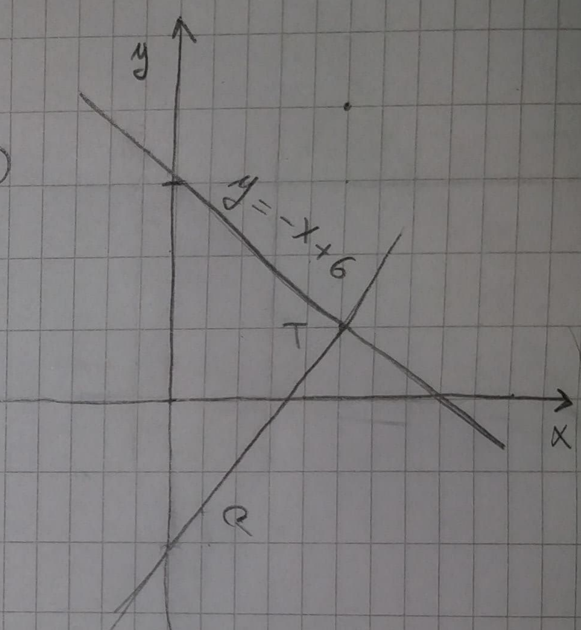
$x + y - 6 = 0$ (I)

$\frac{x}{m} + \frac{y}{-m} = 1$

$\frac{5}{m} + \frac{1}{-m} = 1$

$m = 4$

$y = x - 4$



13. Odsječak pravca p na osi ordinata 3 puta je veći od od njegovog odsječka na osi apscisa. Pravac prolazi točkom $T(3,3)$. Odredi pravac.

$T(3,3)$

$\pm 3m = n$

$\frac{x}{m} + \frac{y}{n} = 1$

$\frac{x}{m} + \frac{y}{3m} = 1$

$\frac{3}{m} + \frac{\cancel{3}}{\cancel{3}m} = 1$

$\frac{4}{m} = 1 \Rightarrow m = 4$

$\frac{x}{4} + \frac{y}{12} = 1$ (S)

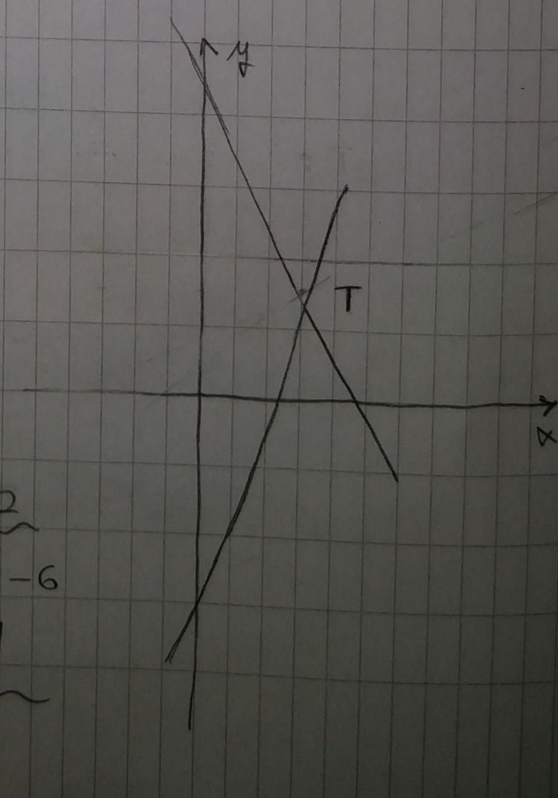
$\frac{3}{m} + \frac{\cancel{3}}{-\cancel{3}m} = 1$

