Penrose darts & kites and the golden ratio

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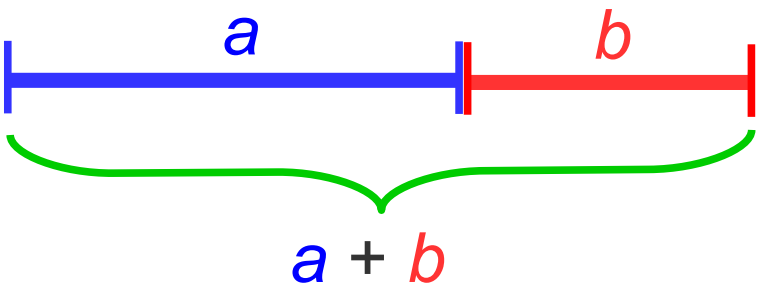
1 golden ratio

1.1 definition



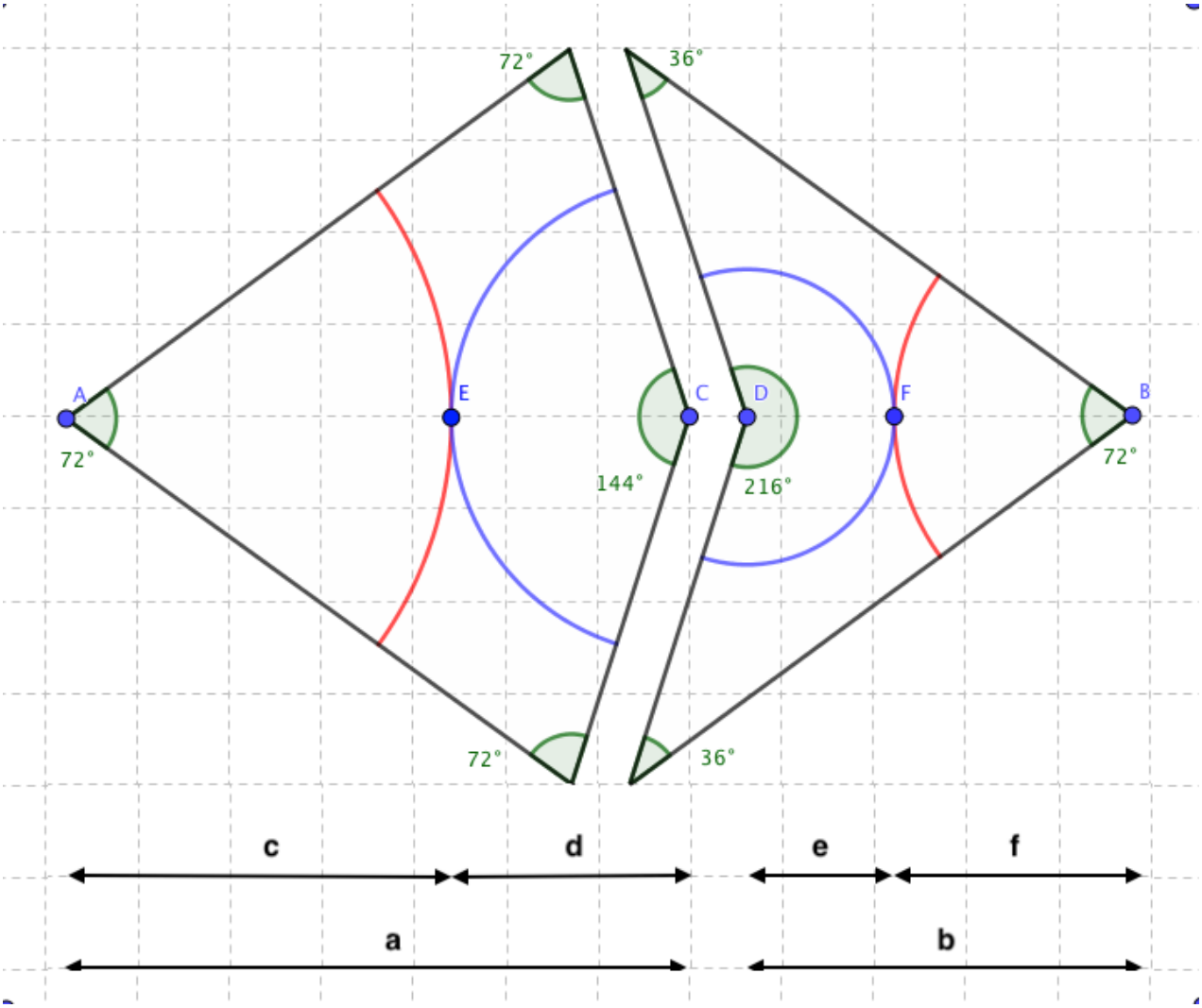
$$\frac{a+b}{a} = \frac{a}{b} \stackrel{def}{=} \Phi$$

