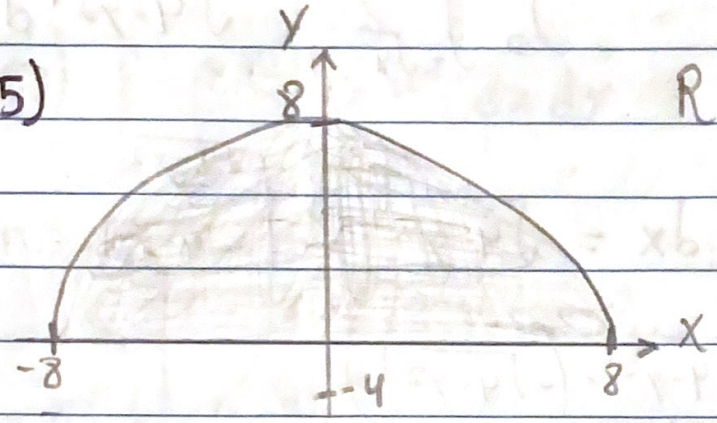
Ejercicios 14.3

Cambio de variables: Coordenadas Polares

En los ejercicios 5 a 8, utilizar las coordenadas Polares para describir la región mostrada.

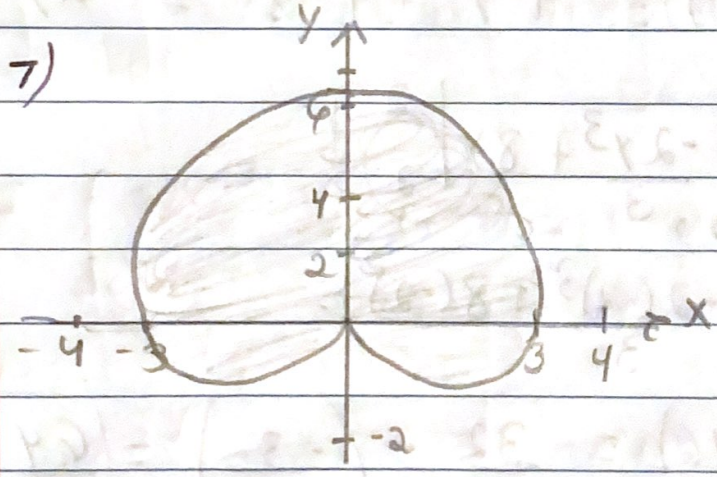
5)



$R: 0 \leq r \leq 8$

$0 \leq \theta \leq \pi$

7)

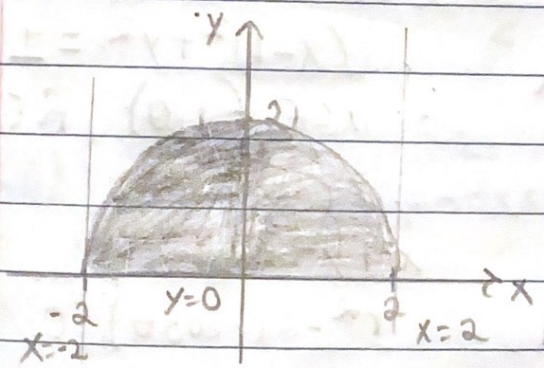


$R: 0 \leq r \leq 3 + 3\operatorname{sen}\theta$

$0 \leq \theta \leq 2\pi$

En los ejercicios 17 a 26, evaluar la integral iterada pasando a Coordenadas Polares.

19) $\int_{-2}^{2}\int_{0}^{\sqrt{4-x^2}} (x^2+y^2)\,dy\,dx$



$0 \le r \le 2$

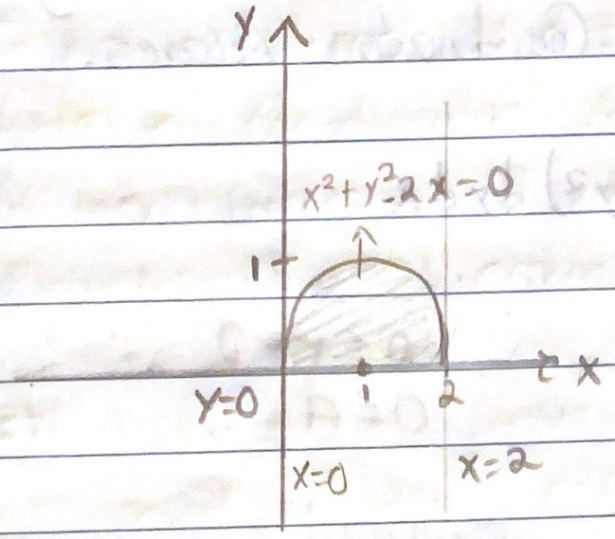
$0 \le \theta \le \pi$

$\int_0^{\pi}\int_0^{2} r^2 \cdot r\,dr\,d\theta$

$I = \int_0^2 r^3\,dr = \frac{r^4}{4}\Big|_0^2 = \frac{2^4}{4} - \frac{0^4}{4} = 4$

