ELECTRICITY AND MAGNETISM REVIEW

> The indented material is supplementary, and you are not accountable for it. This information is just to provide a bit of extra background, to enhance your understanding of the material for which you **are** accountable.

ELECTROSTATIC FORCE (COULOMB'S LAW)

$$F_E = k \frac{(\pm q_1)\ (\pm q_2)}{r^2} \tag{1}$$

The constant *k* is 9×10^9 N m^2 / C^2 and the separation distance *r* (sometimes *d* is used) is measured along the straight line connecting the centers of two charged objects, each small enough to be considered a "point." The charges *q* can be of either sign, so you must select one sign or the other for each charge.

Follow the rules of algebra when multiplying these, and then the force is one of *attraction* if its sign is negative, or the force is one of *repulsion* if its sign is positive. This is because attraction attempts to *reduce* the distance *r* between the objects, while repulsion attempts to *increase* this distance. Remember the rule: "opposites attract, likes repel."

This is, of course, an "inverse-square" law, just like Newton's Law of Gravitation. This means that if we were to double the distance *r*, then the force would be one-fourth (not half) of what it was.

Often, problems will involve electrons and/or protons; their charge magnitude *q* is 1.6×10^{-19} C.

> Note that, to be consistent, we should have used a negative sign in the force-of-gravity equation, since it is always a force of attraction. Note also that the electric force is a vector, and in more advanced study the equation above is written a bit differently, to include the direction of the force in a more general way. When this is done, we can readily find the net, resultant force on a charged object in the presence of several other charged objects, using vector addition. This is the principle of superposition, which is a fancy way of saying that the effects of the several charges are independent of each other.

ELECTRIC FIELD

$$E \equiv k \frac{\pm q_1}{r^2} = \frac{F_E}{+q_2} \tag{2}$$

This definition is based on the force experienced by a small-magnitude, positive "test" charge, which we will call q_2 to be consistent with Eq(1) above. That is, charge q_1 is said to produce an electric "field" in its vicinity. This field exerts no force until some other charged object, here, q_2, comes into its vicinity. As with Coulomb's Law, both objects are assumed to be physically small, i.e., mathematical "points." This is important, since "extended" objects can have much more complicated fields around them.

The electric field is, like the force, a vector quantity. The convention is that, if charge q_1 is positive, we say that the field is directed outward from it, or if it is negative, the field is directed inward toward it. This makes sense, because the "test charge" is assumed to be positive, so if q_1 was also positive, then the resulting force would be one of repulsion, and vice-versa if it was negative.

The units of electric field strength are N / C as we can see from Eq(2). Note that if we have the field *E*, we can easily find the force on some charge *q* in this field, with $F = qE$, which follows directly from Eq(2).

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ELECTRIC POTENTIAL ENERGY

$$PE = k \frac{(\pm q_1)(\pm q_2)}{r} = F_E\, r \tag{3}$$

This is, again, for two "point" charges, and is derived by considering the work done to bring the two charges together from an infinite distance to the distance *r*. Note that this PE can be positive or negative, depending on the signs of the charges, just as with Eq(1).

If the two charges have the same sign, then we must do work (apply a force across a distance) to bring the charges closer together, against the force of repulsion. This is positive work. If the charges are opposite in sign then they will attract each other, and, in a sense, negative work is done (we don't have to apply any external force). Potential energy, of any kind, only exists when there is a force present.

The units of potential energy are, as usual, J, that is, N m, which we can see from Eq(3). Note that potential energy is a **scalar quantity**; it can have a sign (positive or negative) but it does not have a direction. This turns out to be convenient when analyzing more complicated problems.

> Eq(3) is derived using integral calculus. The result, in this case, is simple- just multiply the force by the separation distance *r*. You can imagine that for other kinds of situations, other than simple point charges, these calculations could become very complicated. It's also interesting, and useful, that the electric field can be derived from the electric potential, at any point, by another calculus process, called differentiation.