1.2 calculation

$\phi = (1 + \sqrt{5}) / 2 = 1{,}61803398875$
$\phi = 2 * \sin(54°) = 1{,}61803398875$

2 Penrose darts & kites

2.1 construction

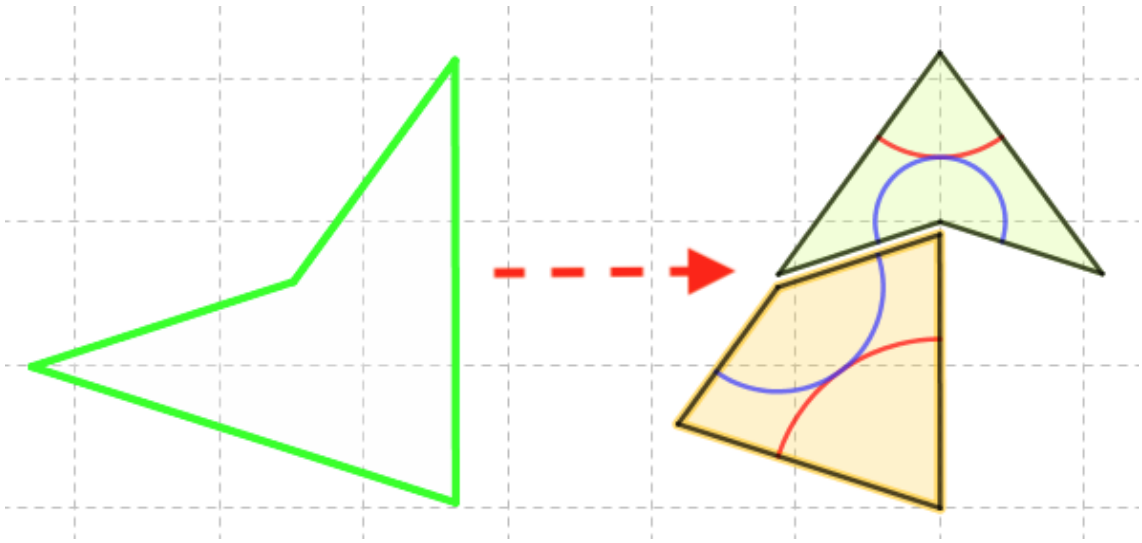


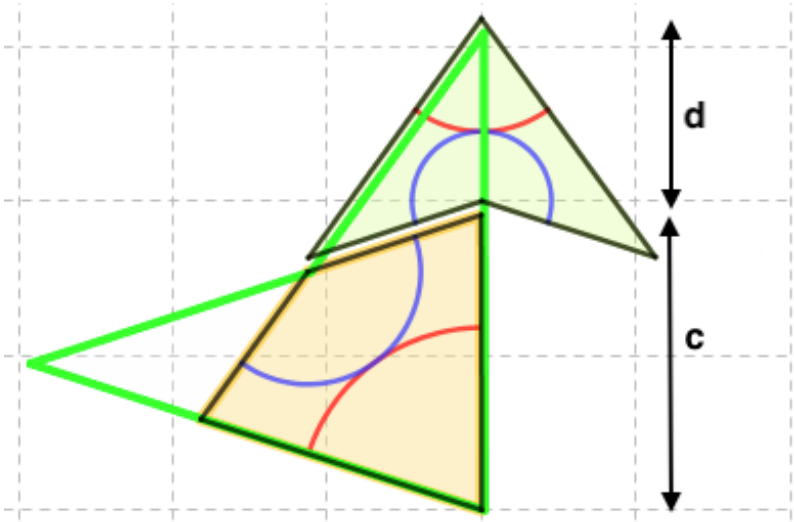
