PROJECTILE MOTION

Line of sight

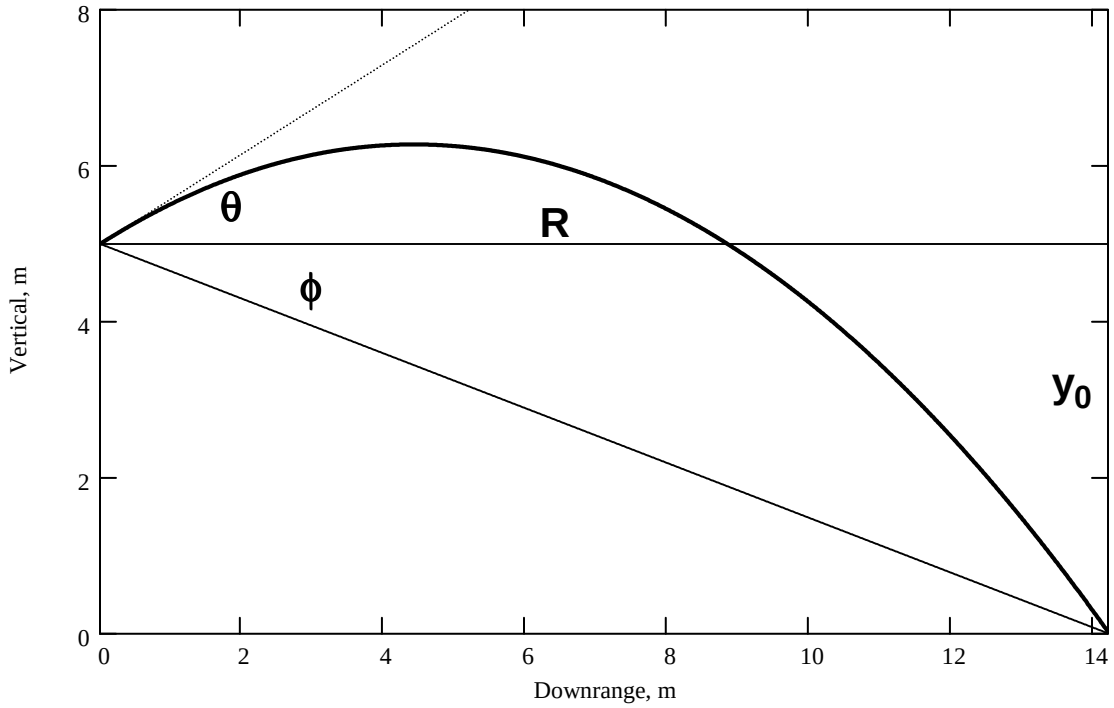
Some problems ask for the angle of the line of sight (LOS) from the "launch" position to the impact position. Consider the trajectory in the plot below. The range is

$$R := \frac{v_0}{g} \cos(\theta) \left[v_0 \sin(\theta) + \sqrt{(v_0 \sin(\theta))^2 + 2\, g\, y_0} \right]$$

This is the x-coordinate of the impact point, whose y-coordinate is of course zero. The launch point is at $(0,y_0)$, so that we have a right triangle with the LOS angle

$$\phi = \operatorname{atan}\left(\frac{-y_0}{R}\right)$$

made at the launch point, positive upward, as usual (counterclockwise).



For the case of hitting a target, at location (X,Y), we have from the same geometry as above

$$\phi = \operatorname{atan}\left(\frac{Y - y_0}{X}\right)$$

Also a special case is a launch from a zero angle, at a nonzero initial height (airplane). This leads to

$$\phi = \operatorname{atan}\left(\sqrt{\frac{g\, y_0}{2\, v_0^2}}\right)$$