INTERNATIONAL BACCALAUREATE

Mathematics: analysis and approaches

MAA

EXERCISES [MAA 2.9]

LOGARITHMS

Compiled by Christos Nikolaidis

O. Practice questions

BASIC PROPERTIES OF LOGARITHMS

1. [Maximum mark: 12] ***[without GDC]***

Write down the following values

$\log_2 1 =$	$\log_2 2 =$	$\log_2 16 =$
$\log_5 1 =$	$\log_5 5 =$	$\log_5 25 =$
$\log_3 1 =$	$\log_3 3 =$	$\log_3 9 =$
$\log_3 27 =$	$\log_3 \frac{1}{3} =$	$\log_3 \sqrt{3} =$

2. [Maximum mark: 9] ***[without GDC]***

Write down the following values

$\log 100 =$	$\log 10 =$	$\log 1 =$
$\log \frac{1}{100} =$	$\log \frac{1}{10} =$	$\log 0.1 =$
$\log 10^{2020} =$	$\log \sqrt{10} =$	$\log \sqrt[3]{10} =$

3. [Maximum mark: 6] ***[without GDC]***

Write down the following values

$\ln 1 =$	$\ln \mathrm{e} =$	$\ln \mathrm{e}^2 =$
$\ln \frac{1}{\mathrm{e}} =$	$\ln \frac{1}{\mathrm{e}^2} =$	$\ln \sqrt{\mathrm{e}} =$

4. [Maximum mark: 6] ***[without GDC]***

Write down the value of x for each of the following equations

$\log_2 8 = x$	$x =$
$\log_2 x = 3$	$x =$
$\log_x 8 = 3$	$x =$

$\log 1000 = x$	$x =$
$\log x = 3$	$x =$
$\ln x = 3$	$x =$

5. [Maximum mark: 6] ***[without GDC]***

Confirm the following properties for $x = 1000$ and $y = 100$

$\log xy = \log x + \log y$	LHS =
	RHS =
$\log \dfrac{x}{y} = \log x - \log y$	LHS =
	RHS =
$\log x^2 = 2\log x$	LHS =
	RHS =

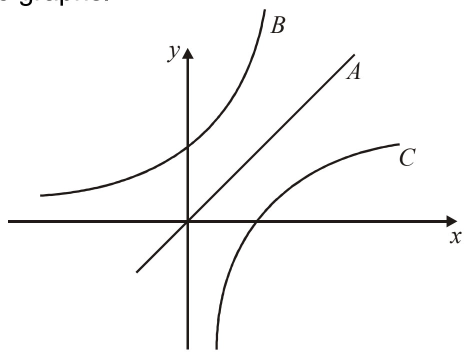
6. [Maximum mark: 8] ***[without GDC]***

Find the following integers

$\log_3 3^5 =$	$\log 10^5 =$
$3^{\log_3 5} =$	$10^{\log 5}$
$3^{2\log_3 5} =$	$10^{2\log 5}$
$3^{3\log_3 5} =$	$10^{3\log 5} =$
$\ln e^5 =$	$\log_a a^5 =$
$e^{\ln 5} =$	$a^{\log_a 5} =$
$e^{2\ln 5} =$	$a^{2\log_a 5} =$
$e^{3\ln 5} =$	$a^{3\log_a 5} =$

7. [Maximum mark: 4] ***[without GDC]***

The diagram shows three graphs.



A is part of the graph of $y = x$, B of the graph of $y = 2^x$,

C is the reflection of graph B in line A. Write down:

(a) the equation of C in the form $y = f(x)$. [2]

(b) the coordinates of the point where C cuts the x-axis. [2]

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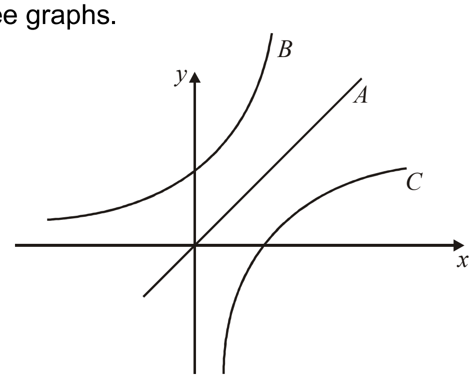
8. [Maximum mark: 4] ***[without GDC]***

The diagram shows three graphs.



