Newton-Raphson Method :

Let x_0 be an approximate value of a real root of the equation f(x) = 0. Let $x_1 = x_0 + h$ be an exact root, where h is very small, positive or negative. Then $f(x_1) = 0$ ----------(1)

Expanding by Taylor's series, $f(x_1) = f(x_0 + h) = f(x_0) + \frac{h}{1!} f'(x_0) + \frac{h^2}{2!} f''(x_0) + ...$

Since $f(x_1) = 0$ and h is very small, neglecting the second and higher order terms of h, we have $f(x_0) + hf'(x_0) = 0 \quad \therefore \quad h = -\frac{f(x_0)}{f'(x_0)}$ provided $f'(x_0) \neq 0$. Hence $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ is a first approximation to the root. Similarly starting with x_1 we get the next approximation to the root given by $x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$. In general $\boxed{x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}, \quad n = 0,1,2,3....}$

This method is called Newton-Raphson's iteration method of finding root.

Algorithm for Newton Raphson method:

Step 1. Read x_0 (the initial guess) and eps (the allowable error)

Step 2. Define $F(x)$

Step 3. Compute $x_1 = x_0 - F(x_0)/F'(x_0)$ and $T = x_1 - x_0$

Step 4. If $|T_0| > eps$, then replace x_0 by x_1 and go to step 3, otherwise print the root is x_1 and terminate the algorithm.

```
%The Newton Raphson Method
clc;
close all;
clear all;
syms x;
f=input('Enter your function here : '); %Enter the Function here
g=diff(f); %The Derivative of the Function
```

```
n=input('Enter the number of decimal places:');
epsilon = 5/10^(n+1)
x0 = input('Enter the intial approximation:');
for i=1:100
     f0=subs(f,x,x0); %Calculating the value of function at x0
     f0_d=subs(g,x,x0); %Calculating value of derivative at x0
     x1=x0-f0/f0_d; % The Formula
err=abs(x1-x0);
if err<epsilon %checking the amount of error at each iteration
    break
end
x0=x1;
end
x1 = x1 - rem(x1,10^-n); %Displaying upto required decimal
places
fprintf('The Root is : %f \n',x1);
fprintf('No. of Iterations : %d\n',i);
```

Example : Find a real root of the equation $x^3 - 3x + 1 = 0$ lying between 1 and 2 to three places of decimal by using Newton-Raphson method.

Enter your function here : x^3-3*x+1
Enter the number of decimal places:5
Enter the intial approximation:3
The Root is : 1.532080
No. of Iterations : 6