Katia Oropeza
A01570452

Composition of functions

10 / 1

1. For each of the following pair of functions, find both $(f \circ g)(x)$ and $(g \circ f)(x)$

a) $f(x) = x^2 - 5$ and $g(x) = \sqrt{x-1}$

$f(g) = x^2 - 5$

$\cancel{\sqrt{x-1}}^{\cancel{2}} - 5$

$x - 1 - 5$

$x - 6$

$g(f) = \sqrt{x-1}$

$\sqrt{x^2 - 5 - 1}$

$\sqrt{x^2 - 6}$

b) $f(x) = (x+1)^2 - 5$ and $g(x) = x + 3$

$f(g) = (x+1)^2 - 5$

$(x+3+1)^2 - 5$

$(x+4)^2 - 5$

$(x+4)(x+4) - 5$

$x^2 + 4x + 4x + 16 - 5$

$x^2 + 8x + 16 - 5$

$x^2 + 8x + 11$

$x^2 + 8x + 11$

$g(f) = x + 3$

$(x+1)^2 - 5 + 3$

$(x+1)(x+1)$

$x^2 + 1x + 1x + 1 - 5 + 3$

$x^2 + 2x + 1$

$x^2 + 2x - 1$

c) $f(x) = 1 - 3x^2$ and $g(x) = x^2$

$f(g) = 1 - 3x^2$

$(1 - 3x^2)^2$

$g(f) = x^2$

$(1 - 3x^2)^2$

$(1 - 3x^2)(1 + 3x$

$1 - 3x^2 - 3x^2 -$

$\boxed{1 - 6x^2 - 9x^4}$

d) $f(x) = x^6$ and $g(x) = \sqrt[3]{x^2 + x}$

$f(g) = x^6$

$\sqrt[3]{(x^2+x)^6}$

$(x^2+x)^2$

$(x^2+x)(x^2+x)$

$x^4 + x^3 + x^3 + x^2$

$\boxed{x^4 + 2x^3 + x^2}$

$g(f) = \sqrt[3]{(x^6)^2 + x^6}$

$= \boxed{\sqrt[3]{x^{12} + x^6}}$

Kana De Loera
ID: 01570452

10

Evaluating functions - Self study 1

I. A) Given the graphs of functions f(x) and g(x) evaluate the following:

$f(0) = -1$

$f(1) = 2$

$g(-4) = 1$

$g(-1) = 2$

$g(4) = 3$

B) Considering the previous graphs of functions f(x) and g(x) answer the following:

i) For which value of x is $f(x) = g(x)$? → intersection

1.2

ii) For which value of x is $g(x) = 0$?

-5

iii) For which value of x is $f(x) = 3.5$?