golden ratio:
a = c + d
b = e + f
(a + b) / a = a / b
(c + d) / c = c / d
(e + f) / f = f / e

2.2 inflation

2.2.1 construction

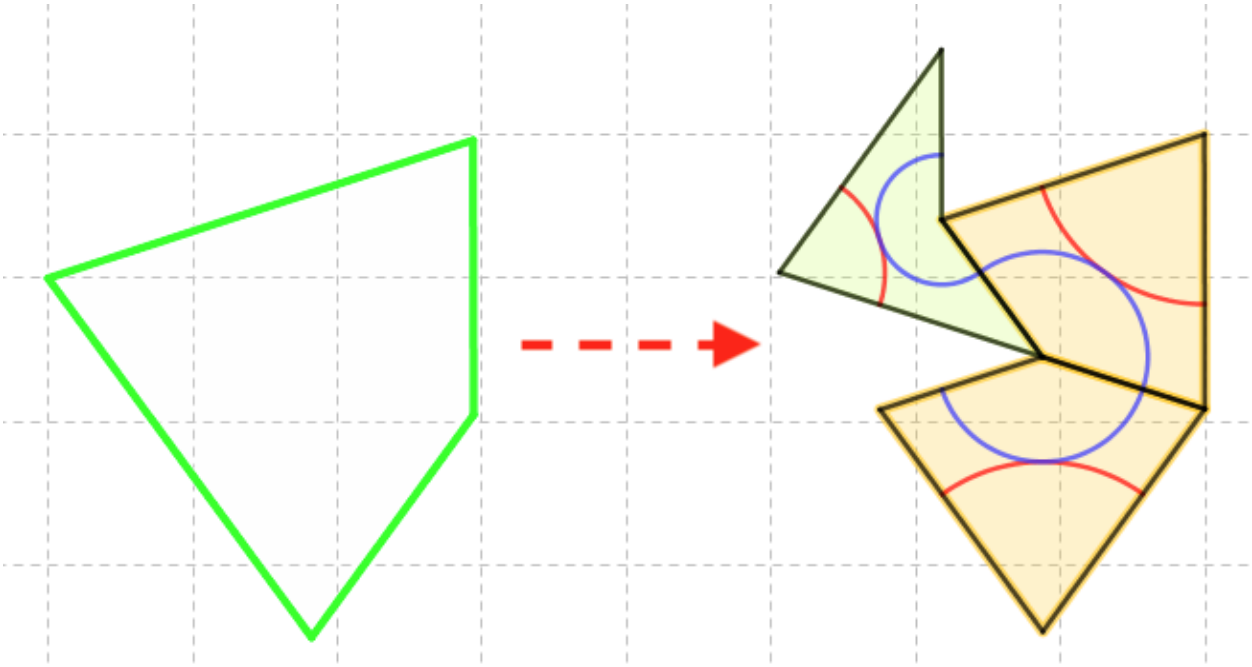
2.2.1.1 a dart is divided to 1 kite and 1 dart