$\int_0^{\pi} 4\,d\theta = 4\theta\Big|_0^{\pi} = 4\pi - 4\cdot 0 = \boxed{4\pi}$

23) $\int_0^2 \int_0^{\sqrt{2x-x^2}} xy \, dy \, dx$



$y = \sqrt{2x-x^2}$

$y^2 = 2x - x^2$

$x^2 + y^2 - 2x = 0$

$(x^2 - 2x + 1) + y^2 = 1$

$(x-1)^2 + y^2 = 1$

C (1,0) , R (1)

R: $0 \leq r \leq 2\cos\theta$

$0 \leq \theta \leq \pi/2$

$x^2 + y^2 - 2x = 0$

$r^2 - 2(r\cos\theta) = 0$

$r^2 - 2r\cos\theta = 0 \div r$

$r - 2\cos\theta = 0$

$r = 2\cos\theta$

$$\int_0^{\pi/2} \int_0^{2\cos\theta} r\cos\theta \, r\,\text{sen}\theta \, dr \, d\theta$$

$$I = \int_0^{2\cos\theta} r^3 \cos\theta \,\text{sen}\theta \, dr$$

$$= \frac{r^4}{4} \cos\theta \,\text{sen}\theta \Big|_0^{2\cos\theta} = \frac{(2\cos\theta)^4 \cos\theta \,\text{sen}\theta}{4}$$

$$= \frac{16\cos^5\theta \,\text{sen}\theta}{4} = 4\cos^5\theta \,\text{sen}\theta$$

$$= \int_0^{\pi/2} 4\cos^5\theta \,\text{sen}\theta \, d\theta$$

$u = \cos\theta$

$du = -\text{sen}\theta \, d\theta$

$$\Big\} -4\int_p^q u^5 \, du = 4\left[\frac{u^6}{6}\right]_p^q$$

$$= -4\left[\frac{\cos\theta^6}{6}\right]_0^{\pi/2}$$

$$= -4\left[\frac{\cos(\pi/2)^6}{6} - \frac{\cos(0)^6}{6}\right]$$

$$= -4\left[0 - \frac{1}{6}\right] = \frac{4}{6} = \boxed{\frac{2}{3}}$$

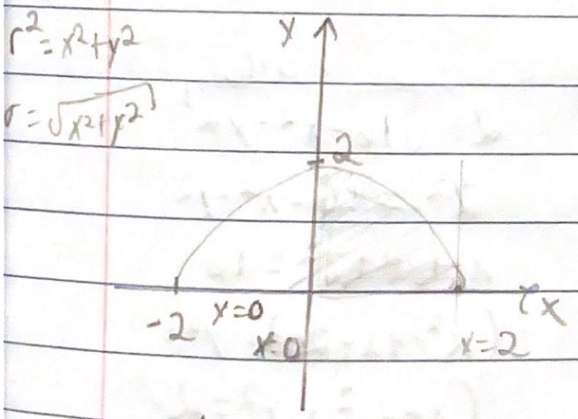
26) $\int_0^2 \int_0^{\sqrt{4-x^2}} \operatorname{sen}\sqrt{x^2+y^2}\, dy\, dx$