1.-5

II. Evaluate the following for the given functions:

$f(x) = 2x + 3$ $g(x) = 5 - x^2$ $h(x) = -\sqrt{x+2}$

$f(-2) = 2(-2) + 3$
$-4 + 3 = -1$
$f(-2) = -1$

$f(\frac{5}{2}) = 2(\frac{5}{2}) + 3$
$5 + 3 = 8$
$f(\frac{5}{2}) = 8$

$f(0) =$
$2(0) + 3$
$0 + 3$
$f(0) = 3$

$g(0) = 5 - 0^2$
$g(0) = 5$

$g(\sqrt{5}) =$
$5 - \sqrt{5}^2 = 0$
$g(\sqrt{5}) = 0$

$g(-2) =$
$5 - (-2^2)$
$5 - 4 = 1$
$g(-2) = 1$

$h(-2) = -\sqrt{-2+2}$
$h(-2) = 0$

$h(2) = -\sqrt{2+2}$
$-\sqrt{4}$

$h(\frac{1}{4}) =$

Katia Oropeza
A01570452

2.03 One to One Functions

10

I. Determine whether the given function is one to one function.

a) $f(x) = \frac{x}{2} - 3$

One to one function

b) $f(x) = x^2 - 3x$

NOT

c) $f(x) = 2\sqrt{x+4}$

One to one function

d) $f(x) = |x| + 2$

NOT

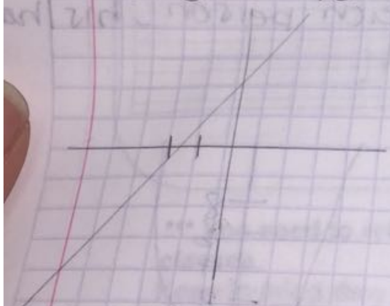
e) $f(x) = \frac{1}{x-2}$

One to one function

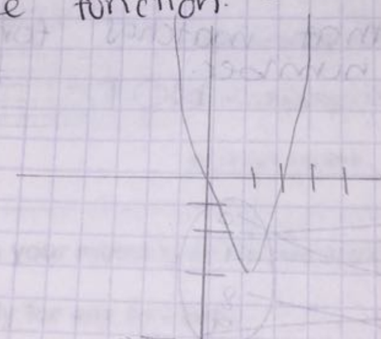
f) $f(x)\ x^2 + 6x - 1$ where $x > -3$

NOT

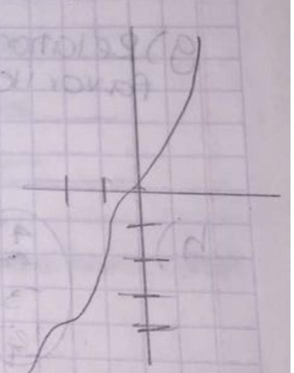
II. Determine whether the given function is a one to one function.



One to one function



NOT



One to one function

III. Determine whether the given relation is one to one function.

a) $\{(-5, 3), (4, 6), (6, 9), (5, 5)\}$

One to one function

) $\{(-4, 0), (0, 2), (1, 3), (3, 2)\}$

One to one function

Algebraic and Transcendental Functions

The Absolute Value $y=|x|$ and its Translations

Tecnológico de Monterrey Preparatoria

202

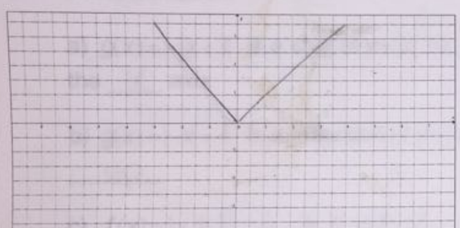
By: Ing. Ziad Najjar

Name: Katia Oropeza ID Number: A01570452 Group: ____

Exploring the function $y=|x|$

Investigate the graph of the function $y=|x|$, and state its characteristics

Fill in the table of values, then graph



x	-3	-2	-1	0	1	2	3
y	3	2	1	0	1	2	3

a) Domain $\mathbb{R}$ b) Range $[0,\infty)$

c) Decreasing in interval —

d) Increasing in interval —

e) Asymptotes —

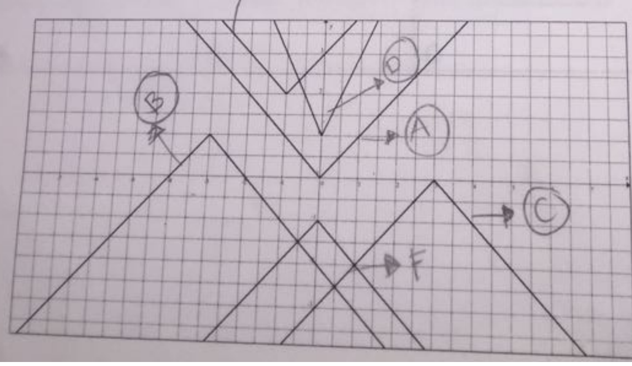
***** You need to refresh your memory of translations we have seen in past classes.**

Keep in mind they apply for any function!

