

1210.

$$a) \lim_{n \rightarrow \infty} \frac{(n+1)^3 - (n-1)^3}{(n+1)^2 + (n-1)^2} = \lim_{n \rightarrow \infty} \frac{n^3 + 3n^2 + 3n + 1 - n^3 + 3n^2 - 3n + 1}{n^2 + 2n + 1 + n^2 - 2n + 1} = \lim_{n \rightarrow \infty} \frac{6n^2 + 2}{2n^2 + 2} \div \frac{n^2}{n^2} = \lim_{n \rightarrow \infty} \frac{6+2 \cdot \frac{1}{n^2}}{2+2 \cdot \frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{6+2 \cdot 0}{2+2 \cdot 0} = 3$$

$$6) \lim_{n \rightarrow \infty} \left( \frac{2n^2}{2n+3} + \frac{1-3n^3}{3n^2+1} \right) = \lim_{n \rightarrow \infty} \frac{6n^4 + 2n^2 + 2n + 3 - 6n^4 - 9n^3}{6n^3 + 2n + 9n^2 + 3} = \lim_{n \rightarrow \infty} \frac{-9n^3 + 2n^2 + 2n + 3}{6n^3 + 9n^2 + 2n + 3} \div \frac{n^3}{n^3} = \lim_{n \rightarrow \infty} \frac{-9+2 \cdot \frac{1}{n} + 2 \cdot \frac{1}{n^2} + 3 \cdot \frac{1}{n^3}}{6+9 \cdot \frac{1}{n} + 2 \cdot \frac{1}{n^2} + 3 \cdot \frac{1}{n^3}} = \lim_{n \rightarrow \infty} \frac{-9+2 \cdot 0 + 2 \cdot 0 + 3 \cdot 0}{6+9 \cdot 0 + 2 \cdot 0 + 3 \cdot 0} = -\frac{3}{2}$$

1213.

$$6) \lim_{n \rightarrow \infty} \frac{(n+2)! - (n+1)!}{(n+3)!} = \lim_{n \rightarrow \infty} \frac{(n+2)(n+1)! - (n+1)!}{(n+3)(n+2)(n+1)!} = \lim_{n \rightarrow \infty} \frac{(n+1)!(n+2-1)}{(n+3)(n+2)(n+1)!} = \lim_{n \rightarrow \infty} \frac{n+1}{n^2 + 5n + 6} \div \frac{n^2}{n^2} = \lim_{n \rightarrow \infty} \frac{\frac{1}{n} + \frac{1}{n^2}}{1 + 5 \cdot \frac{1}{n} + 6 \cdot \frac{1}{n^2}} = \lim_{n \rightarrow \infty} \frac{0+0}{1+0+0} = 0$$

$$S_n = \frac{n}{2}(1+4n-3) = \frac{n}{2}(4n-2) = 2n-n^2$$

$$h) \lim_{n \rightarrow \infty} \left( \overbrace{\frac{1+5+9+\dots+(4n-3)}{2(n+1)}}^{2(n+1)} - n \right) = \lim_{n \rightarrow \infty} \frac{2n^2 + n - 2n^2 - 2n}{2n+2} = \lim_{n \rightarrow \infty} \frac{-3n}{2n+2} \div \frac{n}{n} = \lim_{n \rightarrow \infty} \frac{-3}{2+2 \cdot \frac{1}{n}} = \lim_{n \rightarrow \infty} \frac{-3}{2+2 \cdot 0} = -\frac{3}{2}$$