A is part of the graph of $y = x$, B of the graph of $y = \mathrm{e}^x$,

C is the reflection of graph B in line A. Write down:

(a) the equation of C in the form $y = f(x)$. [2]

(b) the coordinates of the point where C cuts the x-axis. [2]

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9. [Maximum mark: 28] ***[without GDC]***

Let $\log x = a$, $\log y = b$ and $\log z = c$. Express the following in terms of a, b, c .

$\log xy$	
$\log \frac{x}{y}$	
$\log x^3$	
$\log xyz$	
$\log x^2 y$	
$\log \sqrt{x}$	
$\log \frac{xy}{z}$	
$\log(10x)$	
$\log(100x)$	
$\log \frac{y}{10}$	
$\log \frac{y}{100}$	
$\log \frac{xy}{10z}$	
$\log \frac{1}{z}$	
$\log \frac{x^2 y^7}{\sqrt{z}}$	

10. [Maximum mark: 28]

Let $\ln x = a$, $\ln y = b$ and $\ln z = c$. Express the following in terms of a, b, c.

$\ln xy$	
$\ln \frac{x}{y}$	
$\ln x^3$	
$\ln xyz$	
$\ln x^2 y$	
$\ln \sqrt{x}$	
$\ln \frac{xy}{z}$	
$\ln(\mathrm{e}x)$	
$\ln(\mathrm{e}^2 x)$	
$\ln \frac{y}{\mathrm{e}}$	
$\ln \frac{y}{\mathrm{e}^2}$	
$\ln \frac{xy}{\mathrm{e}z}$	
$\ln \frac{1}{z}$	
$\ln \frac{x^2 y^7}{\sqrt{z}}$	

11. [Maximum mark: 26] ***[without GDC]***

Let $\log_5 x = a$, $\log_5 y = b$ and $\log_5 z = c$. Express the following in terms of a, b, c.

$\log_5 xy$	
$\log_5 \frac{x}{y}$	
$\log_5 x^3$	
$\log_5 \sqrt{x}$	
$\log_5 \frac{xy}{z}$	
$\log_5 \frac{xy}{5z}$	
$\log_5 \frac{1}{z}$	
$\log_5 \frac{x^2 y^7}{\sqrt{z}}$	
$\log_{25} x$	
$\log_x 5$	
$\log_x y$	
$\log_z xy$	
$\log_{25} xy$	

LOGARITHMIC EQUATIONS

12. [Maximum mark: 6] ***[without GDC]***

Solve the following equations

$\log_3(x+1)=2$	
$\log(x+1)=2$	
$\ln(x+1)=2$	

13. [Maximum mark: 12] ***[without GDC]***

Solve the equations

$\log_7(x+5)=0$	
$\log_7(x+5)=1$	
$\log(x+5)=0$	
$\log(x+5)=1$	
$\ln(x+5)=0$	
$\ln(x+5)=1$	

14. [Maximum mark: 12] ***[without GDC]***

Solve the equations

$\log(2x)=2$	
$\ln(2x)=2$	
$\log(2x+4)=1$	
$\ln(2x+4)=1$	
$\log(2x-5)=0$	
$\ln(2x-5)=0$	

15. [Maximum mark: 9] ***[without GDC]***

Solve the equations

(a) $\log_2 x + \log_2(x+1) = \log_2 6$ [3]

(b) $\log_2 x + \log_2(x+1) = 1$ [3]

(c) $\log_2(x+5) - \log_2 x = 1$ [3]

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16. [Maximum mark: 12] ***[without GDC]***

Solve the equations

(a) $\log x + \log(x+1) = \log 6$ [4]

(b) $\log x + \log(x+3) = 1$ [4]

(c) $\log(x+18) - \log x = 1$ [4]

17*. [Maximum mark: 10] ***[without GDC]***

Solve the equations

(a) $\log_2(x+14) - 2\log_2 x = 2$ [4]

(b) $\log_4(x+14) - \log_2 x = 1$ by using change of base on $\log_4(x+14)$ [3]

(c) $\log_2(x+14) = 2 + \log_{\sqrt{2}} x$ by using change of base on $\log_{\sqrt{2}} x$ [3]

18*. [Maximum mark: 8] ***[without GDC]***

Solve the following equations

(a) $(\ln x)^2 - \ln x^2 + 1 = 0$ by letting $y = \ln x$ [3]

(b) $\ln x + \dfrac{1}{\ln x} = 2$ by using an appropriate substitution. [2]

(c) $\log x + \dfrac{1}{\log x} = 2$ by using an appropriate substitution. [3]

A. Exam style questions (SHORT)

PROPERTIES OF LOGARITHMS

19. [Maximum mark: 5] ***[without GDC]***

(a) Find $\log_2 32$. [1]

(b) Given that $\log_2\left(\frac{32^x}{8^y}\right)$ can be written as $px+qy$, find the value of p and of q. [4]

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20. [Maximum mark: 5] ***[without GDC]***

Let $\log_{10} P = x$, $\log_{10} Q = y$ and $\log_{10} R = z$. Express $\log_{10}\left(\frac{P}{QR^3}\right)^2$ in terms of x, y and z.