$\frac{3}{m} - \frac{1}{m} = 1$

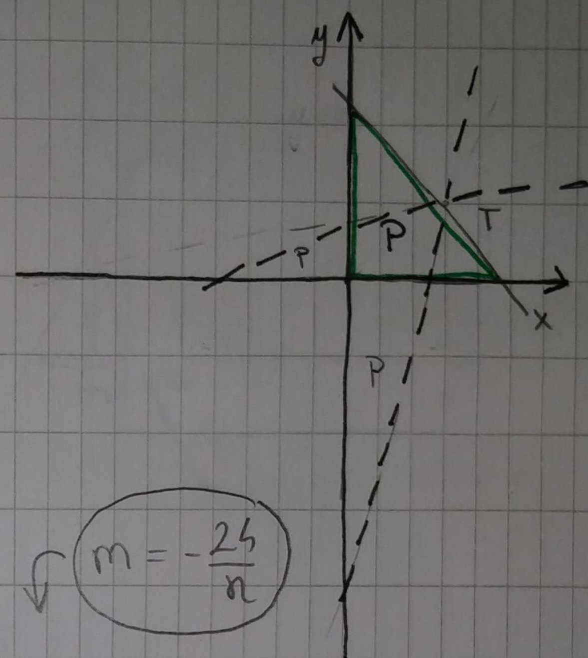
$\frac{2}{m} = 1 \Rightarrow m = 2$

$n = -6$

$\frac{x}{2} + \frac{y}{-6} = 1$



14. Odredi jednadžbu pravca koji prolazi točkom $T(8,6)$, a s koordinatnim osima zatvara trokut površine 12.



$T(8,6)$

$P_\triangle = 12$

$$\frac{|m \cdot n|}{2} = \frac{a \cdot v_a}{2}$$

$$|m| = \frac{24}{|n|} \Rightarrow \frac{x}{\frac{24}{n}} + \frac{y}{n} = 1$$

$$= \frac{\cancel{8}^{1}}{\frac{\cancel{24}_{3}}{n}} + \frac{6}{n} = 1$$

$$= \frac{\frac{1}{1}}{\frac{3}{n}} + \frac{6}{n} = 1$$

$$= \frac{n}{3} + \frac{6}{n} = 1$$

$$= \frac{n^2}{3n} + \frac{18}{3n} = 1$$

$$= \frac{18+n^2}{3n} = 1 \quad / \cdot 3n$$

$$= 18 + n^2 = 3n$$

$$= n^2 - 3n + 18 = 0$$

$$n_{1,2} = \cancel{\frac{3 \pm 3\sqrt{7}i}{2}}$$

nema rješenja

$$m = -\frac{24}{n}$$

$$\frac{x}{-\frac{24}{n}} + \frac{y}{n} = 1$$

$$\frac{\frac{1}{1}}{-\frac{3}{n}} + \frac{6}{n} = 1$$

$$-\frac{n}{3} + \frac{6}{n} = 1$$

$$-\frac{n^2}{3n} + \frac{18}{3n} = 1 \quad / \cdot 3n$$

$$-n^2 + 18 = 3n$$

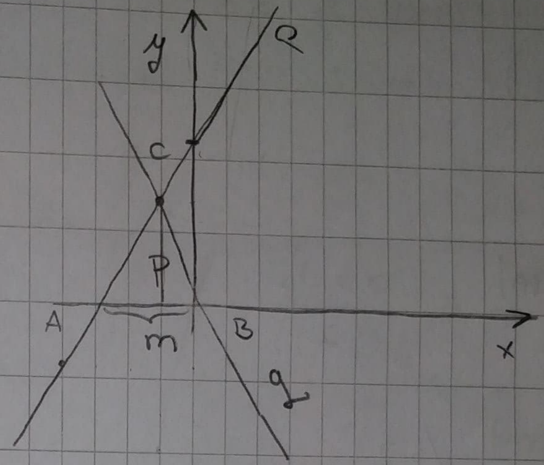
$$-n^2 - 3n + 18 = 0$$

$$n_{1,2} = 3, -6$$

$$\frac{x}{-8} + \frac{y}{3} = 1$$

$$\frac{x}{4} + \frac{y}{-6} = 1$$

15. Ishodištem koordinatnog sustava položi pravac koji će s pravcem $3x - 4y + 18 = 0$ i osi apscisa zatvarati trokut površine 9



$P_\triangle = 9$

$p \dots \; 3x - 4y + 18 = 0$ (1)

$g \dots \; ?$

$T(0,0)$

$$3x - 4y + 18 = 0$$
$$3x + 18 = 4y$$
$$\frac{3}{4}x + \frac{9}{2} = y \quad (E)$$

$$3x - 4y = -18 \quad /:-18$$
$$\frac{3}{-18}x + \frac{-4}{-18}y = 1$$
$$-\frac{1}{6}x + \frac{2}{9}y = 1$$
$$\frac{x}{-6} + \frac{y}{4.5} = 1 \quad (S)$$

$$m = -6$$
$$\frac{|m \cdot v_m|}{2} = P$$
$$P = \frac{\cancel{6}^{3} \cdot v_m}{\cancel{2}} \quad /:3$$
$$\frac{P}{3} = v_m \Rightarrow \boxed{v_m = 3}$$
$$v_m = y_C$$

