EXERCISES [MAA 2.4-2.5]

COMPOSITION – INVERSE FUNCTION

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O.   Practice questions

1.   [Maximum mark: 10]   *[without GDC]*

Let $f(x) = 10 - 2x$ and $g(x) = 5x$. Calculate

(a) $(f \circ g)(x)$ and $(g \circ f)(x)$ [2]

(b) $f^{-1}(x)$ [2]

(c) $g^{-1}(10)$ [2]

(d) $(f^{-1} \circ g)(x)$ and $(g \circ f)^{-1}(x)$ [2]

(e) $(f \circ f)(x)$ and $(g \circ g)(x)$ [2]

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2. [Maximum mark: 6]   *[without GDC]*

Complete the following table with the inverse of each function

Original function	Inverse function
$f(x) = x + 5$	$f^{-1}(x) = x - 5$
$f(x) = x - 5$	
$f(x) = x + 100$	
$f(x) = 3x$	
$f(x) = \dfrac{x}{5}$	
$f(x) = x^3$	
$f(x) = 3x + 100$	

3. [Maximum mark: 4]   *[without GDC]*

Consider the following table of values for f

x	1	2	3	4	5	6
$f(x)$	3	4	5	6	1	2

(a)   Write down the values of

    (i) $f(1)$ (ii) $f^{-1}(1)$ [2]

(b)   Given that $f(x) = 2$ write down the value of x.   [1]

(c)   Given that $f^{-1}(x) = 2$ write down the value of x.   [1]

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4.   [Maximum mark: 8]   *[without GDC]*

The tables below show some values of two functions f and g

x	1	2	3	4
$f(x)$	2	−3	−1	3

x	1	2	3	4
$g(x)$	5	3	1	−3

(a)   Write down the values of $g(3)$, $f^{-1}(3)$.   [2]

(b)   Calculate $(f \circ g)(2)$ [2]

(c)   Calculate $(g \circ g)(3)$ [2]

(d)   Find a solution of the equation $(g \circ f)(x) = 3$.   [2]

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5.   [Maximum mark: 5]   *[without GDC]*

Find the inverse of the function $f(x) = \dfrac{x}{x+5}$

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6. [Maximum mark: 6] **[without GDC]**

Let $f(x) = 2^x$, and $g(x) = \dfrac{x}{x-2}$, $x \neq 2$. Find

(a) $(g \circ f)(3)$; [3]

(b) $g^{-1}(5)$. [3]

METHOD A [by finding $(g \circ f)(x)$ and $g^{-1}(x)$ first]

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METHOD B [directly by definitions: $(g \circ f)(3) = g(f(3))$ and $g^{-1}(5) = x \Leftrightarrow g(x) = 5$]

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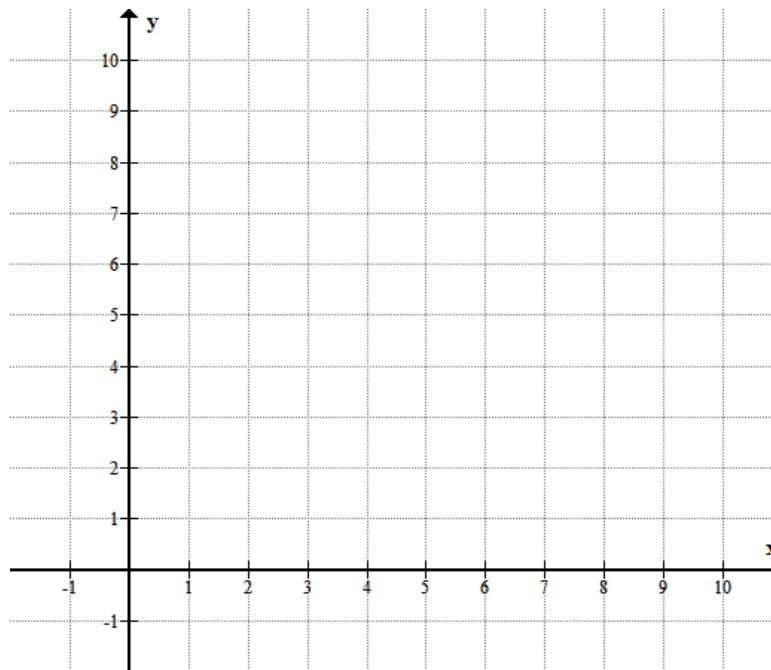...

7. [Maximum mark: 10] *[without GDC]*

The function f is given by $f(x) = x^2$, for $0 \le x \le 3$.

