Reciprocal Quadratic Functions

Main function: $f(x) = ax^2 + bx + c$
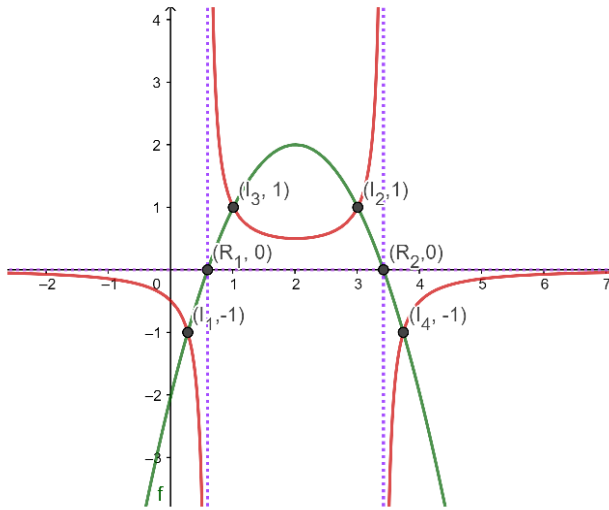
Reciprocal function:

$$\frac{1}{f(x)} = \frac{1}{ax^2 + bx + c}$$

In this example, $a = -1$, $b = 4$, $c = -2$

$$f(x) = -x^2 + 4x - 2$$

$$\frac{1}{f(x)} = \frac{1}{-x^2 + 4x - 2}$$

Roots: $R_1, R_2 = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \dfrac{-4 \pm \sqrt{(4)^2 - 4(-1)(-2)}}{2(-1)} = 2 \pm \sqrt{2}$

Vertical asymptotes: $\qquad x = R_1, \; R_2 = 2 \pm \sqrt{2}$

Horizontal asymptote: $\qquad f(x) = 0$

Intercepts: $\qquad (I_{1-4}, \pm 1), \; I_{1-4} = \dfrac{-b \pm \sqrt{b^2 - 4a(c \pm 1)}}{2a}$

$I_1 = \dfrac{-4 + \sqrt{(4)^2 - 4(-1)(-2+1)}}{2(-1)} = \dfrac{-4 + \sqrt{12}}{-2}$ $\qquad (I_1, -1) = (2 - \sqrt{3}, -1)$

$I_2 = \dfrac{-4 - \sqrt{(4)^2 - 4(-1)(-2-1)}}{2(-1)} = \dfrac{-4 - \sqrt{4}}{-2}$ $\qquad (I_2, 1) = (3, 1)$

$I_3 = \dfrac{-4 + \sqrt{(4)^2 - 4(-1)(-2-1)}}{2(-1)} = \dfrac{-4 + \sqrt{4}}{-2}$ $\qquad (I_3, 1) = (1, 1)$

$I_4 = \dfrac{-4 - \sqrt{(4)^2 - 4(-1)(-2-1)}}{2(-1)} = \dfrac{-4 - \sqrt{12}}{-2}$ $\qquad (I_4, -1) = (2 + \sqrt{3}, -1)$