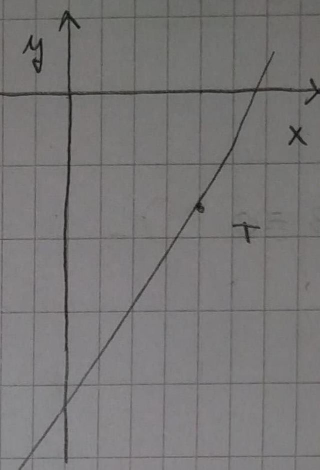
$T(0,0),\ C(x_C, 3)$

$$g \dots \; y = kx$$
$$4 \cdot 3 = 3 \cdot x_C + 18$$
$$12 = 3x_C + 18$$
$$-6 = 3x_C$$
$$x_C = -2$$
$$C(-2, 3)$$
$$y = -\frac{3}{2}x$$

$C(x_C, -3)$

$$g \dots \; y = kx$$
$$4 \cdot -3 = 3x_C + 18$$
$$-12 = 3x_C + 18$$
$$-30 = 3x_C \quad /:3$$
$$x_C = -10$$
$$C(-10, -3)$$
$$y = \frac{3}{10}x$$

16. Točkom $T(4,-3)$ položi pravac tako da površina trokuta što ga taj pravac tvori s koordinatnim osima bude 3.



$T(4,-3)$

$P_\Delta = 3$

$$P_\Delta = \frac{|n \cdot m|}{2} = \frac{a \cdot v_a}{2}$$

$$3 = \frac{|n| \cdot |m|}{2} \quad / \cdot 2$$

$$6 = |n| \cdot |m|$$

$$m = \frac{6}{n}$$

$$\frac{x}{\frac{6}{n}} + \frac{y}{n} = 1$$

$$\frac{\cancel{4}^{2}}{\frac{\cancel{6}_{3}}{n}} + \frac{-3}{n} = 1$$

$$\frac{2n}{3} - \frac{3}{n} = 1$$

$$\frac{2n^2}{3n} - \frac{9}{3n} = 1 \quad / \cdot 3n$$

$$2n^2 - 9 = 3n$$

$$2n^2 - 3n - 9 = 0$$

$$n_{1,2} = 3, -\frac{3}{2}$$

$$m = -\frac{6}{n}$$

$$\frac{x}{-\frac{6}{n}} + \frac{y}{n} = 1$$

$$\frac{\frac{\cancel{4}^{2}}{1}}{-\frac{\cancel{6}_{3}}{n}} + \frac{-3}{n} = 1$$

$$-\frac{2n}{3} - \frac{3}{n} = 1$$

$$-\frac{2n^2}{3n} - \frac{9}{3n} = 1 \quad / \cdot 3n$$

$$-2n^2 - 9 = 3n$$

$$n_{1,2} = \cancel{\frac{-3 \pm 3\sqrt{7}\, i}{4}}$$

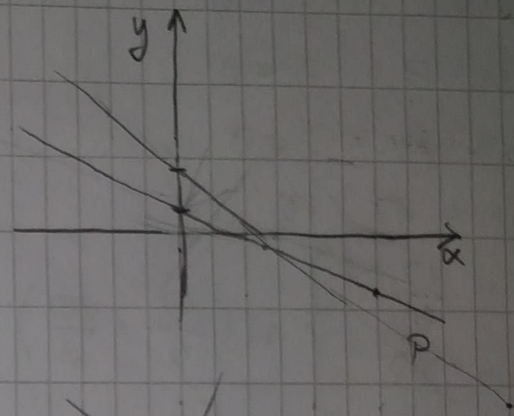
nema rješenja

6.3

29. Ortocentar trokuta kojem su dvije stranice na pravcima $x+3y-1=0$ i $3x+5y-6=0$ ishodište je koordinatnog s. Odredi jednadžbu pravca kojem pripada treća stranica trokuta

p... $x+3y-1=0 \Rightarrow -\frac{1}{3}x+\frac{1}{3}=y$

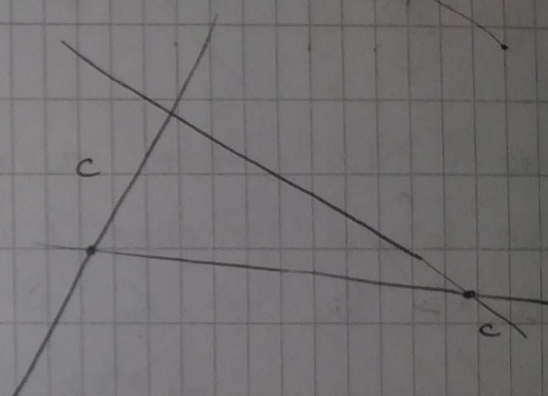
q... $3x+5y-6=0 \Rightarrow -\frac{3}{5}x+\frac{6}{5}=y$



