COLLISION VELOCITY

William C. Evans January 2003

Introduction

In the Washington Post of January 15, 2003, page C3, there was a question about the speed needed to move a 3000 lb car some 4 feet, when that car had been hit by a 5000 lb car, pulling into a driveway. We have these assumptions:

- The collision is inelastic; some energy goes into deformation of the cars.
- The only source of energy is the kinetic energy of the moving car. Its velocity at impact is all that matters, what it was doing prior to that instant (accelerating, braking, etc.) is irrelevant.
- The kinetic energy of the combined cars (Case I) or the second car (Case II) is dissipated by friction.

CASE I The cars combine after the collision into one mass, moving together.

The mass of the larger, moving car is *M*, that of the smaller car is *m*. The speed of the moving car is *u* and that of the combined object (both cars, glommed together), after the collision, is *v*. The speed of the second car before the collision is zero. The fraction of the kinetic energy of the moving car that goes into deformation (smashing) is *f*. With this we have, by conservation of energy:

$$\frac{1}{2} M u^2 \left(1-f\right) = \frac{1}{2}\left(M+m\right) v^2$$

The work done, or energy expended, by the combined mass in moving a distance *d* against the force of friction is

$$\mu\left(M+m\right) g\, d$$

where μ is the (unknown) coefficient of friction, assumed constant. The acceleration due to gravity is *g*; the mass times this acceleration is the normal force, which when multiplied by μ, gives the friction force. Now we can write

$$\frac{1}{2} M u^2 \left(1-f\right) = \mu\left(M+m\right) g\, d$$

since the work done must be supplied by the *available* kinetic energy of the moving car. Solving this for the speed *u*,

$$u\left(\mu, f\right) = \sqrt{\left(1+\frac{m}{M}\right)\frac{2\mu g d}{\left(1-f\right)}} \qquad (1)$$

Using 5000 lb for *M*, 3000 lb for *m*, 32 ft/sec^2 for *g*, and 4 ft for *d*, we can reduce (1) to

$$u\left(\mu, f\right) = 14\sqrt{\frac{\mu}{1-f}} \qquad (2)$$

where in (2) the speed *u* is expressed in mph, as a function of the unknown parameters. Fortunately, both *f* and μ have simple ranges, varying only from 0 to 1 in both cases. We can now plot the speed *u*, letting the "crunch fraction" *f* be the independent variable, and the coefficient of friction μ be a parameter.

This is shown in Fig. 1. The solid curve is for a coefficient of friction of 1; in the the dotted curve it is 0.5. Note that the more friction we assume, the faster the moving car must have been going, to move the combined mass a given distance. Also, as the crunch fraction increases, the faster the moving car had to be going, and as more energy goes into crunching the cars, the speed becomes quite large.

CASE II The stationary car bounces off the moving car and moves independently.

In this case, we assume that the moving car stops quickly after the collision, due to braking, while the second car is moved through the distance *d*, by itself. The moving car will have some kinetic energy after the collision, taking away energy that would have been available to move the second car. We again have deformation, or crunch, energy expended. Conservation of energy will have that

$$\frac{1}{2} M u^2 \left(1-F\right) = \mu m g d$$

where *F* now accounts for both the fraction of energy lost to crunching and to the residual kinetic energy of the moving car, post-collision. Note that the normal force is now much smaller, just due to the mass *m* instead of *M*+*m*. Solving this, we obtain

$$u\left(\mu, F\right) = \sqrt{\frac{m}{M}\frac{2\mu g d}{\left(1-F\right)}} \qquad (3)$$

which, as before, can be reduced to an expression in mph,

$$u\left(\mu, F\right) = 8.5\sqrt{\frac{\mu}{1-F}} \qquad (4)$$

Here we see, for the same coefficient of friction settings as as used in the plot above, that the speeds are lower, since it takes less energy to move the smaller car, by itself, a given distance. In both solutions we see that the Land Rover speeds are reasonable, in the 10-20 mph range, for a car approaching a residential driveway.

