

INTERNATIONAL BACCALAUREATE
Mathematics: analysis and approaches

MAA

EXERCISES [MAA 2.8]

EXPONENTS

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O. Practice questions

1. [Maximum mark: 15] **[without GDC]**

Find the following values in the form of an integer or a fraction $\frac{a}{b}$ of integers.

$3^3 =$	$3^{-2} =$	$3^{-3} =$
$\left(\frac{1}{3}\right)^2 =$	$\left(\frac{2}{3}\right)^2 =$	$\left(\frac{2}{3}\right)^3 =$
$\left(\frac{1}{3}\right)^{-2} =$	$\left(\frac{2}{3}\right)^{-2} =$	$\left(\frac{2}{3}\right)^{-3} =$
$4^{\frac{1}{2}} =$	$4^{\frac{3}{2}} =$	$4^{-\frac{1}{2}} =$
$25^{\frac{1}{2}} =$	$25^{-\frac{1}{2}} =$	$\left(\frac{25}{4}\right)^{\frac{1}{2}} =$

2. [Maximum mark: 6] **[without GDC]**

Find the following values in the form of an integer or a fraction $\frac{a}{b}$ of integers

$3^2 \times 2^3 =$	$3^5 \times 3^{-3} =$	$3^{-5} \times 3^3 =$
$3^0 + 0^3 =$	$\frac{5^4 \times 2^3}{2^2 \times 5^3} =$	$\frac{7^4 \times 7^2}{7 \times 7^7} =$
$3^{-1} \times 2^3 =$	$3 \times 2^{-3} =$	$\frac{2^{-1}}{3^{-1}} =$

3. [Maximum mark: 9] **[without GDC]**

Find the following values in the form of an integer or a fraction $\frac{a}{b}$ of integers

$\frac{30^2}{15^2} =$	$\frac{15^{-2}}{30^{-2}} =$	$\frac{12^{\frac{1}{2}}}{3^{\frac{1}{2}}} =$
$7^{0.3} \times 7^{0.7} =$	$7^{\frac{1}{2}} \times 7^{\frac{3}{2}} =$	$7^{-2} \times 7 =$
$7^2 \times 7^{-1} =$	$\frac{2^3 \times 3^3}{6^3} =$	$\frac{4^3 \times 3^3}{6^3} =$

4. [Maximum mark: 15] **[without GDC]**

Express the following in the form of a single power (x^y)

$a^6 a^2 =$	$a^6 a =$	$(a^{\frac{1}{2}})^6 =$
$(a^6)^2 =$	$(a^2)^6 =$	$\frac{a^6}{a^2} =$
$a^2 a^3 a =$	$\frac{a^2 a^6}{a^5} =$	$\frac{a^2 b^6}{b^2 a^{-3} b^4} =$
$\left(a^{\frac{3}{5}}\right)^5 =$	$\left(a^{\frac{3}{5}}\right)^{10} =$	$(a^{-1})^2 =$
$a^{x+3} a^{1-x} =$	$\frac{a^{n+5}}{a^{n+3}} =$	$\frac{a^{-8}}{a^{-10}} =$

5. [Maximum mark: 8] **[without GDC]**

Given that $A > 0$, $B > 0$, $C > 0$, simplify the expressions

$\frac{A^6 B^3 C^{10}}{C^5 A^2 B} =$
$\frac{A^2 B + AB^3}{AB} =$
$\frac{2A + A(4B) + (2A)^2}{2A} =$
$\frac{A^4 B^3 + A^3 B^4}{A + B} =$

6. [Maximum mark: 5] **[without GDC]**

Consider the following powers of e :

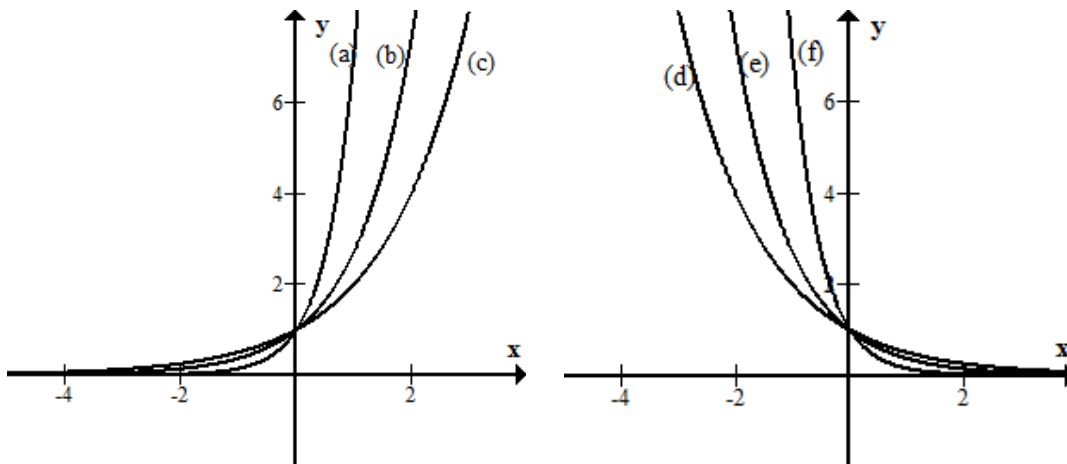
A. e^{-2}	B. $e^{\frac{1}{2}}$	C. e^x	D. e^{-x}	E. $e^{x/2}$	F. e^{2x}	G. e^{x+2}
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Find the equivalent expressions in the table below, by stating the corresponding letter.

