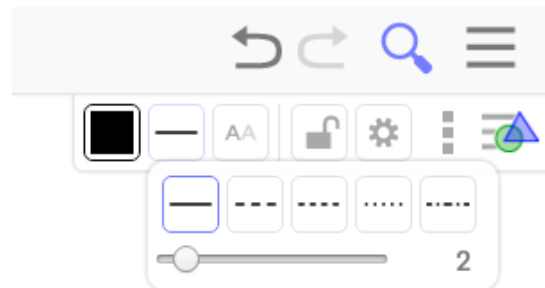


## Altitudes, Angle Bisectors, Perpendicular Bisectors of Triangles GeoGebra Investigation

Go to [www.geogebra.org](http://www.geogebra.org) → Start Creating → Geometry Perspective

- 1) Use the POLYGON tool to construct a triangle. Label its vertices  $A$ ,  $B$ , and  $C$ .
- 2) Use the LINE tool to construct a line that passes through  $A$  and  $B$ . Use this same tool to construct a line that passes through  $B$  and  $C$ . Also, do the same for  $A$  and  $C$ .
- 3) What does it mean for lines to be perpendicular? Explain.
- 4) Construct a line that passes through  $B$  that is perpendicular to  $\overline{AC}$ .
- 5) Construct a point  $D$  (using the INTERSECT tool) at which this line and segment intersect.
- 6) What should the measure of  $\angle BDC$  be? \_\_\_\_\_ Use GeoGebra to now find and display this angle measure.
- 7) Now construct a line through  $A$  that is perpendicular to  $\overline{BC}$ .
- 8) Construct the point of intersection of the *two lines you constructed in steps 4 and 7*. **Label their point of intersection as  $E$ .**
- 9) Now construct a line through  $C$  that is perpendicular to  $\overline{AB}$ .
- 10) Now click on the MOVE tool (arrow). Highlight segments  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$  (the 3 sides of our original  $\triangle ABC$ .) Make these segments appear thicker. (To do this easily, highlight the segment and go to this menu:



- 11) Now select the other three lines on your screen. Go to Display and choose any color that's different from the color of your triangle interior and the color of the sides of the triangle.

- 12) Now drag the vertices of triangle  $ABC$  around. As you change the size and shape of this triangle, study how these 3 lines intersect. *Describe how these three lines meet.* Be specific in your response.
- 11) Is it possible to drag the vertices of the triangle around so point  $E$  lies anywhere **on** the triangle? If it is possible, **describe the nature (type) of the triangle** where this occurs. (*Hint: How would you classify this triangle by its angles?*)
- 12) Is it possible to drag the vertices of the triangle around so point  $E$  lies anywhere **outside** the triangle? If it is possible, **describe the nature (type) of the triangle** where this occurs. (*Hint: How would you classify this triangle by its angles?*)
- 13) Is it possible to drag the vertices of the triangle around so point  $E$  lies anywhere **inside** the triangle? If it is possible, **describe the nature (type) of the triangle** where this occurs. (*Hint: How would you classify this triangle by its angles?*)
- 14) **The intersection of the 3 lines that contain the triangle's altitudes is called the \_\_\_\_\_ of the triangle. This point lies**  
 \_\_\_\_\_ **if the triangle is** \_\_\_\_\_  
 \_\_\_\_\_ **if the triangle is** \_\_\_\_\_  
 \_\_\_\_\_ **if the triangle is** \_\_\_\_\_.

Now put a textbox in your GeoGebra file here and type the bolded sentences above into it. Save this file. Name it **Orthocenter**. Make it **public**.

In a new file of GeoGebra,

- 1) Construct any *segment*  $\overline{AB}$ .
- 2) Suppose we wanted to construct the line that is the **perpendicular bisector** of this segment. What would this line “have to do” in order for us to be able to classify it as this segment’s **perpendicular bisector**? Explain. (*Hint: Think of what these 2 words mean*).
- 3) Use GeoGebra to construct the perpendicular bisector of  $\overline{AB}$ .
- 4) What have we learned about every point on the perpendicular bisector of a segment? Explain briefly and use GeoGebra to quickly verify your assertion.
- 5) **Go to a new GeoGebra “Geometry” file again (no need to save work from 1-4 above)** and construct any triangle. Be sure its vertices are labeled  $A$ ,  $B$ , and  $C$ .
- 6) Construct the *perpendicular bisector* of  $\overline{AC}$ . Then construct the point of intersection of this line with  $\overline{AC}$  and label this point as  $D$ .
- 7) Construct the *perpendicular bisector* of  $\overline{BC}$ . Then construct the point of intersection of this line with  $\overline{BC}$  and label this point as  $E$ .
- 8) So far you’ve constructed 2 perpendicular bisectors of your triangle. *Construct the point at which these 2 perpendicular bisectors intersect* and label this point as  $K$ .
- 9) Now construct the *perpendicular bisector* of  $\overline{AB}$ . Then Construct the point of intersection of this line with  $\overline{AB}$  and label this point as  $F$ .
- 10) As before, select (highlight) segments  $\overline{AB}$ ,  $\overline{BC}$ , and  $\overline{AC}$  and make them **thick** so it’s easier to see the original triangle.
- 11) Now select (highlight) the three perpendicular bisectors of this triangle and change their color different from the colors of the triangle’s interior and sides.
- 12) Now drag the vertices of the triangle below and study the triangle’s 3 perpendicular bisectors carefully. As you change the size and shape of this triangle, study how these 3

perpendicular bisectors intersect. *Describe how these three lines meet.* Be specific in your response.

