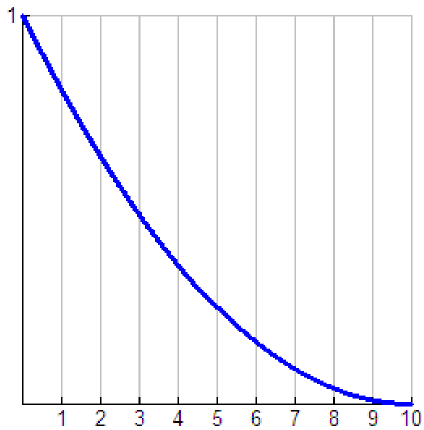
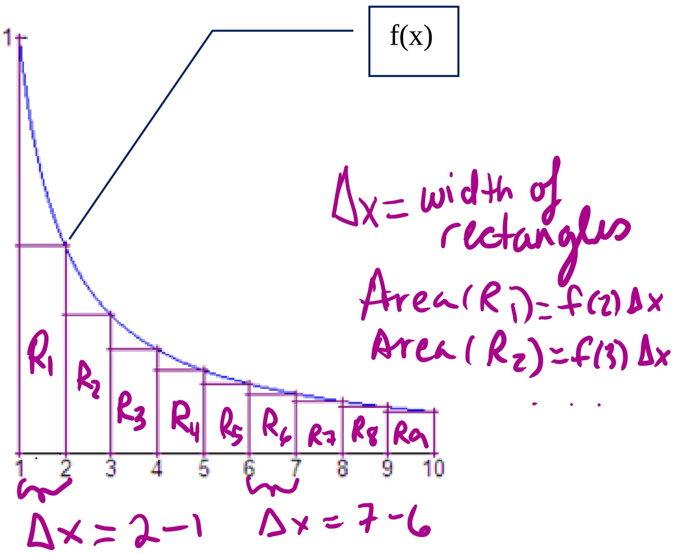
Approximating Area under a Curve
Left and Right Riemann Sums

How would you find the area under this curve on the interval [1,10]?





Right Riemann Sum

An approximation can be found by creating 9 rectangles of width 1. The heights of the rectangles are $f(x)$ for some x in the subinterval bounded by the sides of each rectangle.

In the example above, we are using the value of the right end point of each subinterval, we call this the Right Riemann Sum. In other words, the height of the first rectangle is $f(2)$ because 2 is the right most point on the subinterval [1,2]. The height of the second rectangle is $f(3)$. The area of each rectangle is (1)(f(x)) because the width is 1 and the height is f(x). So, the approximation of the area under the curve is:

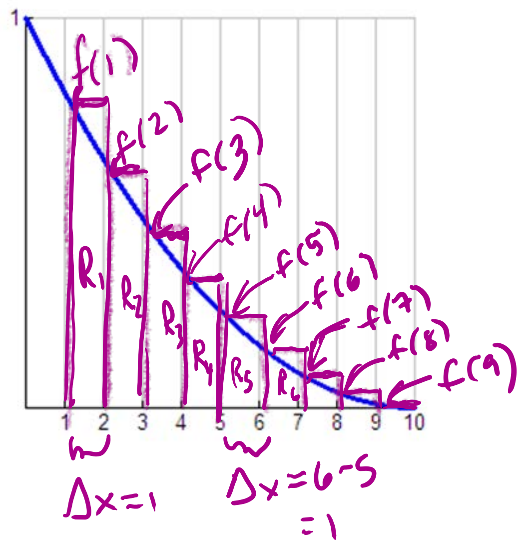
$$f(2)+f(3)+f(4)+\cdots+f(10)=\sum_{k=2}^{10} f(k) = \text{Area}(R_1)+\text{Area}(R_2)+\cdots+\text{Area}(R_9)$$

$$\approx \text{Area under } f \text{ from } 1 \text{ to } 10$$

Since the function is decreasing, the right most point of each subinterval would have a lower value than the rest of the of the points in the subinterval, so the Right Riemann sum would be an underapproximation.

Left Riemann Sum

If we use left most endpoint of each subinterval to find the height of the rectangles, we call it the Left Riemann Sum.



$\text{Area}(R_1) = f(1)\Delta x$

$\text{Area}(R_2) = f(2)\Delta x$

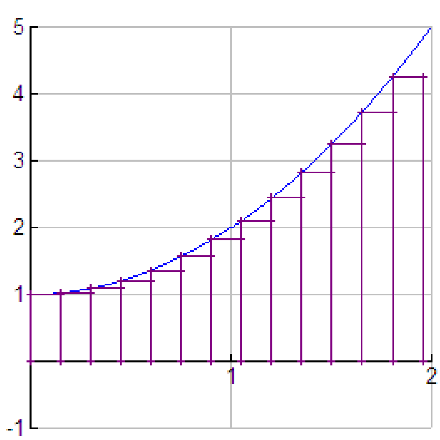
$\vdots$

$$f(1)+f(3)+f(4)+\cdots+f(9)=\sum_{k=1}^{9} f(k) = \text{Area}(R_1)+\cdots+\text{Area}(R_9) \approx \text{Area under curve}$$

Since the function is decreasing, the Left Riemann Sum is an underapproximation of the area under the curve.

Clearly, if we used more subintervals, our approximation would be better.

Question: Below, we see the graph of an increasing function: Is the Riemann Sum and Left Riemann Sum or a Right Riemann Sum? Does it over-approximate or under-approximate the area under the curve?



Answers: Right, Under