$(e^x)^2$	$\frac{1}{e^x}$	$\frac{e^{2x}}{e^x}$	$\frac{1}{e^2}$	\sqrt{e}	$\frac{e^x}{e^{-2}}$	$\sqrt{e^x}$
F						

7. [Maximum mark: 8] **[without GDC]**

(a) The graphs of 6 functions are shown below



Match the graphs (a) (b) (c) (d) (e) and (f) to the following functions

$y = 2^x$	$y = 5^x$	$y = e^x$

$y = 2^{-x}$	$y = 5^{-x}$	$y = e^{-x}$

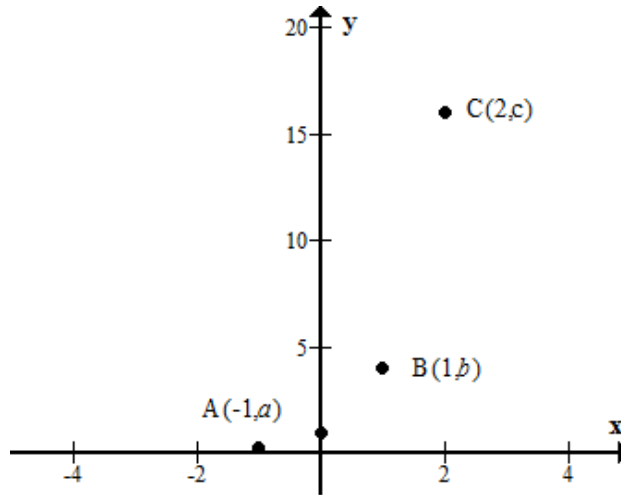
(b) Complete the following table (it contains the common details for all six graphs)

y – intercept	
Horizontal asymptote	
Domain	
Range	

8. [Maximum mark: 10] **[without GDC]**

Let $f(x) = 4^x$.

The points $A(-1,a)$, $B(1,b)$ and $C(2,c)$ of the graph are shown in the diagram below.



- (a) Write down the coordinates of the y -intercept of the graph. [1]
- (b) Write down the value of a in decimal form. [2]
- (c) Write down the values of b and of c . [2]
- (d) Write down the equation of the horizontal asymptote. [1]
- (e) On the diagram above, sketch the graph of f . [2]
- (f) Write down the domain and the range of f . [2]

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9. [Maximum mark: 6] **[without GDC]**

The graph of $f(x) = 2^x + 2$ passes through the points $A(1, a)$, $B(2, b)$ and $C(3, c)$.

- (a) Find the values of a , b and c . [2]
 (b) Sketch the graph of the function. Indicate the y -intercept, the horizontal asymptote and the points A, B, C of the graph. [4]

<p>$a = \dots\dots\dots$</p> <p>$b = \dots\dots\dots$</p> <p>$c = \dots\dots\dots$</p>	
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10. [Maximum mark: 6] **[without GDC]**

The graph of $f(x) = 2^x - 2$ passes through the points $A(1, a)$, $B(2, b)$ and $C(3, c)$.

- (a) Find the values of a , b and c . [2]
 (b) Sketch the graph of the function. Indicate the y -intercept, the horizontal asymptote and the points A, B, C of the graph. [4]

<p>$a = \dots\dots\dots$</p> <p>$b = \dots\dots\dots$</p> <p>$c = \dots\dots\dots$</p>	
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12. [Maximum mark: 15] ***[without GDC]***

Solve the equations

- (a) $25^{x+1} = 5^3$ [3]
- (b) $25^{x+1} = \frac{1}{5^x}$ [3]
- (c) $25^{x+1} = \sqrt{5^x}$ [3]
- (d) $25^{x^2} = 125^x$ [3]
- (e) $7^{x^2-5x} = 1$ [3]

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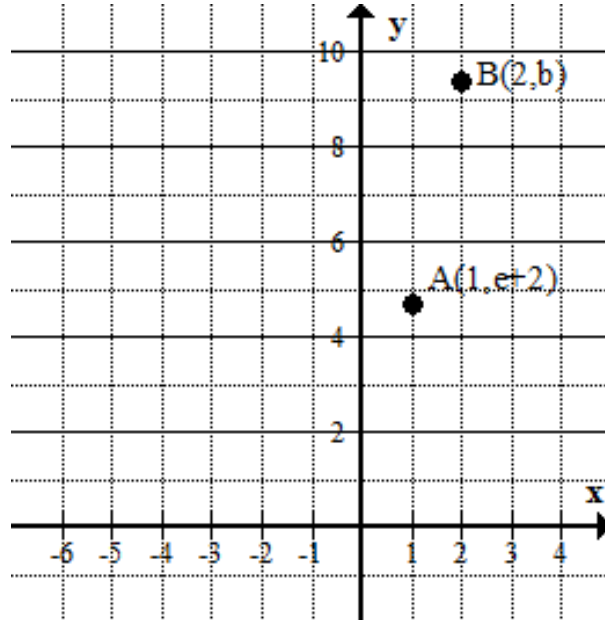
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A. Exam style questions (SHORT)

13. [Maximum mark: 9] **[with GDC]**

Let $f(x) = e^x + 2$. The points $A(1, e+2)$ and $B(2, b)$ of the graph are shown in the diagram below.