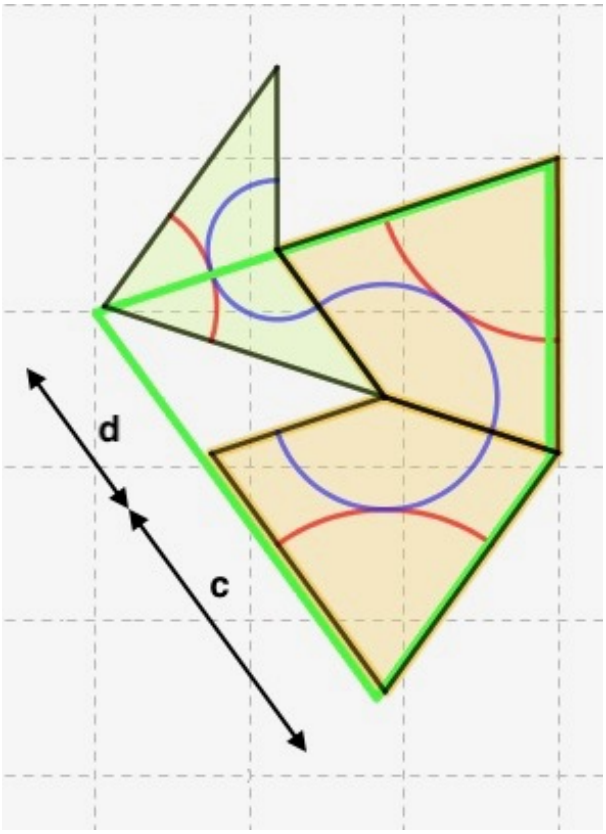




golden ratio:
(c + d) / c = c / d

2.2.1.2 a kite is divided to 2 kites and 1 dart





golden ratio:
(c + d) / c = c / d

3 production with pdk_gen.ggb

3.1 limits

My platform:

- macOS 10.13.6,
- 2,6 GHz Intel Core i5,
- 8 GB memory,
- Intel Iris 1536 MB graphics,
- GeoGebra Classic 5.0.426.0-d

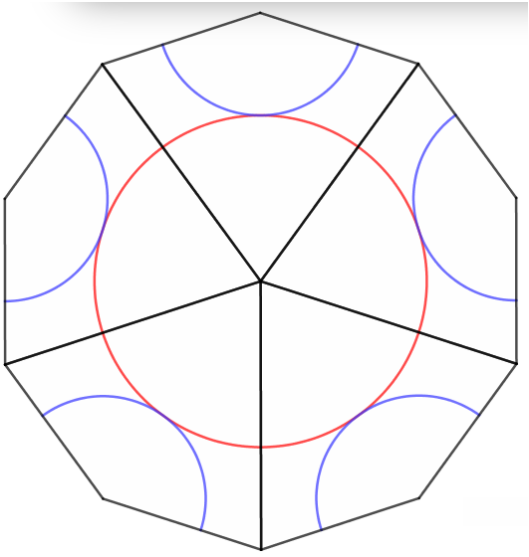
With the parameter *number of start kites = 5*, this applet work well from generation #1 till #8. Although I didn't get any error message not from GG either from OS, my attempts greater than generation #8, didn't succeed. Accessing greater *number of generations* was successfull with decreasing the parameter *number of start kites.*