ELECTRIC POTENTIAL

$$V \equiv \frac{PE}{q_2} = k \frac{\pm q_1}{r} \tag{4}$$

This is confusing, because it sounds similar to the PE, but it is actually the ***PE per unit charge***. Eq(4) applies to a point charge, q_1, which sets up an electric field in its vicinity. As charge q_2 moves with respect to q_1, the potential energy of this *system* (i.e., both charges) changes. For practical work it is simpler to divide this PE by q_2 to get a new quantity, the electric potential *V*, which is (1) independent of the charge at a point at distance *r* from q_1, and is (2) *caused* by the presence of charge q_1.

It isn't very useful to speak of the potential by itself. Just as with gravitational PE, we really need to use potential **differences**. So we should write (we usually just write *V* but we really mean *ΔV)*

$$\Delta V = \frac{\Delta PE}{q} = \frac{W}{q} \tag{5}$$

where *W* is the work done. Be sure to recognize that a potential difference *ΔV* is not the same thing as a difference in potential energy, *ΔPE*, as we can plainly see in Eq(5)! The potential difference in volts is the potential energy in joules *per unit charge*, in coulombs; thus volts = joules / coulomb.

For a uniform electrical field, such as between two parallel plates, separated by a distance *Δd*, we have

$$\Delta V = E\, \Delta d \tag{6}$$

and we can combine Eq(5) and (6) to give the useful relations

$$\Delta PE = q\, E\, \Delta d = F_E\, \Delta d = W \tag{7}$$

where the last part recognizes that $qE = F$, and then we have the work done, i.e., force times distance, equal to the change in PE, as we have seen before (with gravitation).

ELECTRIC CURRENT, RESISTANCE, POWER

$$I \;\equiv\; \frac{dq}{dt} \quad\Rightarrow\quad I \approx \frac{\Delta q}{\Delta t}$$

Electrical current is motion of charges that is created by an applied electric field. Quantitatively it is the *rate* of this motion, or the net charge passing some point in a circuit per unit time. The units will then be coulombs per second, which we call amperes (A). "Current flow" is redundant; current *is* a flow, of charge.

Much of the energy put into the charges (electrons, usually) goes into randomly-directed kinetic energy and thus collisions with the atoms in the conductor material; this energy raises the temperature of the material, rather than increasing the amount, or speed, of the charge flow. This situation is accounted for in the "resistance" of the conducting material,

$$R \;\propto\; \frac{length}{area}$$

that is, the resistance varies directly with the length, and inversely as the cross-sectional area of a conductor. Most conductors take the form of wires, which of course are elongated circular cylinders.

Ohm's Law actually says that the resistance of a conductor does not depend on the current through it, nor on the applied voltage. If this is so, we can write the simple relation that is usually *called* Ohm's Law:

$$V = I\,R \tag{8}$$

Here, the units are volts = (amperes)(ohms). Be aware that this linear relation does not hold for all possible conducting materials (or over all voltage or current ranges)! In some cases, increasing the current causes changes in resistance, perhaps via temperature effects.

As for power, we can write several relations; the first part uses the idea of power as the rate of change of energy, and Eq(5):

$$P = \frac{\Delta PE}{\Delta t} = \frac{q\,\Delta V}{\Delta t} \Rightarrow I\,V \qquad\qquad P \;=\; I\,V \;=\; \frac{V^2}{R} \;=\; I^2\,R \tag{9}$$

These equivalent expressions give the power in a circuit, which can be thought of as the *rate of energy transfer* in the circuit. The units of power are watts, so a watt = (ampere)(volt); power = (current)(voltage).

Usually, a circuit's purpose is to transfer energy from a source, like a battery, to a sink, like a light bulb. Electrical energy is converted to light (and heat) energy, and some is lost to the heating of the conducting wires. The lower the resistance of those wires, the less heating loss there will be, for a given current.

The energy used over some time interval (T_1 to T_2) is given by

$$energy \;=\; \int_{T_1}^{T_2} P(t)\,dt$$

if the power is not constant across this interval. If it *is* constant, then we just have

$$energy = P\,(T_2 - T_1)$$

There is a device at your home that, in effect, does the integral above. It is the "kilowatt-hour meter" that is read each month by the "power" company. But really, they are selling *energy*, not power! This integration is necessary because the power use of your home is far from constant. The kWh meter reading always increases, unless your home generates electrical energy and sends it out so that the meter would run backward. Watch the disk in the meter in the summer, when the A/C comes on...