- (a) Write down value of b
 - (i) in **exact** form
 - (ii) correct to 3s.f.
- (b) Write down, correct to 3.s.f.,
 - (i) the value of $f(1)$
 - (ii) the value of $f^{-1}(6)$
- (c) On the diagram above, sketch the graph of f . Indicate the y -intercept and the horizontal asymptote of the graph.
- (d) Write down the domain and the range of f .

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14. [Maximum mark: 5] **[without GDC]**

Find the **exact** solution of the equation $9^{2x} = 27^{(1-x)}$.

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15. [Maximum mark: 5] **[without GDC]**

Solve the equation $9^{x-1} = \left(\frac{1}{3}\right)^{2x}$

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16. [Maximum mark: 5] **[without GDC]**

Solve the equation $25^{x^2} = \sqrt{5}$

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17. [Maximum mark: 5] **[without GDC]**

Solve the equation $4^{x^2} = 8^x$.

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18*. [Maximum mark: 5] **[without GDC]**

Solve the equation $5^{x^2-3} = \left(\frac{1}{25}\right)^{2x-1}$.

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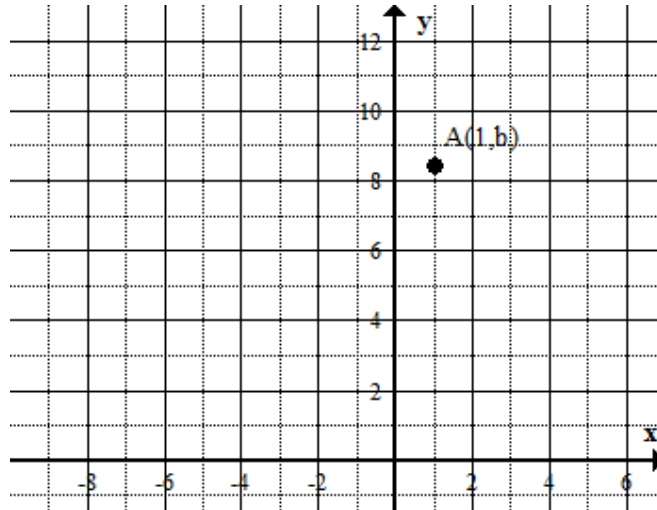
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B. Exam style questions (LONG)

19. [Maximum mark: 12] **[without GDC]**

For each of the following functions sketch the graph and complete the table

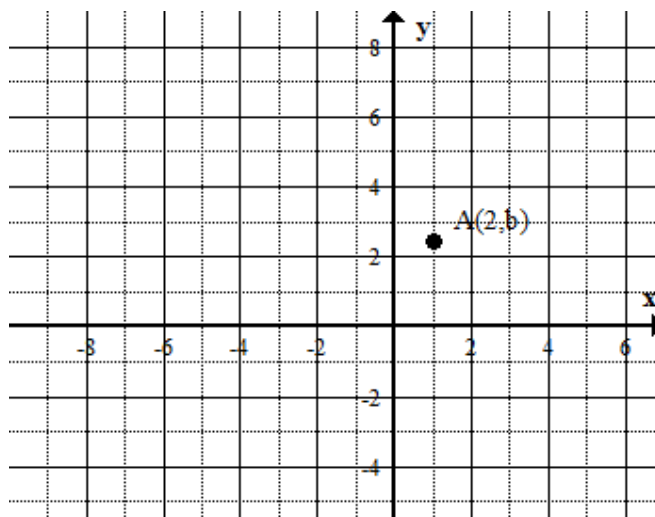
(a) $f(x) = 2e^x + 3$ (the point A lies on the curve).



y - intercept:	
horizontal asymptote:	Exact value of b :
Domain:	Range:

[6]

(b) $f(x) = 2e^x - 3$ (the point A lies on the curve).



y - intercept:	
horizontal asymptote:	Exact value of b :
Domain:	Range:

[6]

20*. [Maximum mark: 12] **[with GDC]**

Consider the function $y = f(x)$ with $f(x) = 10e^{0.3x} + 5$

- (a) Write down the domain of f . [1]
- (b) Find the y -intercept of the graph. [2]
- (c) Find (i) $f(5)$ correct to 3sf. (ii) $f^{-1}(100)$ correct to 3sf. [3]
- (d) Find the first integer value of x for which the value of y will exceed 120. [2]
- (e) Find the value of $f(-20)$ and deduce the equation of the horizontal asymptote of the graph. [2]
- (f) Write down the range of f . [2]

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