I. Match each graph to its corresponding function, by writing the letter of function on graph:

A) $y=|x|$ ✓ B) $y=-|x+3|+1$ ✓ C) $y=-|x-3|$ ✓

D) $y=2|x|+1$ ✓ E) $y=|x+1|+2$ ✓ F) $y=-|-x|-1$ ✓



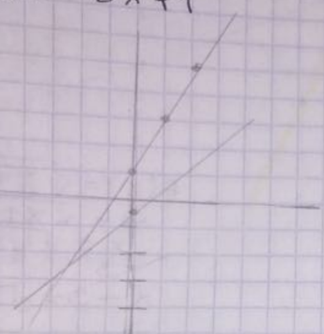
3.04
Graphing Inverse Functions

Katia Oropeza
A01570452

10

I. Find the inverse function of f(x) algebraically. Graph both f(x) and its inverse function $f^{-1}(x)$ (on the same plane) in addition to clearly showing the axis of reflection $y=x$

a) $f(x) = 2x+1$



$f(x) = 2x+1$
$y = 2x+1$
$x = 2y+1$
$-1 \quad -1$

$\frac{x-1}{2} = \frac{2y}{2}$

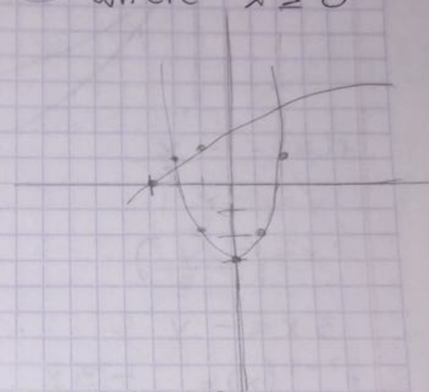
$\frac{x-1}{2} = y$

$\boxed{f^{-1}(x) = \frac{x-1}{2}}$

$\rightarrow mx+b$

slope: $\frac{1}{2}$, y-intercept: $-\frac{1}{2}$

b) $f(x) = x^2 - 3$ where $x \geq 0$



$f(x) = x^2 - 3$
$y = x^2 - 3$
$x = y^2 - 3$
$+3 \quad +3$
$x+3 = y^2$
$\sqrt{x+3} = y$

$\boxed{f^{-1}(x) = \sqrt{x+3}}$

Algebraic and Transcendental Functions

Inverse Functions

3.06

By: Ing. Ziad Najjar

Name: Katia De Loera ID Number: A01510452 Group: ____

I. Answer the following regarding the given functions:

a) Given the function $f(x)=3x+1$ Find: $f(2)=$ $3(2)+1$ $6+1=\underline{7}$ $f^{-1}(7)=$ $\frac{7-1}{3}=\frac{6}{3}=\underline{2}$ $f^{-1}(-8)=$ $\underline{-3}$ $x=3y+1$ $\frac{-8-1}{3}=-3$ $\frac{x-1}{3}=y$	b) Given the function $f(x)=\frac{x}{2}+7$ Find: $x=\frac{y}{2}+7$ $f^{-1}(x)=$ $(x-7)2=\frac{y}{2}$ $\underline{2x-14=y}$ $f^{-1}(5)=$ $2(5)-14=$ $10-14=\underline{-4}$
c) Given the function $f(x)=\sqrt{2x+1}$ Find: $f^{-1}(x)=$ $x=\sqrt{2y+1}$ $x-1=\sqrt{2y}$ $\frac{x-1}{2}=\sqrt{y}\rightarrow\frac{(x-1)^2}{2}=y$ $\underline{f^{-1}(x)=\frac{x^2-1}{2}}$ $f^{-1}(7)=$ $\frac{7^2-1}{2}=\frac{49-1}{2}=\underline{24}$	d) Given $g(x)=7-x^2$, *where* $x\le 0$ Find: $g^{-1}(x)=$ $x=7-y^2$ $x-7=-y^2$ $\sqrt{x-7}=-y$ $-\sqrt{x-7}=y$ $\underline{f^{-1}(x)=-\sqrt{x-7}}$ $g^{-1}(-9)=$ $-\sqrt{-9-7}$ $\underline{=-\sqrt{-16}}$