ortocentar = sjecište okomica na stranice

$H(0,0)$

$-\frac{1}{3}x+\frac{1}{3} = -\frac{3}{5}x+\frac{6}{5} \quad /\cdot 15$



$-5x+5 = -9x+18$

$4x = 13 \quad /:4x$

$x = \frac{13}{4}$ —— $C\left(\frac{13}{4}, -\frac{3}{4}\right)$

$y = -\frac{1}{3}\cdot\frac{13}{4}+\frac{1}{3}$

$y = -\frac{13}{12}+\frac{1}{3} \Rightarrow \frac{-13+4}{12}$

$= -\frac{\cancel{9}^{3}}{\cancel{12}_{4}} = -\frac{3}{4}$

$k_{vc} = \frac{-\frac{3}{4}}{\frac{13}{4}} = -\frac{13}{3}$

$y_{vc} = -\frac{13}{3}x$

$V_a = 3x$

$V_b = \frac{5}{3}x$

A $\Rightarrow$ $3x = -\frac{3}{5}x+\frac{6}{5}$

$\left(\frac{1}{3}, 1\right)$

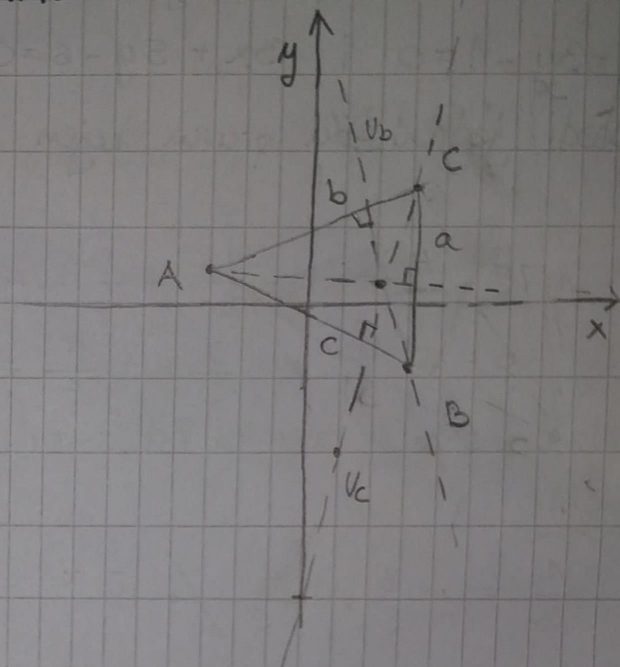
$x = \frac{1}{3}$

$y = \frac{1}{3}\cdot 3 \Rightarrow 1$

c $\Rightarrow$ $y-1 = \frac{13}{3}\left(x-\frac{1}{3}\right)$

$39x-9y-4=0$

30. Na pravcima $y = 4x - 8$ i $y = -2x + 9$ dvije su visine trokuta ABC. Ako je $A(-3, 1)$, odredi vrhove B i C.



$V_c \ldots\ 4x - 8 = y$

$V_b \ldots\ -2x + 9 = y$

$4x - 8 = -2x + 9$

$4x + 2x = 9 + 8$

$6x = 17 \ /:6$

$$\left.\begin{aligned} x &= \frac{17}{6} \\ y &= \frac{10}{3} \end{aligned}\right\} H\left(\frac{17}{6}, \frac{10}{3}\right)$$

za C su potrebne a i b ili V_c i b ili V_c i a

za B su potrebne c i a ili V_b i c ili V_b i a

$a \Rightarrow V_a \Rightarrow A$ i H

$$y = kx + l$$

$$y = \frac{\frac{10}{3} - 1}{\frac{17}{6} + 3} x + l$$

$$y = \frac{2}{5} x + l$$

$$1 = -\frac{6}{5} + l$$

$$l = \frac{11}{5}$$

$$V_a \Rightarrow \boxed{y = \frac{2}{5}x + \frac{11}{5}} \Rightarrow a \Rightarrow y = -\frac{5}{2}x + l$$