- 13) Is it possible to drag the vertices of the triangle around so point  $K$  lies anywhere ***outside*** the triangle? If it is possible, ***describe the nature (type) of the triangle*** where this occurs. (*Hint: How would you classify this triangle by its angles?*)
- 14) Is it possible to drag the vertices of the triangle around so point  $K$  lies anywhere ***on*** the triangle? If it is possible, ***describe the nature (type) of the triangle*** where this occurs. (*Hint: How would you classify this triangle by its angles?*)
- 15) Is it possible to drag the vertices of the triangle around so point  $K$  lies anywhere ***inside*** the triangle? If it is possible, ***describe the nature (type) of the triangle*** where this occurs. (*Hint: How would you classify this triangle by its angles?*)
- 16) Use GeoGebra to now measure the lengths  $KA$ ,  $KB$ , and  $KC$ . (To measure  $KA$ , select both points  $K$  and  $A$ . Then choose Measure and then Distance. *Now do this for the other two distances.*) Then drag the vertices of the triangle around again. What interesting phenomenon do you notice? Describe.

17) *Now construct a circle with center  $K$  that passes through point  $A$ . (To do this, select the circle tool and then select point  $K$ . Move away from  $K$  and then select point  $A$ .) Drag the vertices of the triangle around. What interesting phenomenon do you notice about this circle? Be specific in your description. Draw a quick sketch too.*

18) Why should your observation from (16) allow your observation in (17) to occur? Explain.

19) **The intersection of a triangle's 3 \_\_\_\_\_ is called the \_\_\_\_\_ of the triangle. It is only point that is \_\_\_\_\_ from the triangle's 3 \_\_\_\_\_.**  
**Because of this it is also the \_\_\_\_\_ of the \_\_\_\_\_ we can \_\_\_\_\_ about the triangle.**

*Now put a textbox in your GeoGebra file here and type the bolded sentences above into it. Save this file. Name it **Circumcenter**. Make it **public**.*

In a new file GeoGebra,

- 1) Construct any triangle. Label its vertices  $A$ ,  $B$ , and  $C$ .
- 2) We're going to construct the triangle's 3 **angle bisectors** now. Drawn below is a line that serves as an angle bisector of  $\angle TUV$ . Complete the following sentence.

*A line is an angle bisector of an angle if...*

- 3) Highlight the vertices  $A$ ,  $B$ , and  $C$  **in that order**. Then go to Construct and choose Angle Bisector. What angle did you just bisect? (Use 3 letters to name the angle).
- 4) Construct the bisector of  $\angle ACB$  now.
- 5) So far, you've constructed 2 angle bisectors of this triangle. Construct *the point at which these two angle bisectors intersect* and label this point  $I$ .
- 6) Construct the bisector of the remaining angle.
- 7) Drag the vertices of your triangle around. As you change the size and shape of this triangle, study how these 3 angle bisectors intersect. **Describe how these three rays meet. Be specific in your response.**
- 8) Is it possible to drag the vertices of the triangle around so point  $I$  lies anywhere **outside** the triangle? If it is possible, **describe the nature of the triangle** where this occurs.
- 9) Now drag the vertices of your triangle around so that your triangle is large and covers most of your screen.

- 10) Now construct a line that passes through  $I$  and is perpendicular to  $\overline{AB}$ . Construct the point at which this line intersects  $\overline{AB}$ . Label this point  $D$ .
- 11) Now construct a line that passes through  $I$  and is perpendicular to  $\overline{AC}$ . Construct the point at which this line intersects  $\overline{AC}$ . Label this point  $E$ .
- 12) Now construct a line that passes through  $I$  and is perpendicular to  $\overline{BC}$ . Construct the point at which this line intersects  $\overline{BC}$ . Label this point  $F$ .
- 13) Use GeoGebra to measure and display the distances  $ID$ ,  $IE$ , and  $IF$ . Then drag the vertices of this triangle around. **What do you notice about these three lengths?**
- 14) Select the compass (circle) tool. Then construct a circle with center  $I$  that contains point  $D$ .
- 15) Drag the vertices of your triangle around now. What interesting phenomenon do you notice? *Describe this phenomenon as best you can in your own words. Be specific. Include a sketch too.*
- 16) **The intersection of a triangle's 3 \_\_\_\_\_ is called the \_\_\_\_\_ of the triangle. It is the only point inside the triangle that is \_\_\_\_\_ from the triangle's 3 \_\_\_\_\_.**

*Now put a textbox in your GeoGebra file here and type the bolded sentences above into it. Save this file. Name it **Incenter**. Make it **public**.*