Number of start kites	Number of generations
5	8
2	9
1	10

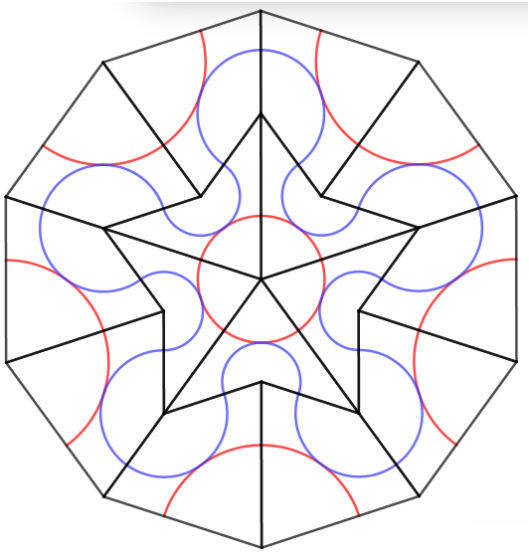
3.2 products

3.2.1 successfull attempts with *number of start kites = 5*

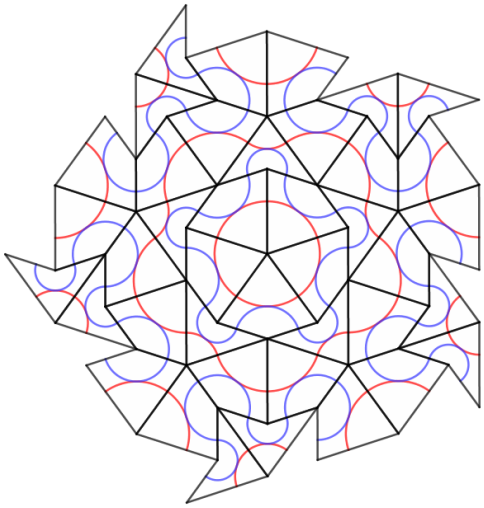
Generation #1



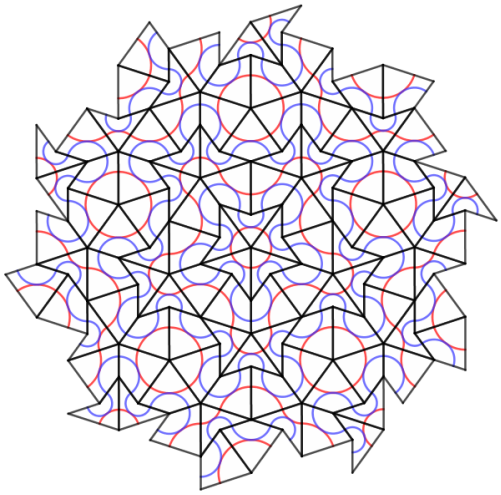
Generation #2



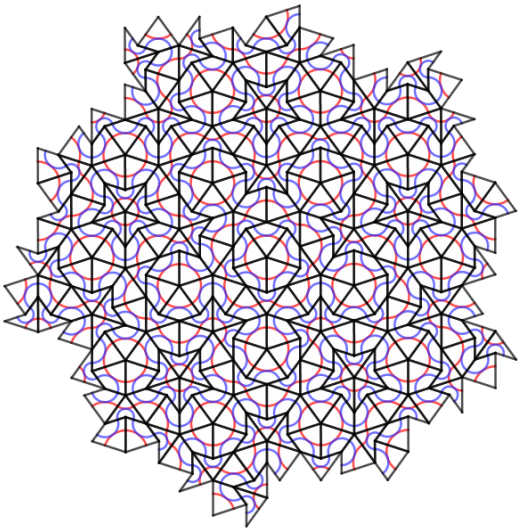
Generation #3



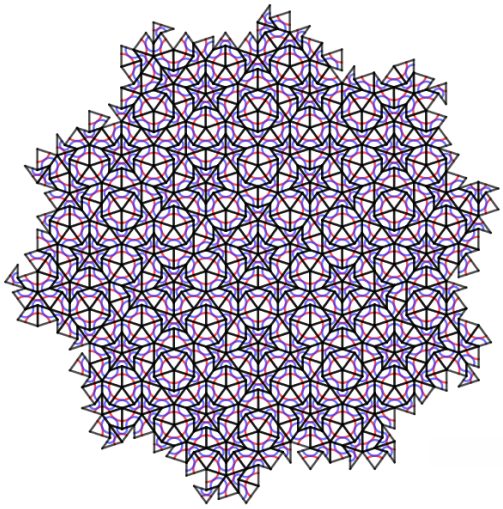
Generation 4



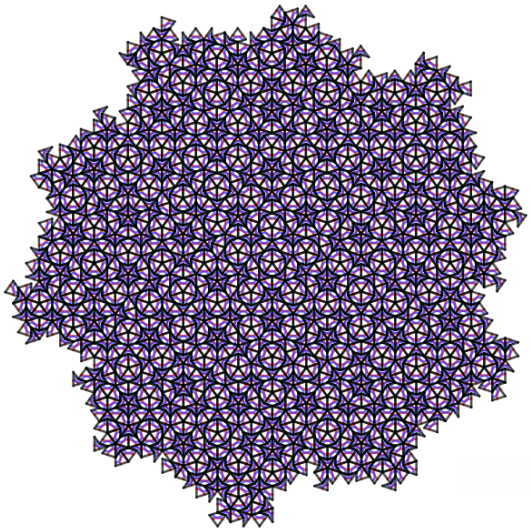
Generation #5



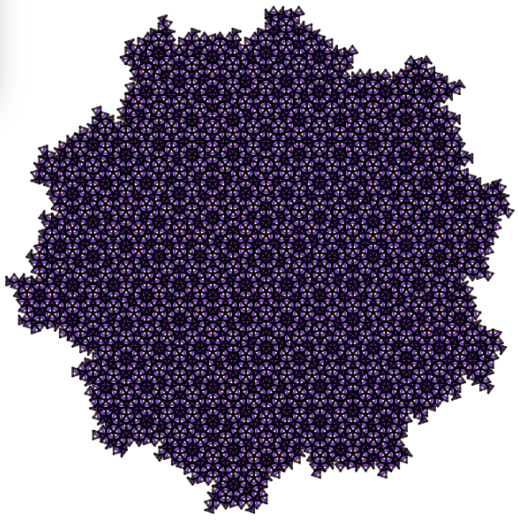
Generation #6



Generation #7

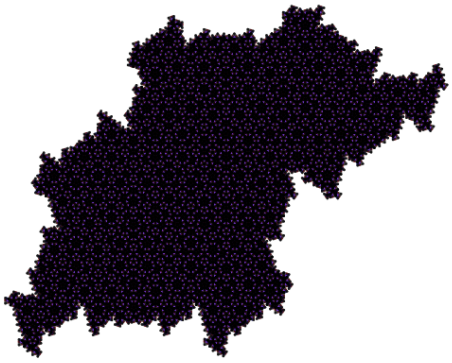


Generation #8



3.2.2 successfull attempts with *number of start kites = 2*

Generation #9



3.2.3 successfull attempts with *number of start kites = 1*

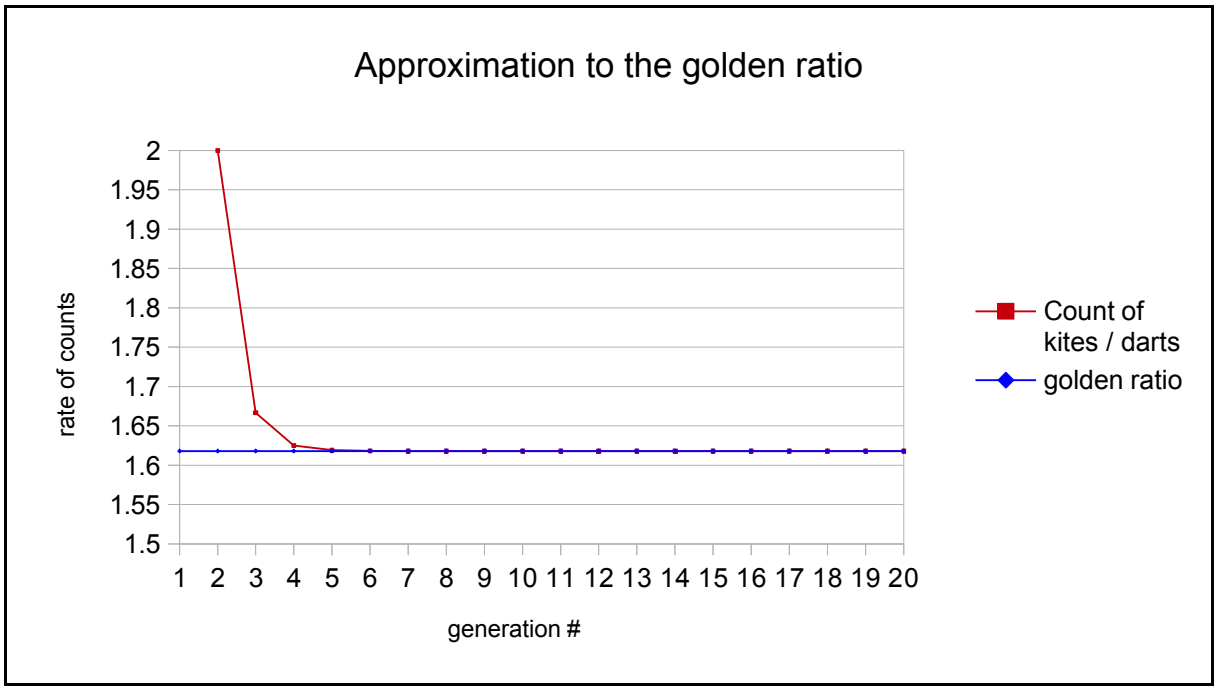
Generation #10

3.3 statistics

1st 3 columns are created by ***pdk_counter.sh***

Generation #	Count of darts	Count of kites	Count of kites / darts
1	0	5	
2	5	10	2
3	15	25	1.6666666667
4	40	65	1.625
5	105	170	1.619047619