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►***Find effective resistance of circuit:*** (10)

$$R_{eff} = R_{series} + \sum R_{parallel} \qquad R_{series} = \sum_{i=1}^{n} R_i \qquad R_{parallel} = \frac{1}{\sum_{j=1}^{m} \frac{1}{R_j}} = \left(R_1^{-1} + R_2^{-1} + R_3^{-1} + \cdots + R_m^{-1}\right)^{-1}$$

We use the equations above to combine the resistors; there are *n* series resistors, and there may be several parallel groups, each of *m* resistors, where *m* may be different for each parallel group. The last expression, for the resistance of a parallel group, is for use on your calculators (use the x^{-1} key, and don't forget the parentheses). A couple of special cases for parallel resistors:

$$R_{parallel} = \frac{R_1 R_2}{R_1 + R_2} \quad \text{m} = 2\,\text{only} \qquad R_{parallel} = \frac{R_{equal}}{m} \quad \text{all m equal}$$

►***Find total current in circuit:***

$$I_{total} = \frac{V}{R_{eff}} \tag{11}$$

This of course is just Ohm's Law, solved for the current. *V* is in volts, *I* is in amperes, *R* is in ohms.

►***Find total power dissipation in circuit:***

$$P_{total} = I_{total} V = \frac{V^2}{R_{eff}} \tag{12}$$

►***Find voltage drop (potential difference) across resistor(s):***

$$\Delta V = I_{total} R \tag{13}$$

Here, *R* is either the resistance of a given series resistor in the circuit, or the combined resistance of a parallel group. If the latter, then this voltage drop is the same for all resistors in that group. *The sum of the voltage drops around the circuit must equal the supply voltage.* This can be a check of your calculations.

►***Find current through and power dissipation in the j-th resistor in a parallel group:***

$$I_j = I_{total} \frac{R_{parallel}}{R_j} \qquad P_j = \frac{\left(I_{total} R_{parallel}\right)^2}{R_j} \tag{14}$$

►***Find current through and power dissipation in the i-th resistor in series:***

$$I_i = I_{total} \qquad P_i = I_{total}^2 R_i \tag{15}$$

The sum of the power dissipated in all the individual resistors, whether configured in series or parallel, around the circuit must equal the total power dissipation of the circuit, found above, in Eq(12). This is also a good check on your calculations.

MAGNETIC FORCE

$$F_{\text{mag, charge}} = q\,v\,B\,\sin(\theta) \qquad (16)$$

This is the magnitude of the force due to a magnetic field of strength *B*, in Tesla, N / (A m), on a charge *q* (coulombs) moving with velocity *v* (m / s), which is directed at some angle *θ* to the direction of the magnetic field. For most problems it is stated that the motion is perpendicular to the field, so that the sine term has no effect; sin(90) = 1. If it isn't explicitly stated otherwise, assume *θ* is 90 degrees. Clearly, the magnitude of this force is greatest when the charge moves at right angles (perpendicular) to the field B.

If we consider that the current in a length *L* of wire that is immersed in a magnetic field is the movement of many charges along the wire, it can be shown that for this case we will have

$$F_{\text{mag, wire}} = L\,I\,B\,\sin(\theta) \qquad (17)$$

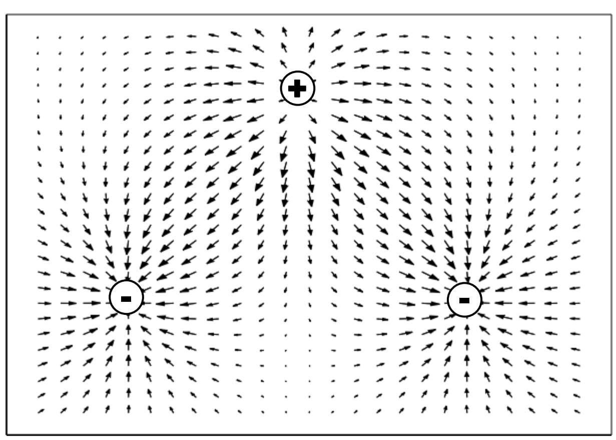
where *I* is the current in the wire. From a dimensional analysis we can see how this works: *qv* is coulombs-meters/sec and *LI* is meters-amperes, or meters-coulombs/sec.