(a) Sketch the graph of f. [2]

(b) State the domain and the range of f. [2]

(c) Find the inverse function f^{-1}. [2]

(d) On the same axes with f, sketch the graph of f^{-1} [2]

(e) State the domain and the range of f^{-1}. [2]

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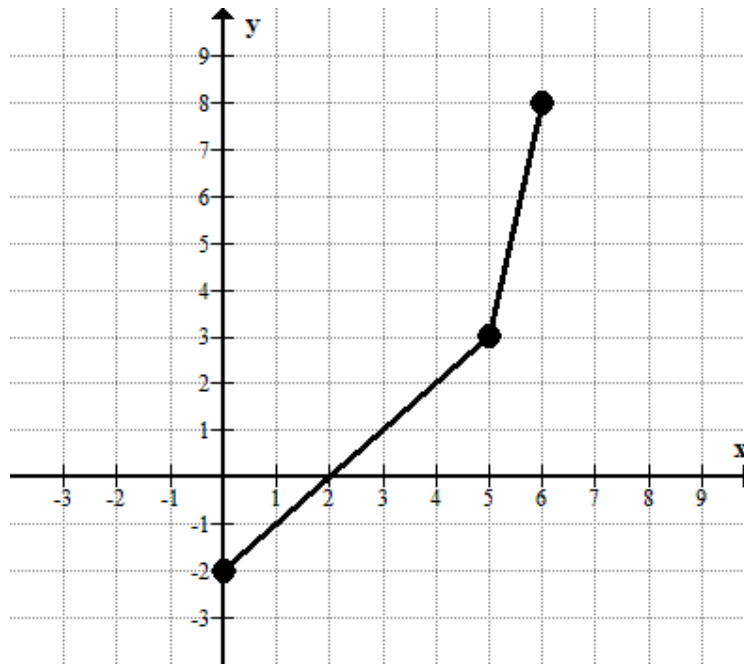
8. [Maximum mark: 12]   *[without GDC]*

The graph of the function f is shown below.

(a)   Write down the values of

(i) $f(0)$ (i) $f(2)$ (ii) $f(4)$ [3]

(b)   Write down the values of

(i) $f^{-1}(3)$ (ii) $f^{-1}(8)$ (iii) $f^{-1}(-1)$ (iv) $f^{-1}(0)$ [4]

(c)   Write down the solution of the equation $f(x) = 0$ [1]

(d)   Write down the solution of the equation $f(x) = 2$ [1]

(e)   On the diagram above sketch the graph of f^{-1} [3]

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9*. [Maximum mark: 8]   *[without GDC]*

Let $f(x) = \dfrac{2x-3}{3x-2}$. Show that

(a) $f^{-1} = f$ (i.e. the function is self-inverse)   [4]

(b) $(f \circ f)(x) = x$ [4]

10*. [Maximum mark: 7]   *[without GDC]*

The function f is defined by $f : x \mapsto x^3$.

(a) Find an expression for f^{-1}. [2]

(b) Find an expression for $g(x)$ in terms of x in each of the following cases

   (i) $(f \circ g)(x) = x + 1$;

   (ii) $(g \circ f)(x) = x + 1$. [5]

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11**. [Maximum mark: 6]   *[without GDC]*

(a) Given that $f \circ g = h$, express f in terms of g and h. [2]

(b) Given that $f \circ g = h$, express g in terms of f and h. [2]

(c) Given that $f \circ g \circ h = k$, express g in terms of f, h and k. [2]

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A. Exam style questions (SHORT)

12. [Maximum mark: 6]   *[without GDC]*

Let $g(x) = 3x - 2$, $h(x) = \dfrac{5x}{x-4}$, $x \neq 4$.

(a)   Find an expression for $(h \circ g)(x)$. Simplify your answer.   [3]

(b)   Solve the equation $(h \circ g)(x) = 0$.   [3]

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13. [Maximum mark: 6]   *[without GDC]*

The functions f and g are defined by $f : \mapsto 3x$, $g : x \mapsto x + 2$.

(a)   Find an expression for $(f \circ g)(x)$.   [2]

(b)   Show that $f^{-1}(18) + g^{-1}(18) = 22$.   [4]

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14. [Maximum mark: 6]   *[without GDC]*

Let $f(x) = 2x + 1$ and $g(x) = 3x^2 - 4$. Find

(a) $f^{-1}(x)$;   [2]

(b) $(g \circ f)(-2)$;   [2]

(c) $(f \circ g)(x)$.   [2]

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15. [Maximum mark: 4]   *[without GDC]*

The function f is defined by $f: x \mapsto \sqrt{3 - 2x}$, $x \le \dfrac{3}{2}$

Evaluate $f^{-1}(5)$.