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21. [Maximum mark: 5] ***[without GDC]***

Let $a=\log x$, $b=\log y$, and $c=\log z$. Write $\log\left(\dfrac{x^2\sqrt{y}}{z^3}\right)$ in terms of a, b and c.

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22. [Maximum mark: 5] ***[without GDC]***

Let $p=\log_{10} x$, $q=\log_{10} y$ and $r=\log_{10} z$. Write $\log_{10}\left(\dfrac{x}{y^2\sqrt{z}}\right)$ in terms of p, q and r.

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23. [Maximum mark: 6] ***[without GDC]***

Let $\ln a = p$, $\ln b = q$. Write the following expressions in terms of p and q.

(a) $\ln a^3 b$ [3]

(b) $\ln\left(\frac{\sqrt{a}}{b}\right)$ [3]

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24. [Maximum mark: 6] ***[without GDC]***

Given that $p = \log_a 5$, $q = \log_a 2$, express the following in terms of p and/or q.

(a) $\log_a 10$ [2]

(b) $\log_a 8$ [2]

(c) $\log_a 2.5$ [2]

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25. [Maximum mark: 6] ***[without GDC]***

(a) Let $\log_c 3 = p$ and $\log_c 5 = q$. Find an expression in terms of p and q for

(i) $\log_c 15$; (ii) $\log_c 25$. [4]

(b) Find the value of d if $\log_d 6 = \frac{1}{2}$. [2]

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26. [Maximum mark: 6] ***[without GDC]***

Given that $\log_5 x = y$, express each of the following in terms of y.

(a) $\log_5 x^2$ (b) $\log_5\left(\frac{1}{x}\right)$ (c) $\log_{25} x$ [2+2+2]

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27. [Maximum mark: 4] ***[without GDC]***

If $\log_a 2 = x$ and $\log_a 5 = y$, find in terms of x and y, expressions for

(a) $\log_2 5$. [2]

(b) $\log_a 20$. [2]

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28. [Maximum mark: 6] ***[without GDC]***

Find the **exact** value of x in each of the following equations.

(a) $5^{x+1} = 625$ [3]

(b) $\log_a (3x+5) = 2$ [3]

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LOGARITHMIC EQUATIONS

29. [Maximum mark: 6] ***[without GDC]***

Solve $\log_2 x + \log_2(x-2) = 3$, for $x > 2$

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30. [Maximum mark: 6] ***[without GDC]***

(a) Given that $\log_3 x - \log_3(x-5) = \log_3 A$, express A in terms of x. [2]

(b) Hence or otherwise, solve the equation $\log_3 x - \log_3(x-5) = 1$. [4]

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31. [Maximum mark: 6] ***[without GDC]***

Solve the equation $\log_3(x+2) = 1 + \frac{\log_3 x}{2}$

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32. [Maximum mark: 4] ***[without GDC]***

Solve the equation $\log_9 81 + \log_9 \frac{1}{9} + \log_9 3 = \log_9 x$.

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33. [Maximum mark: [5]] ***[without GDC]***

Solve the equation $\log(10x+20)-2\log x=1$

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34. [Maximum mark: [5]] ***[without GDC]***

Solve the equation $\log_2(4x)+2\log_2 x=5$

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35. [Maximum mark: 6] ***[without GDC]***

Solve, for x, the equation $\log_2(5x^2 - x - 2) = 2 + 2\log_2 x$.

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36. [Maximum mark: 6] ***[without GDC]***

Solve the equation $\log_{16} \sqrt[3]{100 - x^2} = \frac{1}{2}$.

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37. [Maximum mark: 5] ***[with GDC]***

Solve the equation $\log_{27} x = 1 - \log_{27}(x - 0.4)$.