6	275	445	1.6181818182
7	720	1 165	1.6180555556
8	1 885	3 050	1.6180371353
9	4 935	7 985	1.6180344478
10	12 920	20 905	1.6180340557
11	33 825	54 730	1.6180339985
12	88 555	143 285	1.6180339902
13	231 840	375 125	1.618033989
14	606 965	982 090	1.6180339888
15	1 589 055	2 571 145	1.6180339888
16	4 160 200	6 731 345	1.6180339888
17	10 891 545	17 622 890	1.6180339887
18	28 514 435	46 137 325	1.6180339887
19	74 651 760	120 789 085	1.6180339887
20	195 440 845	316 229 930	1.6180339887

golden ratio:
(Count of kites + Count of darts) / Count of kites = Count of kites / Count of darts



4 pdk_counter.sh

```
#!/bin/ksh

#************** pdk_counter.sh **************
#Define Variables
(( MaxGenNum = 20 )) # max number of generations
(( KiteNumber1st = 5 )) # number of 1st kites

#************** GENERATION #1
(( GenNum = 1 ))
(( CountOfDartsThisGeneration = 0 ))
(( CountOfKitesThisGeneration = KiteNumber1st ))
echo "GenNum    count of darts          count of kites"
echo $GenNum "       " $CountOfDartsThisGeneration "
" $CountOfKitesThisGeneration

#************** GENERATION # >1
while [[ $GenNum -lt $MaxGenNum ]];
do
(( CountOfDartsPrevGeneration = CountOfDartsThisGeneration ))
(( CountOfKitesPrevGeneration = CountOfKitesThisGeneration ))

# * BEGIN darts and kites of actual generation originated from
kites (1 kite is devided to 2 kites and 1 dart) **********
(( CountDartsFromKites = CountOfKitesPrevGeneration ))
(( CountKitesFromKites = CountOfKitesPrevGeneration*2 ))

# * BEGIN darts and kites of actual generation originated from
darts (1 dart is devided to 1 kites and 1 dart) *********
(( CountDartsFromDarts = CountOfDartsPrevGeneration ))
(( CountKitesFromDarts = CountOfDartsPrevGeneration ))

(( CountOfDartsThisGeneration = CountDartsFromKites +
CountDartsFromDarts ))
(( CountOfKitesThisGeneration = CountKitesFromKites +
CountKitesFromDarts ))

echo $GenNum "       " $CountOfDartsThisGeneration "
" $CountOfKitesThisGeneration
(( GenNum++ ))
done
```