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16. [Maximum mark: 6]   *[without GDC]*

Let $f(x) = \sqrt{x+4}$ $x \geq -4$ and $g(x) = x^2$, $x \in \mathbb{R}$.

(a)   Find $(g \circ f)(3)$ [2]

(b)   Find $f^{-1}(x)$ [3]

(c)   Write down the domain of f^{-1}.   [1]

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17. [Maximum mark: 5]   *[without GDC]*

Let $f(x) = x^3 - 4$ and $g(x) = 2x$.

(a)   Find $(g \circ f)(-2)$.   [2]

(b)   Find $f^{-1}(x)$ [3]

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18. [Maximum mark: 6]   *[without GDC]*

Consider the functions $f(x) = 2x$ and $g(x) = \dfrac{1}{x-3}$, $x \neq 3$.

(a) Calculate $(f \circ g)(4)$.   [2]

(b) Find $g^{-1}(x)$ [3]

(c) Write down the domain of g^{-1}.   [1]

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19. [Maximum mark: 4]   *[without GDC]*

Two functions f, g are defined as follows:

$$f : x \mapsto 3x + 5 \qquad\qquad g : x \mapsto 2(1 - x)$$

Find

(a) $f^{-1}(2)$;   [2]

(b) $(g \circ f)(-4)$.   [2]

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20. [Maximum mark: 6]   *[without GDC]*

Two functions f and g are defined as follows:

$$f(x) = \cos x, \qquad g(x) = 2x + 1$$

(a)   Find $(g \circ f)(x)$.   [2]

(b)   Find $(f \circ g)(x)$.   [2]

(c)   Find $(g^{-1} \circ g)(x)$ and $(g \circ g^{-1})(x)$.   [2]

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21. [Maximum mark: 6]   *[without GDC]*

Consider the functions $f : x \mapsto 4(x-1)$ and $g : x \mapsto \dfrac{6-x}{2}$.

(a)   Find g^{-1}.   [2]

(b)   Solve the equation $(f \circ g^{-1})(x) = 4$.   [4]

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22. [Maximum mark: 5]   *[without GDC]*

Let $f(x) = x^2$ and $g(x) = 2x - 3$.

(a) Find $g^{-1}(x)$.   [2]

(b) Find $(f \circ g)(4)$.   [3]

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23. [Maximum mark: 5]   *[without GDC]*

Let $f(x) = 7 - 2x$ and $g(x) = x + 3$.

(a) Find $(g \circ f)(x)$.   [2]

(b) Write down $g^{-1}(x)$.   [1]

(c) Find $(f \circ g^{-1})(5)$.   [2]

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24. [Maximum mark: 5]   *[without GDC]*

Let $f(x) = 3x$, $g(x) = 2x - 5$ and $h(x) = (f \circ g)(x)$.

(a) Find $h(x)$. [2]

(b) Find $h^{-1}(x)$. [3]

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25. [Maximum mark: 6]   *[with GDC]*

Let $f(x) = \sqrt{x}$, and $g(x) = 2^x$.

(a) Find $(f^{-1} \circ g)(x)$ [4]

(b) Solve the equation $(f^{-1} \circ g)(x) = 16$ [2]

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26. [Maximum mark: 6]   *[without GDC]*

Consider the functions f and g where $f(x) = 3x - 5$ and $g(x) = x - 2$.

Solve the equation $(f^{-1} \circ g)(x) = (g^{-1} \circ f)(x)$.

27. [Maximum mark: 6]   *[without GDC]*

Let $f(x) = \dfrac{4}{x+2}$, $x \neq -2$ and $g(x) = x - 1$. If $h = g \circ f$, find

(a) $h(x)$ [2]

(b) $h^{-1}(x)$ [4]

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28. [Maximum mark: 4]   *[without GDC]*

Given functions $f : x \mapsto x + 1$ and $g : x \mapsto x^3$, find the function $(f \circ g)^{-1}$

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29. [Maximum mark: 5]   *[without GDC]*

If $f(x) = \dfrac{x}{x+1}$, for $x \neq -1$ and $g(x) = (f \circ f)(x)$, find

(a) $g(x)$.   [3]

(b) $(g \circ g)(2)$.   [2]

30. [Maximum mark: 5]   *[without GDC]*

The function f is defined as $f(x) = \dfrac{3x-4}{x+2}$, $x \neq -2$.

(a)   Find an expression for $f^{-1}(x)$.   [4]

(b)   Write down the domain of f^{-1}.   [1]

31*. [Maximum mark: 9]   *[with / without GDC]*

The function f is given by $f(x) = x^2 - 6x + 13$, for $x \geq 3$.