$y = \sqrt{4-x^2}$

$x^2+y^2 = 4$

$r^2 = x^2+y^2$

$r = \sqrt{x^2+y^2}$



R: $0 \le r \le 2$

$0 \le \theta \le \pi/2$

$$\int_0^{\pi/2} \int_0^2 \operatorname{sen}(r)\, r\, dr\, d\theta$$

$$I = \int_0^2 \operatorname{sen}(r)\, r\, dr$$

$u = r \qquad dv = \operatorname{sen} r\, dr$

$du = dr \qquad v = -\cos r$

$= -r(\cos r) + \int \cos(r)\, dr$

$= -r\cos(r) + \operatorname{sen}(r)$

$-r\cos(r) + \operatorname{sen}(r)\Big|_0^2$

$-2\cos(2) + \operatorname{sen}(2) - 0$

$\operatorname{sen}(2) - 2\cos(2)$

$$\int_0^{\pi/2} \operatorname{sen}(2) - 2\cos(2)\, d\theta$$

$(\operatorname{sen}(2) - 2\cos(2))\theta\Big|_0^{\pi/2}$

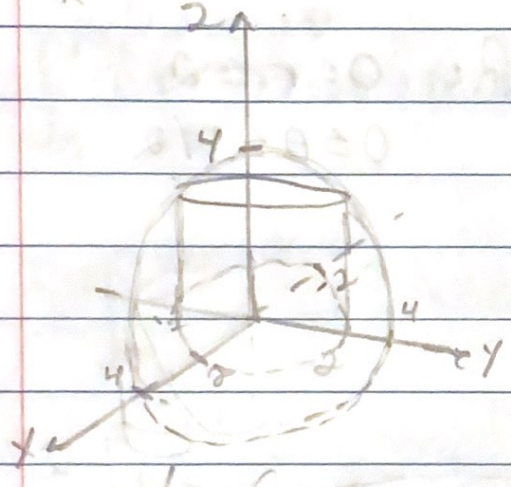
$(\operatorname{sen}(2) - 2\cos(2)\,\pi/2) = \frac{\pi}{2}(\operatorname{sen}(2) - 2\cos(2))$

Bóveda de Viviani

Volumen.

En los ejercicios 33 a 38, utilizar una integral doble en coordenadas polares para hallar el volumen del sólido limitado o acotado por las gráficas de las ecuaciones.

37) interior al hemisferio $z = \sqrt{16 - x^2 - y^2}$ e interior al cilindro $x^2 + y^2 - 4x = 0$



$z = \sqrt{16 - x^2 - y^2}$

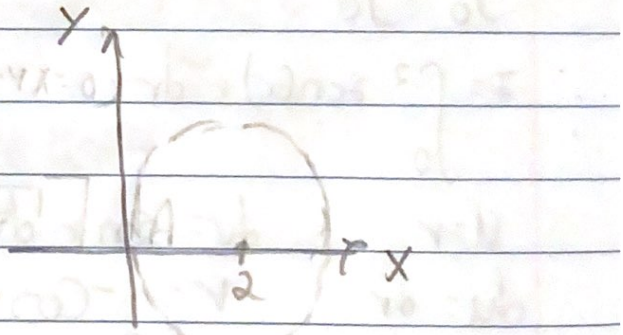
$z^2 = 16 - x^2 - y^2$

$x^2 + y^2 + z^2 = 16$

$(x^2 - 4x + 2) + y^2 =$

$(x-2)^2 + y^2 = 2$

$C(2, 0) \quad r = 1.41$



$x^2 + y^2 - 4x = 0$

$r^2 - 4(r\cos\theta) = 0$

$r2 - 4r\cos\theta = 0$

$r - 4\cos\theta = 0$

$r = 4\cos\theta$

R: $0 \leq r \leq 4\cos\theta$

$0 \leq \theta \leq \frac{\pi}{2}$

$$V = 2\int_0^{\frac{\pi}{2}}\int_0^{4\cos\theta} \sqrt{16-r^2}\, r\, dr\, d\theta$$

$$I = \int_0^{4\cos\theta} \sqrt{16-r^2}\, r\, dr = -\int_p^9 u^2\, du = -\frac{u^3}{3}\Big|_p^9 = -\frac{\sqrt{16-r^2}^3}{3}\Big|_0^{4\cos\theta}$$

$$= -\left[\frac{\sqrt{16-(4\cos\theta)^2}^3}{3} - \frac{\sqrt{16-0}^3}{3}\right] = -\left[\frac{\sqrt{16(1-\cos^2\theta)}^3}{3} - \frac{64}{3}\right]$$

$$= -\left[\frac{64(\operatorname{sen}^3\theta) - 64}{3}\right] = \frac{-64\operatorname{sen}^3\theta}{3} + \frac{64}{3} =$$

$$2\int_0^{\frac{\pi}{2}} \frac{-64}{3}\operatorname{sen}^3\theta + \frac{64}{3}\, d\theta = \frac{-128}{3}\int_0^{\frac{\pi}{2}} \operatorname{sen}^3\theta + 2\int \frac{64}{3}\, d\theta$$

$$\frac{-128}{3}\left[\frac{2}{3}\right] + \frac{128}{3}\left[\theta\right]_0^{\frac{\pi}{2}} = \frac{-128}{3}\left[\frac{2}{3}\right] + \left[\frac{128\pi}{6}\right]$$

(fórmula de wallies)

$$= \frac{-256}{9} + \frac{64\pi}{3} \approx \frac{64}{9}(3\pi - 4)u^3$$