DOMAĆI

1080. a) $1+2+3+\cdots+n=\frac{n(n+1)}{2}$

(1) $a_n=n \quad a_1=S_1=F_1=1$

(2) $S_n=F_n \to S_{n+1}=F_{(n+1)} \qquad a_{n+1}=n+1$

$1+2+3\ldots+n+n+1=\frac{(n+2)(n+1)}{2}$

$$\frac{n(n+1)}{2}+n+1=\frac{(n+2)(n+1)}{2}$$

$$\frac{n(n+1)+2(n+1)}{2}=\frac{(n+2)(n+1)}{2}$$

$$\frac{(n+1)(n+2)}{2}=\frac{(n+2)(n+1)}{2}$$

1081.b) $\frac{3}{1\cdot2}+\frac{7}{2\cdot3}+\cdots+\frac{n^2+n+1}{n(n+1)}=\frac{n(n+2)}{n+1}$

$$S_n=F_{(n)}$$

$$a_n=\frac{n^2+n+1}{n(n+1)}$$

(1) n=1

$$S_n=F_{(n)} \to S_{n+1}=F_{(n+1)}$$

$$F_{n+1}=\frac{(n+1)(n+2+1)}{n+1+1}=\frac{(n+1)(n+3)}{n+2}$$

$$S_{n+1}=\frac{(n+1)^2+n+1+1}{(n+1)(n+1+1)}=\frac{n^2+2n+1+n+2}{(n+1)(n+2)}$$

$$=\frac{n^2+3n+3}{(n+1)(n+2)}$$

$$\frac{3}{1\cdot2}+\frac{7}{2\cdot3}+\cdots+\frac{n^2+n+1}{n(n+1)}+\frac{n^2+3n+3}{(n+1)(n+2)}=\frac{(n+1)(n+3)}{n+}$$

Crvenom bojom je označena indukcijska hipoteza

$$\frac{n(n+2)}{n+1}+\frac{n^2+3n+3}{(n+1)(n+2)}=\frac{(n+1)(n+3)}{n+2}$$

$$\frac{n(n+2)^2+n^2+3n+3}{(n+1)(n+2)}=\frac{(n+1)(n+3)}{n+2}$$

$$\frac{n^3+4n^2+4n+n^2+3n+3}{(n+1)(n+2)}=\frac{(n+1)(n+3)}{n+2}$$

Pomnožimo izraz sa (n+1)(n+2) ,ne trebaju uslovi jer n pripada N

$$n^3+4n^2+4n+n^2+3n+3=(n+1)^2(n+3)$$

$$n^3+5n^2+7n+3=(n^2+2n+1)(n+3)$$

$$n^3+5n^2+7n+3=n^3+2n^2+n+3n^2+6n+3$$

$$n^3+5n^2+7n+3=n^3+5n^2+7n+3$$

$$S_{n+1}=F_{(n+1)}$$