In a more advanced physics course we would learn that the equations above are based on one of two types of vector products; these are used extensively in physics. When we multiply vector quantities we can get either a scalar result, or a vector result. The scalar, or "dot" product, is something we used in calculating work, although we didn't call the product by that name. This was when the result was the product of the two vectors' magnitudes, multiplied by the cosine of the angle between them.

The vector, or "cross" product, is what we are really doing with the magnetic field calculations. This is written like so, where the bold letters are vectors:

$$\mathbf{F}_{mag} = q\,\mathbf{v} \times \mathbf{B}$$

The *magnitude* of the resulting vector is as we have written above. The *direction* of that vector is found using a "right-hand rule," and the product vector, here the force **F**, is perpendicular to the plane containing the two vectors being multiplied. This is very important-- since the force is perpendicular to the velocity, then no work can be done by the magnetic field, and so the object's speed cannot be changed by this field. Only the *direction* of the motion of a charged particle can be changed by a magnetic field! It turns out, also, that the sign of the charge *q* does affect the direction of **F** for a point charge, but, interestingly, it has no effect for the wire (*University Physics*, 9th, p881).



Vector electric field for two negative charges and one positive charge.

E & M EQUATION SUMMARY

Electrostatics k = 9 x 10^9 N m^2 / C^2 e = 1.6 x 10^{-19} C

$$F_E = k\,\frac{(\pm q_1)\,(\pm q_2)}{r^2} \qquad E \equiv k\,\frac{\pm q_1}{r^2} = \frac{F_E}{+q_2} \qquad PE = k\,\frac{(\pm q_1)(\pm q_2)}{r} = F_E\, r$$

Potential difference

$$V \equiv \frac{PE}{q_2} = k\,\frac{\pm q_1}{r} \qquad \Delta V = \frac{\Delta PE}{q} = \frac{W}{q} \qquad \Delta V = E\,\Delta d \qquad \Delta PE = q\,E\,\Delta d = F_E\,\Delta d = W$$

Current, Ohm's Law, Power, Energy

$$I \approx \frac{\Delta q}{\Delta t} \qquad V = I\,R \qquad P = I\,V = \frac{V^2}{R} = I^2\,R \qquad \Delta energy = P\,\Delta t \quad \text{(constant P only)}$$

Basic DC circuit analysis (single voltage source)

$$R_{eff} = R_{series} + \sum R_{parallel} \qquad R_{series} = \sum_{i=1}^{n} R_i \qquad R_{parallel} = \frac{1}{\sum_{j=1}^{m} \frac{1}{R_j}} = \left(R_1^{-1} + R_2^{-1} + R_3^{-1} + \cdots + R_m^{-1}\right)^{-1}$$

$$R_{parallel} = \frac{R_1\,R_2}{R_1 + R_2} \quad \text{m = 2 only} \qquad R_{parallel} = \frac{R_{equal}}{m} \quad \text{all m equal}$$

$$I_{total} = \frac{V}{R_{eff}} \qquad P_{total} = I_{total}\,V = \frac{V^2}{R_{eff}} \qquad \Delta V = I_{total}\,R$$

$$I_j = I_{total}\,\frac{R_{parallel}}{R_j} \qquad P_j = \frac{\left(I_{total}\,R_{parallel}\right)^2}{R_j} \qquad I_i = I_{total} \qquad P_i = I_{total}^2\,R_i$$

current, power in j-th parallel resistor *current, power in i-th series resistor*

Magnetic force

$$F_{\text{mag, charge}} = q\,v\,B\,\sin(\theta) \qquad F_{\text{mag, wire}} = L\,I\,B\,\sin(\theta)$$

assume θ is 90 degrees unless otherwise specified

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