(a)   Show that f may also be written in the form $f(x) = (x-3)^2 + 4$.   [2]

(b)   **Hence** find the inverse function f^{-1}.   [3]

(c)   State the domain and the range of f.   [2]

(d)   State the domain and the range of f^{-1}.   [2]

32*. [Maximum mark: 7]   *[without GDC]*

The functions f and g are both defined in the interval [-4,4] and g is invertible.

Some values of the functions are given below.

x	1	3	4
$f(x)$	3	2	1

x	1	3	4
$g(x)$	4	1	-3

(a) Calculate $(f \circ f)(1)$.   [2]

(b) Calculate $(g^{-1} \circ f)(4)$.   [2]

(c) Find a solution of the equation $(f \circ g)(x) = 1$.   [2]

(d) Calculate $(g^{-1} \circ g)(2)$.   [1]

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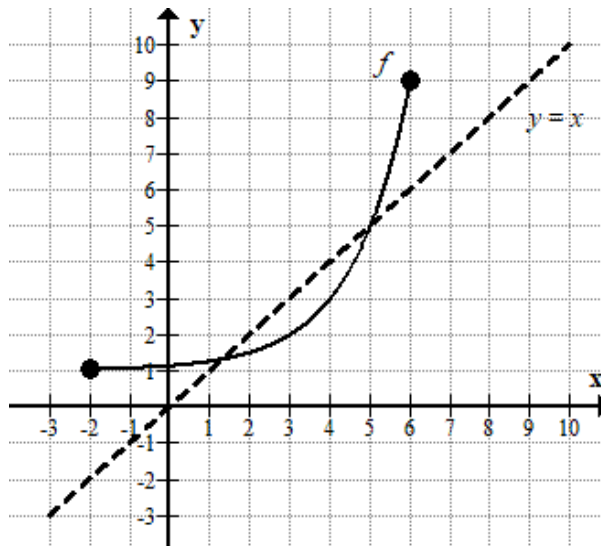...

33. [Maximum mark: 4]   *[without GDC]*

The graph of the function f is shown below. On the same diagram sketch the graph of the function f^{-1}.

34*. [Maximum mark: 6]   *[with / without GDC]*

The functions $f(x)$ and $g(x)$ are given by $f(x) = \sqrt{x-2}$ and $g(x) = x^2 + x$.

The function $(f \circ g)(x)$ is defined for $x \in \mathbb{R}$, **except** for the interval $]a, b[$

(a)   Calculate the value of a and of b.   [5]

(b)   Find the range of $f \circ g$.   [1]

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35**. [Maximum mark: 5]   *[with GDC]*

Consider the functions

$$f(x) = 3x^2 - 12x + 7, \; x \geq a \quad \text{and} \quad g(x) = 3x^2 - 12x + 7, \; x \leq b.$$

(a)   Given that f^{-1}, g^{-1} exist, find

(i) the smallest possible value of a.   (ii) the largest possible value of b.   [3]

(b)   Within the domains determined in (a), find

(i) $f^{-1}(7)$.   (ii) $g^{-1}(7)$ [2]

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36*. [Maximum mark: 6] **[without GDC]**

The function f is defined by $f : x \mapsto x^3 - 1$

(a) Find $g(x)$ given that $(f \circ g)(x) = 2x + 1$; [3]

(b) Find $g(x)$ given that $(g \circ f)(x) = 2x + 1$. [3]

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37.** [Maximum mark: 6] **[without GDC]**

Let f and g be two functions. Given that $(f \circ g)(x) = \dfrac{x+1}{2}$ and $g(x) = 2x - 1$,

find $f(x-3)$.

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38*. [Maximum mark: 6]   *[without GDC]*

The function f is defined for $x \le 0$ by $f(x) = \dfrac{x^2 - 1}{x^2 + 1}$. Find an expression for $f^{-1}(x)$.

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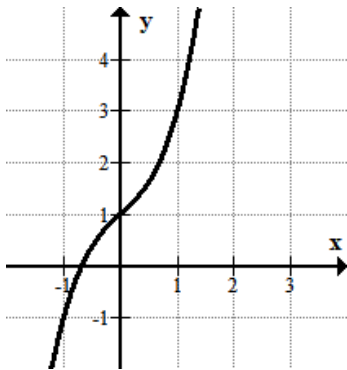..