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38*. [Maximum mark: 6] ***[without GDC]***

Solve the equation $2\log_3(x-3) + \log_{\frac{1}{3}}(x+1) = 2$

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39*. [Maximum mark: 6] ***[without GDC]***

Solve the equation $\log_2 x = \log_4 (x+6)$

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40*. [Maximum mark: 6] ***[without GDC]***

Solve, $|\ln(x+3)| = 1$. Give your answers in **exact** form.

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41. [Maximum mark: 6] ***[without GDC]***

Solve $2(\ln x)^2 = 3\ln x - 1$ for x. Give your answers in **exact** form.

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42.** [Maximum mark: 6] ***[without GDC]***

Solve the equation $9\log_x 5 = \log_5 x$

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43.** [Maximum mark: 6] ***[without GDC]***

Solve the equation $9\log_5 x = 25\log_x 5$. Express your answers in the form $5^{\frac{p}{q}}$, $p, q \in Z$.

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44.** [Maximum mark: 7] ***[without GDC]***

Solve the equation $9\log_8 x = 6 + 8\log_x 8$

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45.** [Maximum mark: 7] ***[without GDC]***

Solve the simultaneous equations: $2^{x^2} = 4^y$ and $\log_x y = \dfrac{3}{2}$

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46.** [Maximum mark: 7] ***[without GDC]***

Solve the simultaneous equations: $8^y = 4^{2x+3}$ and $\log_2 y = \log_2 x + 4$

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47.** [Maximum mark: 7] ***[without GDC]***

Solve the simultaneous equations $\log_2(y-1) = 1 + \log_2 x$ and $2\log_3 y = 2 + \log_3 x$

48.** [Maximum mark: 7] ***[without GDC]***

Solve the simultaneous equations $\log_2 x - \log_4 y = 4$, $\log_2 (x - 2y) = 5$

LOGARITHMS AND SEQUENCES

49*. [Maximum mark: 8] ***[without GDC]***

Calculate the following sums

(a) $\ln 2 + \ln 2^2 + \ln 2^3 + \cdots + \ln 2^{10}$ [3]

(b) $\ln 2 + (\ln 2)^2 + (\ln 2)^3 + \cdots + (\ln 2)^{10}$ [3]

(c) $\ln 2 + (\ln 2)^2 + (\ln 2)^3 + \cdots$ (infinite sum) [2]

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50*. [Maximum mark: 6] ***[without GDC]***

Find $\sum_{r=1}^{50} \ln\left(2^r\right)$, giving the answer in the form $a\ln 2$, where $a \in \mathbb{Q}$.

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51*. [Maximum mark: 6] ***[without GDC]***

Find an expression for the sum of the first 35 terms of the series

$$\ln x^2 + \ln\frac{x^2}{y} + \ln\frac{x^2}{y^2} + \ln\frac{x^2}{y^3} + \cdots$$

giving your answer in the form $\ln\dfrac{x^m}{y^n}$, where $m, n \in \mathbb{N}$.

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LOGARITHMS AND FUNCTIONS

52. [Maximum mark: 6] ***[without GDC]***

The function f is defined for $x > 2$ by $f(x) = \ln x + \ln(x-2) - \ln(x^2-4)$.

(a) Express $f(x)$ in the form $\ln\left(\dfrac{x}{x+a}\right)$. [2]

(b) Find an expression for $f^{-1}(x)$. [4]

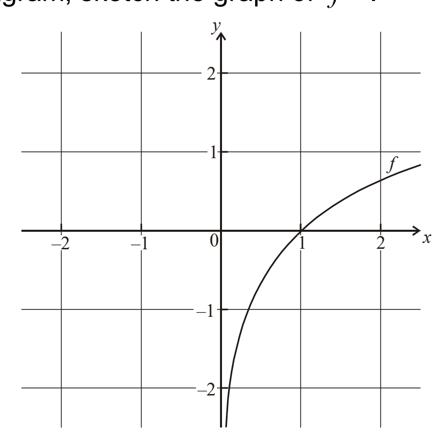
53. [Maximum mark: 6] ***[without GDC]***

Let $f(x) = \log_a x$, $x > 0$.

(a) Write down the value of (i) $f(a)$ (ii) $f(1)$ (iii) $f(a^4)$ [3]

(b) The diagram below shows part of the graph of f.

On the same diagram, sketch the graph of f^{-1}.



[3]

54. [Maximum mark: 7] ***[without GDC]***

Let $f(x) = k\log_2 x$.

