An Introduction to Quadratic Equations

Housekeeping

An equation which is quadratic <u>in one variable</u> can be generalised, in standard form, as $y = ax^2 + bx + c$ for $a \neq 0, b \in \mathbb{R}, c \in \mathbb{R}$. Typically a, b, and c will all be integers.

Open https://www.geogebra.org/m/txfbbxyv There are four available functions on the Geogebra app associated with this document.

- $f: x \rightarrow x^2$
- $g: x \rightarrow ax^2, b = c = 0$
- $h: x \rightarrow 1x^2 + bx, c = 0$
- $p: x \rightarrow 1x^2 + c, b = 0$

When we perform an experiment it is better if we follow the KISS principle (= **keep it simple, stupid** KISS principle - Wikipedia). With this in mind, play with only one function at a time (g or h or p). Use the checkboxes to make each function visible / invisible. Compare the results of moving the associated slider to the most basic function $y = x^2$.

What? How? Why?

The questions below should get you started on this topic. You will probably think of some others on your own.

1) What is the name given to the shape of the graph of $y = x^2$?

2) Have you seen or recognised this shape in real life?

3) What happens as a varies? Consider:

   i) When a is negative

   ii) When a is zero

   iii) When a is positive

   iv) When a is positive, but less than one

   v) When a is positive and less than one.

   vi) Overall, how would you describe the effect of parameter a?

4) What happens when c varies? Is there a name we could give to parameter c?

5) Move the appropriate slider to observe the graph as b varies.

   i) There is one interesting feature of the graph of $y = 1x^2 + bx$ that is <u>invariant</u> under b. Do you recognise it?

   ii) Describe in words what happens to the graph when b varies.

   iii) Does this behaviour seem more complicated that when either a or c varies?

   iv) Do you know a technique to re-express the quadratic equation in a different form that makes this complexity more apparent?

Reflection

In your opinion, does Geogebra make learning about parabolas easier or more difficult? Why or why not?

Are you interested in what goes on 'under the hood' (to borrow an automobile metaphor) in Geogebra? Or do you just want to learn the maths?

If you wish to learn more more on experimental design see https://www.coursera.org/lecture/experimentation/1a-why-experiments-are-so-important-LPgaq

Vocabulary

Every word in a vocabulary list is defined in other words. Unless some words are considered primitives this will lead to an infinite regression.   My decision to define a word or leave it undefined is based on experience; hence, by no means foolproof.   Your feedback is welcome.

constant term: there is no variable in this term (or equivalently the variable is written to the zeroth power, such as cx^0) Since $x^0 = 1$ the factor 'x^0' is often omitted

invariant: a property which remains unchanged after an operation has applied. (Above, when the linear term, x, has been multiplied by b.)

linear term: the term in which the variable is written to the first power, such as bx^1. Since $x^1 = x$ the exponent '1' is often omitted.

parameter: a value which can vary between equations, but is fixed for a given equation. c is a parameter in the family of equations $y = x^2 + c$. For example c is fixed at '3' in the equation $y = x^2 + 3$.

quadratic: an equation of degree 2 (not 4, as you might possibly expect). The name is from the Latin word quadratus meaning "square."

quadratic <u>in one variable</u>: Only one of the variables is degree two. For comparison, the equation of a circle is sometimes given as $(x - h)^2 + (y - k)^2 = r^2$. The circle equation is quadratic in two variables. (Why two, where there are three squared terms?)

Real numbers, $\mathbb{R}$: all the numbers on a number line.   Real numbers include counting numbers, fractions, decimals, and other numbers such as $\sqrt{2}$ and π which students (but not professional mathematicians) consider 'unusual.'

term: Operationally, an expression separated from other expressions by the addition (or subtraction operation). On the page, an expression separated from other expressions by addition or subtraction signs. Terms are generally composed of a string of factors.

variable: a placeholder for quantities that may change in the given expression or equation. For $y = x^2 + c$ we expect to be told or be given a specific value for c, such as c = 3 but we expect x and y to take on many (possibly infinitely many) values.

Feedback

All constructive feedback is welcomed.