II. Demonstrate algebraically the given pair of functions are the inverse functions of each other:

Always root = exponent

a) $f(x)=\frac{x-7}{3}$ *and* $g(x)=3x+7$ $x=\frac{y-7}{3}$ $x=3y+7$ $x+7=\frac{y}{3}$ $x-7=3y$ $(x+7)3=y$ $\frac{x-7}{3}=y$	b) $f(x)=\sqrt[3]{2x+5}$ and $g(x)=\frac{x^3-5}{2}$ $x=\sqrt[3]{2y+5}$ $x=\frac{y^3-5}{2}$ $(x)^3=\sqrt{2y+5}$ $x(2)=y^3-5$ $x^3-5=2y$ $2x+5=y^3$ $\frac{y^3-5}{2}=y$ $\sqrt[3]{2x+5}=y$

Tecnológico de Monterrey Preparatoria

The Impact of the Absolute Value Function $y=|f(x)|$ upon the Function $y=f(x)$

4.11

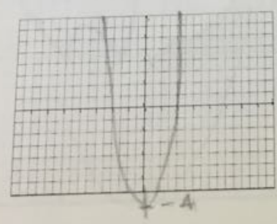
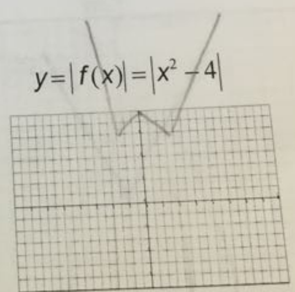
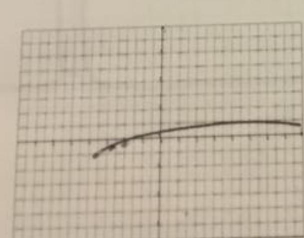
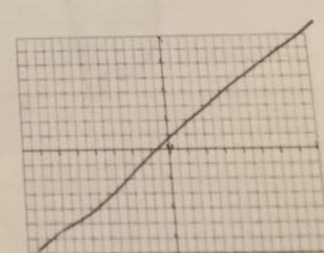
By: Ing. Ziad Najjar

Name: Kana Oropeza ID Number: A01570452 Group: ____

Exploring the impact of the function $y=|f(x)|$ upon $y=f(x)$

<u>**Part 1** (in pairs) – Team up with one of your classmates</u>

Using a graphing utility, member 1 solves #1,2 and member 2 solves #3,4

	Graph $f(x)$	Graph $\lvert f(x)\rvert$
#1	$f(x)=x^2-4$ 	$y=\lvert f(x)\rvert=\lvert x^2-4\rvert$ 
#2	$f(x)=\sqrt{x+4}-2$ 	$y=\lvert f(x)\rvert=\lvert\sqrt{x+4}-2\rvert$ 

PREPA Tec

Algebraic and Transcendental Functions

Tecnológico de Monterrey Preparatoria

Piecewise Functions

4.12

By: Ing. Ziad Najjar

Name: Katia Oropeza ID Number: A01570452 Group: ____

I. Watch the following video whose link is provided

Link1: http://www.virtualnerd.com/algebra-2/linear-equations-functions/absolute-value-piecewise-functions/piecewise/piecewise-function-graphing

In case it´s still not clear, watch the following

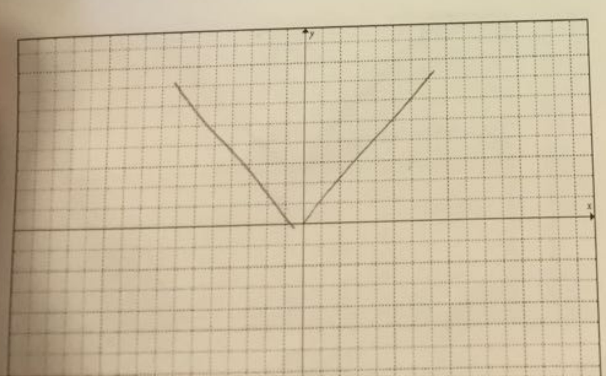
Link2: https://www.youtube.com/watch?v=Gtyf0xCB0kc

In a previous class, you got acquainted with the function $y=|x|$

a) Graph it on the given plane

b) Write the function $y=|x|$ as a piecewise function!

