PABSON
SEE PRE BOARD EXAM - 2078 (PC231)

Time: 3:00 hrs Optional Mathematics **F.M. :** 100

Candidates are required to write their answers according to the instructions given.
Attempt all questions.

Group 'A' $[5 \times (1+1) = 10]$

1. (a) Under what condition the inverse function of a function is possible?
   (b) If $(x-m)$ is a factor of polynomial $p(x)$, what is the value of $p(m)$?

2. (a) Write down the notational representation of right hand limit.
   (b) Evaluate: $\begin{vmatrix} 1 & -2 \\ 3 & 4 \end{vmatrix}$

3. (a) If the intersection plane is parallel to the axis of cone, then what conic does it form?
   (b) What is the formula to find the angle between the lines represented by $ax^2 + 2hxy + by^2 = 0$?

4. (a) Express: $\sin 2P + \sin 2Q$ into product form.
   (b) If $2\cos A = \sqrt{3}$, what is the value of A for $0° \leq A \leq 90°$?

5. (a) If O be the origin and $\overrightarrow{OA} + \overrightarrow{OB} = 8\vec{i} + 12\vec{j}$, then find the position vector of M which is the mid-point of AB.
   (b) In the adjoining figure, P' is the inverse of P. If OP × OP' = 25 cm^2 then find the radius of the inversion circle.

Group 'B' $[13 \times 2 = 26]$

6. (a) If the polynomial $x^3 + 6x^2 + kx + 10$ is divided by $(x+2)$, the remainder is 4, find the value of $'k'$ using the remainder theorem.
   (b) State factor theorem. By using factor theorem, show that $(x-1)$ is a factor of $3x^3 + 2x - 5$.
   (c) Write the standard equation of a parabola and find the vertex of $y = x^2 - 4x + 5$.

7. (a) If matrix $A \begin{pmatrix} 4 & -7 \\ -1 & 2 \end{pmatrix}$, find the inverse matrix A^{-1} of A.
   (b) In the equations $2x + 3y = 12$ and $x + y = 5$, the value of $D = -1$, find the value of 'x' using Cramer's rule.

8. (a) Find the slopes of two straight lines $3x + 4y + 5 = 0$ and $6x + 8y + 7 = 0$ and write the relationship between them.
   (b) Find the single equation for the pair of lines represented by $3x + 2y = 0$ and $2x - 3y = 0$.

9. (a) If $\sin\frac{\theta}{3} = \frac{3}{5}$, find the value of $\sin\theta$.
   (b) Prove that: $\frac{\sin 80° + \sin 10°}{\cos 10° - \cos 80°} = \cot 35°$
   (c) Solve: $4 - 3\sec^2\theta = 0$ $[0° \leq \theta \leq 90°]$

10. (a) Find the angle between two vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a}| = 2, |\vec{b}| = 6$ and $\vec{a}.\vec{b} = 6$.
    (b) The position vectors of M and N are $5\vec{i} + 3\vec{j}$ and $2\vec{i} + 3\vec{j}$ respectively. Find the position vector of the point P which divides MN in the ratio 2:1.
    (c) The first quartile of a data is 70 and third quartile is 25 more than the double of the first quartile. Find the coefficient of the quartile deviation of the Data..

Group 'C' $[11 \times 4 = 44]$

11. If $f(x) = 2x + 5, g(x) = \frac{3x-1}{2}$ and $fog(x) = f(x)$ then find the value of x.

12. Find the maximum value of $P = 3x + 2y$ under the constraints:

$$2y \geq x - 1, x + y \leq 4, x \geq 0, y \geq 0$$

13. Examine the continuity or discontinuity of

$$f(x) = \begin{cases} 4x - 1 & \text{for } x < 1 \\ 7x - 4 & \text{for } x \geq 1 \end{cases} \text{ at } x = 1$$

by calculating left hand limit, right hand limit and functional value.

14. Solve by matrix method:

$$\frac{2}{x} + \frac{5}{y} = 1 \text{ and } \frac{3}{x} + \frac{2}{y} = \frac{19}{20}$$

15. Find the equation of the perpendicular bisector of the line segment joining the points $(4, -5)$ and $(-8, 9)$.

16. If $A + B + B = \pi^\circ$, Prove that:

$$\sin^2 \frac{A}{2} + \sin^2 \frac{B}{2} - \sin^2 \frac{C}{2} = 1 - 2\cos \frac{A}{2} \cos \frac{B}{2} \sin \frac{C}{2}$$

17. Solve: $[0^\circ \leq \theta \leq 360^\circ]$

$$\sqrt{3} \sin \theta + \cos \theta = \sqrt{2}$$

18. The angles of elevation of the top of a tower observed from two points on the same plane are found to be complementary. If the points are at a distance of 20m and 45m from foot of the tower, find the height of the tower.

19. ΔPQR having the vertices $P(3, 4), Q(2, 1)$ and $R(4, 2)$ is translated by $T = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$. The image so formed is enlarged by $E[(0, 0), 2]$. Writing the co-ordinates of the vertices of images thus obtained, represent ΔPQR and its images in the same graph paper.

20. Find the mean deviation from mean and its coefficient from given data.

Marks	0 - 10	10-20	20-30	30-40	40-50
Frequency	5	8	15	10	6

21. Find the standard deviation from given data:

Marks	10-20	10-30	10-40	10-50	10-60
Frequency	8	20	35	44	50

Group 'D' $[4 \times 5 = 20]$

22. A person pays a loan of Rs 45750 in monthly installments, each installment being more than the former by Rs 50. The amount of first installment is Rs 800. In how many installments will the entire amount be paid ? Give reason.

23. Find the equation of the circle which passes through the point $(1, 4)$ and whose equations of two diameters are $x - y = 1$ and $2x + 3y = 7$.

24. In a triangle PQR, $\angle$QPR $= 90^\circ$ and S is a mid-point of QR, prove by the vector method that: SP = QS = SR

25. A square $ABCD$ with the vertices $A(0, 3), B(1, 1), C(3, 2)$ and $D(2, 4)$ is transformed into the parallelogram $A'B'C'D'$ by 2×2 matrix so that the vertices of the parallelogram are $A'(3, 0), B'(1, -1), C'(2, 3)$ and $D'(4, -2)$. Find the 2×2 matrix. Which transformation does the matrix denote?

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