(a) Given that $f^{-1}(1) = 8$, find the value of k. [3]

(b) Find $f^{-1}\left(\frac{2}{3}\right)$. [4]

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55*. [Maximum mark: 7] ***[without GDC]***

Let $f(x) = 3\ln x$ and $g(x) = \ln 5x^3$.

(a) Express $g(x)$ in the form $f(x) + \ln a$, where $a \in \mathbb{Z}^+$. [4]

(b) The graph of g is a transformation of the graph of f. Give a full geometric description of this transformation. [3]

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56. [Maximum mark: 7] ***[without GDC]***

Let $f(x)=\log_3\sqrt{x}$, for $x>0$.

(a) Show that $f^{-1}(x)=3^{2x}$. [2]

(b) Write down the range of f^{-1}. [1]

Let $g(x)=\log_3 x$, for $x>0$.

(c) Find the value of $\left(f^{-1}\circ g\right)(2)$, giving your answer as an integer. [4]

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57. [Maximum mark: 6] ***[with GDC]***

Let $f(x)=\ln(x+2)$, $x>-2$ and $g(x)=\mathrm{e}^{(x-4)}$, $x>0$.

(a) Write down the x-intercept of the graph of f. [1]

(b) (i) Write down $f(-1.999)$.

(ii) Write down $g(4)$ [3]

(c) Find the coordinates of the point of intersection of the graphs of f and g. [2]

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B. Exam style questions (LONG)

58. [Maximum mark: 11] ***[without GDC]***

The functions $f(x)$ and $g(x)$ are defined by $f(x)=\mathrm{e}^{x}$ and $g(x)=\ln(1+2x)$

(a) Write down $f^{-1}(x)$. [1]

(b) Find $g^{-1}(x)$. [3]

(c) Find (i) $(g\circ f)(x)$ (ii) $(f\circ g)(x)$ [4]

(d) Find $(f\circ g)^{-1}(x)$ [3]

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59*. [Maximum mark: 16] ***[without GDC]***

Let $f(x)=\log_3 \frac{x}{2}+\log_3 16-\log_3 4$, for $x>0$.

(a) Show that $f(x)=\log_3 2x$. [2]

(b) Find the value of $f(0.5)$ and of $f(4.5)$. [3]

The function f can also be written in the form $f(x)=\frac{\ln ax}{\ln b}$.

(c) (i) Write down the value of a and of b.

(ii) Hence on graph paper, **sketch** the graph of f, for $-5\le x\le 5$, $-5\le y\le 5$, using a scale of 1 cm to 1 unit on each axis.

(iii) Write down the equation of the asymptote. [6]

(d) Write down the value of $f^{-1}(0)$ [1]

The point A lies on the graph of f. At A, $x=4.5$.

(e) On your diagram, sketch the graph of f^{-1}, noting clearly the image of point A. [4]

60.** [Maximum mark: 20]

The first 4 terms of a geometric sequence $\{u_n\}$ are $5,\ 15,\ 45,\ a$

(a) Find the value of a [1]

Consider the new sequence $\{v_n\}$: $\ln 5, \ln 15, \ln 45, \ln a$

(b) Write down the values of the terns $v_1,\ v_2,\ v_3,\ v_4$ correct to 3 s.f. [3]

(c) Find the differences $v_2 - v_1,\ v_3 - v_2,\ v_4 - v_3$ using the values found in (b).
What do you deduce? [3]

(d) Repeat the process (b) to (c) for the new sequence $\{w_n\}$ by using $\log$, the logarithm to the base 10, instead of $\ln$. Do you obtain a similar result? [4]

Consider now a geometric sequence $\{u_n\}$ with first term a and common ratio 3.

(e) Write down the first three terms of the sequence in terms of a. [2]

Define a new sequence $\{v_n\}$ as above (by using $\ln$).

(f) Show, by using its first three terms that $\{v_n\}$ is an arithmetic sequence. State the common difference. [4]

The general term of a geometric sequence is given by $u_n = u_1 r^{n-1}$

(g) State a proposition which can be derived by the process above and use the n^{th} term and the $(n+1)^{\text{th}}$ term to support your statement. [3]

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61. [Maximum mark: 13] ***[without GDC]***

Solve the following equations.

(a) $\log_x 49 = 2$. [3]

(b) $\log_2 8 = x$ [2]

(c) $\log_{25} x = -\frac{1}{2}$ [3]

(d) $\log_2 x + \log_2 (x-7) = 3$ [5]

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