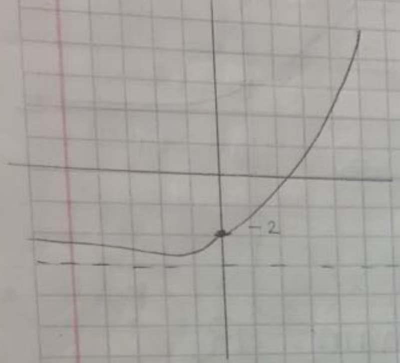
Answer: $y=\begin{cases} |-x| & x \le 0 \end{cases}$



Katia Oropeza A0157 0452

5.03

a) $y = 2x - 3$



Domain: $\mathbb{R}$

Range: $(-3, \infty)$

Asymptote: $y = -3$

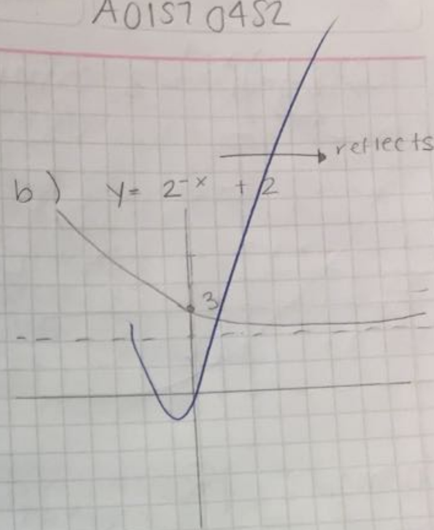
Characteristic point: $(0, -2)$

Concavity: Up

Decreasing: NO

Increasing: Yes

b) $y = 2^{-x} + 2$



Domain: $\mathbb{R}$

Range: $(2, \infty)$

Asymptote: $y = 2$

characteristic point: $(0, 3)$

Concavity: Up

Decreasing: NO

Increasing: Yes

Algebraic and Transcendental Functions

5.07

Logarithmic Functions-Concept and Evaluations

Tecnológico de Monterrey Preparatoria

By: Ing. Ziad Najjar

Name: Karla Oropeza ID Number: A01570457 Group:

When trying to find the inverse function of the exponential function $y = a^x$, we notice we end up with $x = a^y$, and now the problem is isolating the new y !

This is precisely why we need this new notation of logarithms!

The function $y = Log_a x$ is called the logarithmic function with base a.
We say $y = Log_a x$ if and only if $x = a^y$. You will notice that this will occur only for $a > 0$, $a \neq 1$ where x > 0

It is important to state out the importance of two particular cases of the base a being:

1) When the base a=10, $y = Log_{10} x$,the notation is $y = Log(x)$, and the function is called the **common logarithmic** function.

2) When the base a=e (the number e) , $y = Log_e x$,the notation is $y = Ln(x)$, and the function is called the **natural logarithmic** function.

Before we start exploring their graphs, it is important, you are able to write a logarithmic statement into an exponential form and the other way around.