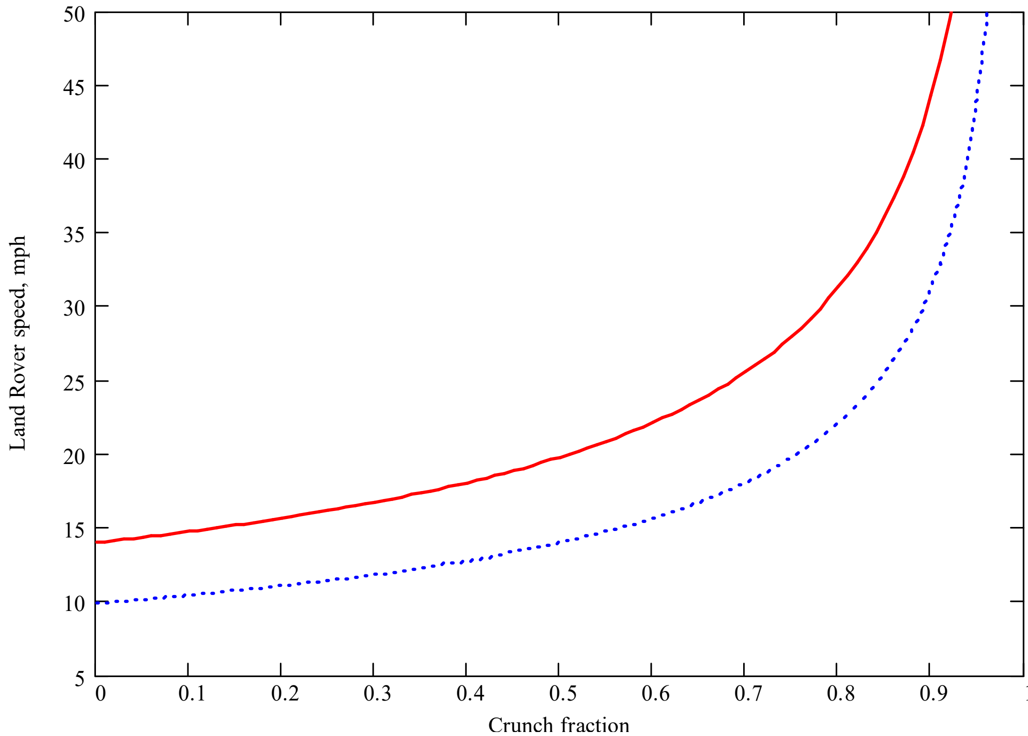

Figure 1. Speed of moving car vs. inelastic energy loss fraction.

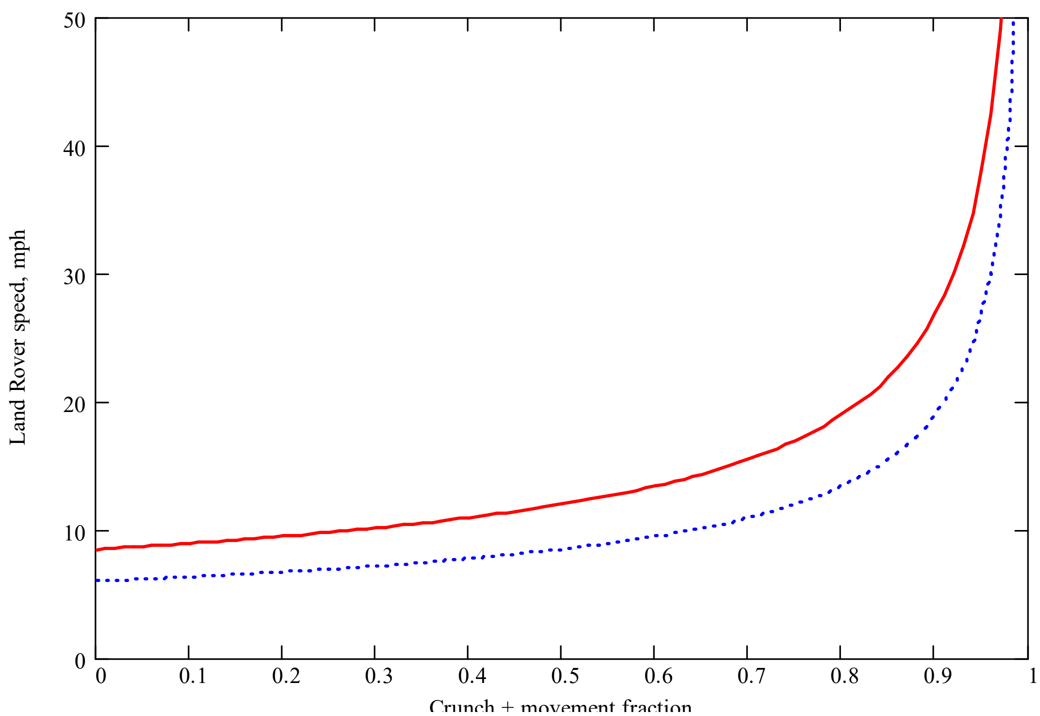

Figure 2. Speed of moving car vs. energy loss fraction.