Matematička indukcija - Domaći rad (Milošević Sara III3)

1080.

a) $\underbrace{1+2+3+\cdots+n}_{Sn} = \underbrace{n*\frac{(n+1)}{2}}_{F(n)}$

1) $S_1 = F_{(1)}$ (?)

$a_1 = S_1$

$a_1 = 1 \rightarrow S_1 = 1$

$F_{(1)} = \frac{1*(1+1)}{2} = 1 \rightarrow S_1 = F_{(1)} \quad (T)$

2) $1+2+3+\cdots+n = n*\frac{(n+1)}{2} \rightarrow \underbrace{1+2+3+\cdots+n}_{Sn} + (n+1) = \underbrace{\frac{(n+1)(n+2)}{2}}_{F(n+1)}$

po indukcijskoj hipotezi

$\frac{n*(n+1)}{2} + \frac{2*(n+1)}{2} = \frac{n^2+n+2n+2}{2} = \frac{n^2+3n+2}{2} = \frac{(n+1)(n+2)}{2} = F_{(n+1)}$

1081.

b) $\underbrace{\frac{3}{1*2} + \frac{7}{2*3} + \cdots + \frac{n^2+n+1}{n*(n+1)}}_{Sn} = \underbrace{\frac{n*(n+2)}{n+1}}_{F(n)}$

1) $S_1 = F_{(1)}$ (?)

$a_1 = S_1$

$a_1 = \frac{3}{2*1} = \frac{3}{2}$

$F_{(1)} = \frac{1*(1+2)}{1+1} = \frac{3}{2} \rightarrow S_1 = F_{(1)} \quad (T)$

2) $\frac{3}{1*2} + \frac{7}{2*3} + \cdots + \frac{n^2+n+1}{n*(n+1)} = \frac{n*(n+2)}{n+1} \rightarrow$

$\underbrace{\frac{3}{1*2} + \frac{7}{2*3} + \cdots + \frac{n^2+n+1}{n*(n+1)}}_{Sn} + \frac{(n+1)^2+(n+1)+1}{(n+1)(n+2)} = \frac{(n+1)(n+3)}{(n+2)}$

po indukcijskoj hipotezi

$\frac{n*(n+2)}{n+1} + \frac{(n+1)^2+(n+2)}{(n+1)(n+2)} = \frac{n*(n+2)^2+(n+1)^2+(n+2)}{(n+1)(n+2)} =$

$\frac{(n+2)(n^2+2n+1)+(n+1)^2}{(n+1)(n+2)} = \frac{(n+2)(n+1)^2+(n+1)^2}{(n+2)(n+1)} =$

$\frac{(n+1)^2*(n+2+1)}{(n+2)(n+1)} = \frac{(n+1)(n+3)}{(n+2)} = F_{(n+1)}$