Examples:

Writing $Log_2 32 = 5$ into exponential form yields $2^5 = 32$

Writing $Log(1000) = 3$ into exponential form yields $10^3 = 1000$

Writing $7^3 = 343$ into logarithmic form yields $Log_7 343 = 3$

Writing $e^4 = 54.5982$ into logarithmic form yields $\ln(54.5982) = 4$

6.12

Katia
Oropeza
A01570452

1) $\log_2(2x-3) = \log_2(x+4)$

$2x-3 = x+4$

$2x - x = 4+3$

$\boxed{x = 7}$

2) $\log_2 x + \log_2(x+2) = \log_2(x+6)$

$x(x+2) = x+6$

$x^2+2x = x+6$

$x^2 + x - 6 = 0$

$(x+3)$ $\quad x-2=0$

$\boxed{x=-3}$ $\quad \boxed{x=2}$

3) $\log_4 x - \log_4(x-1) = \frac{1}{2}$

$\frac{\log_4 x}{\log_4 x - 1} = 1/2$

$\frac{x}{x-1} = 4^{1/2}$

$\frac{x}{x-1} = 2$ $\quad x = 2x-2$ $\quad x-2x = -x =$

$\boxed{x=2}$

4) $\log(x+4) - \log x = \log(x+2)$

$\frac{x+4}{x} = x+2$

$x+4 = x+2(x)$

$x+4 = x^2+2x$

$x = x^2+2x-4=0$

$\boxed{x = -1 \pm \sqrt{5}}$

6.05

Algebraic and Transcendental Functions

Tecnológico de Monterrey Preparatoria

Solve Different Base Exponential Equations

Gathered by: Ing Marcela Treviño, from Larson, R. Algebra and Trigonometry, 8th edition.

Name Katia Orozpeza ID Number: A01570452 Group: ______

Solve the following exponential equations

1. $5^{3x+1} = 10^{x-2}$

$\ln(5^{3x+1}) = \ln(10^{x-2})$

$(3x+1)\ln(5) = (x-2)(\ln(10))$

$3x\ln(5) + \ln(5) = x\ln(10) - 2\ln(10)$

$3x\ln(5) - x\ln(10) = -2\ln(10) - \ln(5)$

$x(3\ln(5) - \ln(10)) = -(2\ln(10) + \ln(5))$

$\boxed{x = \frac{-\ln(500)}{\ln(\frac{25}{5})}}$

2. $6^{x-2} = 3^{x+1}$

$\ln(6^{x-2}) = \ln(3^{x+1})$

$(x-2)\ln(6) = (x+1)\ln(3)$

$\boxed{x = \log_2(108)}$

3. $9^{x-4} = 4^{4-x}$

$x - 4 = 4\log_3(2) - \log_3(2)x$

$x + \log_3(2)x = 4\log_3(2) + 4$

$1 + \log_3(2) = 4\log_3(2) + 4$

$\boxed{x = \frac{4\log_3(2) + 4}{\log_3(2) + 4}}$

4. $e^{2x} - e^{x} - 20 = 0$

$(e^x)^2 - (e^x) - 20 = 0$

$(e^x - 5)(e^x + 4) = 0$

$e^x - 5 = 0$

$e^x = 5$

$\boxed{x = \ln 5}$

$e^x + 4 = 0$

$e^x = -4$

$\boxed{x = \ln(-4)}$

5. $e^{2x} + 9e^{x} - 36 = 0$

$(e^x)^2 + 9(e^x) - 36 = 0$

$x^2 + 9x - 36$

x	12
x	-3

$e^x = -12$

No solution

$e^x = 3$

$e^x = 1$

$\boxed{x = \ln(3)}$

6. $e^{2x} - 11e^{x} + 24 = 0$

$e^{2x} - 11e^x + 24 = 0$

$(e^x)^2 - 11e^x + 24 = 0$

$x^2 - 11x + 24$

x	-8
x	-3

$e^x = 8$

$e^x = 3$

Katia Oropeza
A01570952

7.1 Compound Interest

1 a) $P\left(1+\frac{r}{n}\right)^{nt}$ $\quad P\left(1+\frac{0.95}{4}\right)^{4t}$

