

$$\begin{aligned}
1210. \text{ a) } \lim_{n \rightarrow \infty} \frac{(n+1)^3 - (n-1)^3}{(n+1)^2 + (n-1)^2} &= \lim_{n \rightarrow \infty} \frac{n^3 + 3 \cdot n^2 + 3 \cdot n + 1 - (n^3 - 3 \cdot n^2 + 3 \cdot n - 1)}{n^2 + 2 \cdot n + 1 + n^2 - 2 \cdot n + 1} = \\
&= \lim_{n \rightarrow \infty} \frac{n^3 + 3 \cdot n^2 + 3 \cdot n + 1 - n^3 + 3 \cdot n^2 - 3 \cdot n + 1}{n^2 + 2 \cdot n + 1 + n^2 - 2 \cdot n + 1} = \lim_{n \rightarrow \infty} \frac{6 \cdot n^2 + 2}{2 \cdot n^2 + 2} = \\
\lim_{n \rightarrow \infty} \frac{6 + 2 \cdot \frac{1}{n^2}}{2 + 2 \cdot \frac{1}{n^2}} &= \frac{\lim_{n \rightarrow \infty} 6 + 2 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^2}}{\lim_{n \rightarrow \infty} 2 + 2 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^2}} = \frac{6 + 2 \cdot 0}{2 + 2 \cdot 0} = \frac{6}{2} = 3
\end{aligned}$$

$$\begin{aligned}
\text{v) } \lim_{n \rightarrow \infty} \left(\frac{2 \cdot n^2}{2 \cdot n + 3} + \frac{1 - n^3}{3 \cdot n^2 + 1} \right) &= \lim_{n \rightarrow \infty} \frac{2 \cdot n^2 \cdot (3 \cdot n^2 + 1) + (1 - n^3) \cdot (2 \cdot n + 3)}{(2 \cdot n + 3) \cdot (3 \cdot n^2 + 1)} = \\
&= \lim_{n \rightarrow \infty} \frac{6 \cdot n^4 + 2 \cdot n^2 + 2 \cdot n + 3 - 6 \cdot n^4 - 9 \cdot n^3}{6 \cdot n^3 + 2 \cdot n + 9 \cdot n^2 + 3} = \\
\lim_{n \rightarrow \infty} \frac{-9 \cdot n^3 + 2 \cdot n^2 + 2 \cdot n + 3}{6 \cdot n^3 + 9 \cdot n^2 + 2 \cdot n + 3} &= \lim_{n \rightarrow \infty} \frac{-9 + 2 \cdot \frac{1}{n} + 2 \cdot \frac{1}{n^2} + 3 \cdot \frac{1}{n^3}}{6 + 9 \cdot \frac{1}{n} + 2 \cdot \frac{1}{n^2} + 3 \cdot \frac{1}{n^3}} = \\
\lim_{n \rightarrow \infty} \frac{\lim_{n \rightarrow \infty} (-9) + 2 \cdot \lim_{n \rightarrow \infty} \frac{1}{n} + 2 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^2} + 3 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^3}}{\lim_{n \rightarrow \infty} 6 + 9 \cdot \lim_{n \rightarrow \infty} \frac{1}{n} + 2 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^2} + 3 \cdot \lim_{n \rightarrow \infty} \frac{1}{n^3}} &= -\frac{9}{6} = -\frac{3}{2}
\end{aligned}$$

$$\begin{aligned}
1211. \text{ b) } \lim_{n \rightarrow \infty} \frac{(n+2)! - (n+1)!}{(n+3)!} &= \lim_{n \rightarrow \infty} \frac{(n+2) \cdot n! - (n+1) \cdot n!}{(n+3) \cdot n!} = \lim_{n \rightarrow \infty} \frac{n! \cdot (n+2 - n-1)}{n! \cdot (n+3)} = \\
\lim_{n \rightarrow \infty} \frac{1}{n+3} &= \lim_{n \rightarrow \infty} \frac{\frac{1}{n}}{1 + 3 \cdot \frac{1}{n}} = \frac{\lim_{n \rightarrow \infty} \frac{1}{n}}{\lim_{n \rightarrow \infty} 1 + 3 \cdot \lim_{n \rightarrow \infty} \frac{1}{n}} = \frac{0}{1 + 0} = 0
\end{aligned}$$

$$S_n = \frac{n}{2} \cdot (1 + 4 \cdot n - 3) = \frac{n \cdot (4 \cdot n - 2)}{2} = \frac{2 \cdot n \cdot (2 \cdot n - 1)}{2} = 2 \cdot n^2 - n$$

$$\begin{aligned}
1213. \text{ đ) } \lim_{n \rightarrow \infty} \left(\frac{1+5+9+\dots+(4 \cdot n-3)}{2 \cdot (n+1)} - n \right) &= \lim_{n \rightarrow \infty} \left(\frac{2 \cdot n^2 - n}{2 \cdot n + 2} - n \right) = \\
\lim_{n \rightarrow \infty} \frac{2 \cdot n^2 - n - n \cdot (2 \cdot n + 2)}{2 \cdot n + 2} &= \\
\lim_{n \rightarrow \infty} \frac{2 \cdot n^2 - n - 2 \cdot n^2 - 2 \cdot n}{2 \cdot n + 2} &= \lim_{n \rightarrow \infty} \frac{-3 \cdot n}{2 \cdot n + 2} = \lim_{n \rightarrow \infty} \frac{-3}{2 + 2 \cdot \frac{1}{n}} = \frac{\lim_{n \rightarrow \infty} (-3)}{\lim_{n \rightarrow \infty} 2 + 2 \cdot \lim_{n \rightarrow \infty} \frac{1}{n}} = \frac{-3}{2 + 2 \cdot 0} = \\
-\frac{3}{2} &
\end{aligned}$$