$c \Rightarrow A(-3,1) \text{ i } 4x - 8 = y$

$\Rightarrow y_c = -\frac{1}{4}x + l$

$1 = \frac{3}{4} + l$

$l = \frac{1}{4}$

$y_c = -\frac{1}{4}x + \frac{1}{4}$

$w \Rightarrow A(-3,1) \text{ i } -2x + 9 = y$

$\Rightarrow y_b = \frac{1}{2}x + l$

$1 = -\frac{3}{2} + l$

$l = \frac{5}{2}$

$y_w = \frac{1}{2}x + \frac{5}{2}$

$C \Rightarrow 4x - 8 = \frac{1}{2}x + \frac{5}{2}$

$4x - \frac{1}{2}x = \frac{5}{2} + 8$

$\frac{7}{2}x = \frac{21}{\cancel{2}} \;/\cdot \frac{2}{7}$

$x = 3$

$y = 12 - 8$

$y = 4$

$C(3,4)$

$B \Rightarrow -2x + 9 = -\frac{1}{4}x + \frac{1}{4}$

$-2x + \frac{1}{4}x = \frac{1}{4} - 9$

$-\frac{7}{4}x = -\frac{35}{4} \;/\cdot -\frac{4}{7}$

$x = 5$

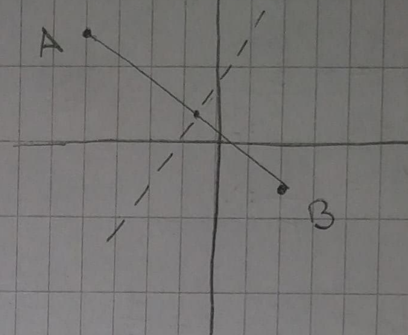
$y = -10 + 9 = -1$

$B(5,-1)$

38. Kolika je površina trokuta što je simetrala dužine $\overline{AB}$, $A(-4,3)$, $B(2,-1)$ tvori s koordinatnim osima.

$A(-4,3)$

$B(2;-1)$



$k = -\frac{2}{3}$

$y = -\frac{2}{3}x + l$

$3 = -\frac{8}{3} + l$

$\frac{8}{3} + \frac{9}{3} = l$

$l = \frac{17}{3}$

$y = -\frac{2}{3}x + \frac{17}{3}$

$y = \frac{3}{2}x + \frac{5}{2}$

$y_\Delta = \frac{3}{2}x + l \Rightarrow 1 = -\frac{3}{2} + l \Rightarrow l = \frac{5}{2}$

$x_S = \frac{-4+2}{2} = -1$ $\quad y_S = \frac{3-1}{2} = 1$ $\quad S(-1,1)$

$y = \frac{3}{2}x + \frac{5}{2}$

$-\frac{3}{2}x + y = \frac{5}{2} \quad / \cdot \frac{2}{5}$

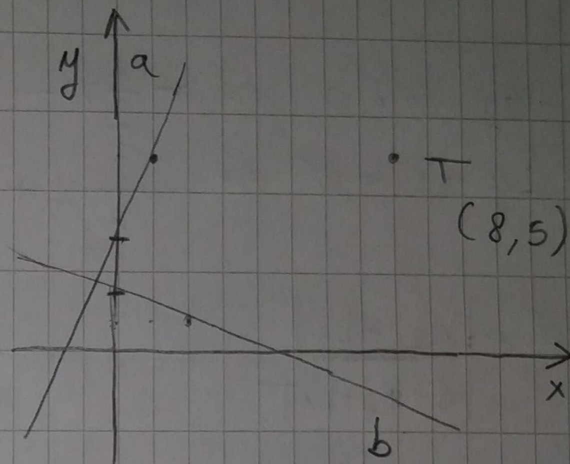
$-\frac{6}{10}x + \frac{2}{5}y = 1$

$\frac{x}{-\frac{10}{6}} + \frac{y}{\frac{5}{2}} = 1$

$P_\Delta = \frac{\left|-\frac{10}{6} \cdot \frac{5}{2}\right|}{2} = \frac{-\frac{\cancel{50}^{25}}{12}}{\frac{2}{1}} = \boxed{\frac{25}{12}}$

6.5

9. Dvije stranice pravokutnika leže na pravcima $x+2y-3=0$ i $2x-y+3=0$. Ako je jedan vrh pravokutnika točka $(8,5)$, kolika je površina pravokutnika?



$a \ldots\ 2x-y+3=0 \Rightarrow y=2x+3$

$b \ldots\ x+2y-3=0 \Rightarrow y=-\frac{1}{2}x+\frac{3}{2}$

$T(8,5)$

$$d(T,a)=\frac{|A\cdot x_0+B\cdot y_0+C|}{\sqrt{A^2+B^2}}$$

$$=3\sqrt{5}$$

$$d(T,b)=\frac{|A\cdot x_0+B\cdot y_0+C|}{\sqrt{A^2+B^2}}$$

$$=\frac{14\sqrt{5}}{5}$$

$$P_{\square}=a\cdot b=3\sqrt{5}\cdot\frac{14\sqrt{5}}{5}$$

$$=\frac{42\cdot\cancel{5}}{\cancel{5}}=42$$