$\frac{2}{\frac{1+0.095}{4}} = 4t$ $\quad \boxed{t = 8.75}$

(b) $A = Pe^{rt}$ $\quad 2 = 1e^{0.095t}$ $\quad \frac{\ln(2)}{0.095} = t$

$2 = e^{0.095t}$

$t = 7.3$ years

2 Pe^{rt} $\quad (10000)e^{rt}$

$\ln(1000) = .14t$

$\frac{\ln(1000)}{14}$ $\quad \boxed{r = 9.1\%}$

3 $\frac{\ln(1000)}{14} =$ $\quad t = 49.34$ years

4 Pe^{rt} $\quad 2000 = 1000e^{r3}$ $\quad \frac{\ln(2)}{3} = r$

$2 = e^{r3}$

$\boxed{r = 23.1\%}$

5 a) $P = \$200,000$

b) $Pe^{rt} = 200,000e^{0.06(15/12)}$

$A = \$218834.85$

Algebraic and Transcendental Functions

7.02B

Modeling Exp and Log Functions: Population Growth

Tecnológico de Monterrey Preparatoria

Created by: *Ing. Ziad Najjar and Ing. Patricia Chapa*

Name Katia de Loera ID Number: A01570452 Group: ____

Solve real-life problems that can be modeled as exponential or logarithmic functions.

POPULATION GROWTH PROBLEMS

1) The earth's population is growing at an annual rate of 1.9%. If the Earth's population is described by the exponential model $P(t) = 6.25(1.019)^t$ in which the number of persons is measured in billions and t in years (considering t = 0 for the year 2007)

a) Find the Earth's population for the year 2010

b) Find when the Earth's population will be 7 billions of persons

$P(t) = 6.25(1.019)^3 = P(t) = 6.61$ billion

$P(t) = 6.25 \quad (1.019)^6 = P(t) = 6.99$ billion

in (2013)

2) Suppose that the fish population P(t) in a lake is attacked by a disease, therefore the fish population is given by: (time is measured in weeks)

$$P = 8350e^{-.04t}$$

a) Find the initial fish population in the lake (t = 0)

b) What is the fish population after 8 weeks?

c) Is the number of fish increasing or decreasing? How is that related to the given equation?

d) When would the fish population be 2,500 fish

$8350e^{-0.4(}$

a) 8350 fish c) decreasing

b) 340 fish d) in 13 weeks

3) Before a parachute opens, a skydiver's velocity in meters/second is given

$$v(t) = 50\left(1 - 2^{-0.3t}\right)$$

$50(1-$

a) Find the initial velocity of the skydiver (t = 0)

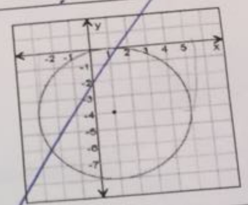
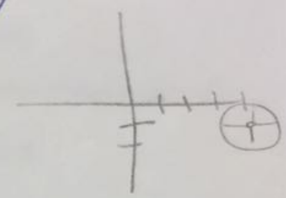
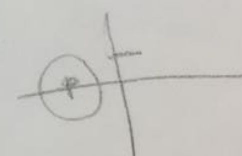
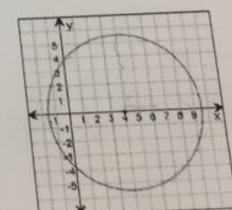
b) Find the velocity after 1 minute

a) $50(1.2)^{(-0.3)(1)} = 0$ b) 9

8.03 Kana Oropeza $d = \sqrt{(x}$

Nombre

Fill up the table by completing the missing information.

Elements, radius and center.	Reduced equation	Graph
1) center: $(1,-4)$ radius: 4	$(x-1)^2+(y+4)^2$	
2) $C(4,-2)$ and radius= $r = \frac{3}{5}$	$(x-4)^2+(y+2)^2$ $9/25 = 0.36$	
3) $C(-2,0)$ $R=\sqrt{3}$	$(x+2)^2+y^2=3$ $-2,0$	
4) center: $(4,0)$ $r=\sqrt{25}=5$	$(x-4)^2+y^2=25$ $=5$	
5) $C(1,-2)$ and passes through point $(4,-2)$	$(x-1)^2+(x+2)^2$ $=9$