39**. [Maximum mark: 6]   *[with GDC]*

The following diagram shows part of the graph of $f(x) = x^3 + x + 1$

(a)   Find (i) $f^{-1}(1)$ (ii) $f^{-1}(0)$ (iii) $f^{-1}(2)$.   [3]

(b)   Solve $f(x) = f^{-1}(x)$.   [3]

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B. Exam style questions (LONG)

40. [Maximum mark: 12]   *[with / without GDC]*

The function f is given by $f(x) = 2x^2 - 8x + 1$ for $x \le 2$.

(a) Find $(f \circ f)(0)$. [2]

(b) Express the function in the form $f(x) = a(x - h)^2 + k$. [3]

(c) State the range of f. [1]

(d) Find the inverse function f^{-1}. [4]

(e) State the domain and the range of f^{-1}. [2]

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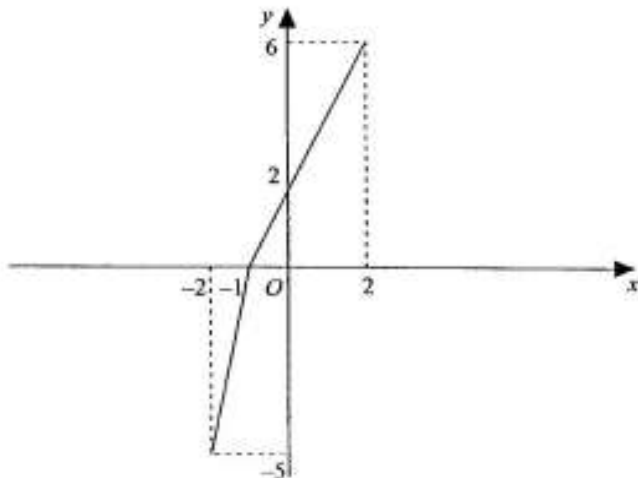
41. [Maximum mark: 10]   *[without GDC]*

The function $f(x)$ is defined by $f(x) = \dfrac{3x-1}{x-3}$, $x \ne 3$.

(a)   Show that f is a self-inverse function, that is $f^{-1} = f$ [4]

(b)   **Hence** find, in terms of k, the value of $(f \circ f)(k)$, where $k \ne 3$.   [1]

The figure below shows a sketch of a one-to-one function $g(x)$ defined over the domain $-2 \le x \le 2$. The graph of $y = g(x)$ consists of two straight line segments and the range of $g(x)$ is $-5 \le g(x) \le 6$.

(c)   Find the value of $(f \circ g)(-2)$.   [2]

(d)   On the same diagram above, sketch the graph of the inverse function $y = g^{-1}(x)$ and state its domain.   [3]

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42**. [Maximum mark: 13]   *[without GDC]*

(a)   The function f is defined by $f(x) = (x+2)^2 - 3$.

The function g is defined by $g(x) = ax + b$, where a and b are constants.

Find the value of a, $a > 0$ and the corresponding value of b, such that

$$f(g(x)) = 4x^2 + 6x - \frac{3}{4}.$$ [8]

(b)   The functions h and k are defined by $h(x) = 5x + 2$ and $k(x) = cx^2 - x + 2$

respectively. Find the value of c are such that $h(k(x))$ has equal roots.   [5]

43**. [Maximum mark: 10]   *[without GDC]*

Let $f(x) = \sqrt{x+1} + 1$ and $g(x) = x^2$.

(a) Solve the equation $(g \circ f)(x) = 1$. [3]

(b) Find f^{-1} in a simplified form. [3]

(c) Find the function $h(x)$ given that $h \circ f = g$. [2]

(d) Find the function $k(x)$ given that $f \circ k = g$. [2]

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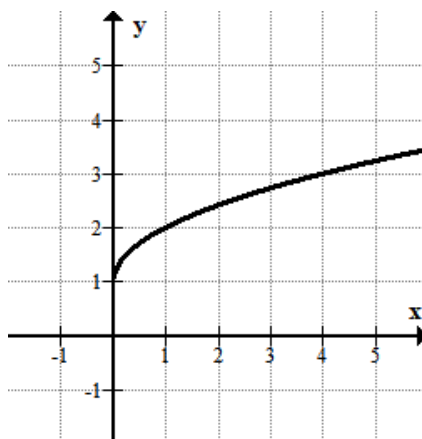
44**. [Maximum mark: 11]   *[without GDC]*

The following diagram shows part of the graph of the function f is defined by

$f(x) = 1 + \sqrt{x}$,   for $x \geq 0$

(a)   Find an expression for $f^{-1}(x)$.   [3]

(b)   State the domain and the range of $f^{-1}(x)$.   [2]

(c)   On the same diagram above sketch the graph of $f^{-1}(x)$.   [2]

(d)   Find the **exact** value of the solution of the equation $f(x) = f^{-1}(x)$. [4]

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